Name: _____ _____

Midterm 1 Quiz (61 points)

Time Limit[1]: 1 Hour.

You must show all of your work!

1. Bo and Samuel own Neverland Water Park, a highly successful theme park with aquatic rides. To better understand how well their business is doing, they hire Jesse to do some market research. Jesse determines that the number of people, N (in hundreds), who come to the park on a given day is a continuous function of the temperature, T (in ^oF) that day. Below is some of the date Jesse gathered. [13 points]

T	60	65	70	75	80	85	90	95
N	1	15	25	33	38	42	44	41

A) What are the units of $\frac{dN}{dT}$? [2 points]

B) Estimate $\frac{dN}{dT}$ when $T = 70$, $T = 75$, and $T = 90$. [3 points]

C) Using your answer from A), how many people do you expect will come to the park on a day where the temperature is 77^oF? [3 points]

D) Using your answer from A), estimate $\frac{d^2N}{dT^2}$ when $T = 70$. [2 points]

E) Is N an invertible function of T? Justify your answer. [3 points]

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[1] This is how long I think you should spend on this to be 'on pace' to finish a full midterm in 90 minutes.

2. True or False (circle[2] **TRUE** if the statement is ALWAYS TRUE and circle **FALSE** otherwise). [10 points; 1 point each]

TRUE **FALSE** If $f(x)$ is a cubic polynomial, then $f(x)$ has 3 roots.

TRUE **FALSE** For any A and B greater than 0, $log(A+B) = log(A) + log(B)$.

TRUE **FALSE** The function $g(y) = tan(y)$ is monotone.

TRUE **FALSE** Every invertible function is monotone.

TRUE **FALSE** If $a(2) = 2$ and $a(10) = -6$, then a has a root between 2 and 10.

TRUE **FALSE** If $p(q)$ is differentiable for $q = 7$, then $p(q)$ is continuous at $q = 7$.

TRUE **FALSE** The sum of two odd functions is an odd function.

TRUE **FALSE** If $l'(j)$ is concave down, then $l''(j) < 0$.

TRUE **FALSE** $3e^x$ can be obtained from e^x by a horizontal shift.

TRUE **FALSE** $\displaystyle\lim_{x \to 0} \frac{|x|}{x} = 1$

3. Fun with graphs! [13 points]

A) On the axes below, sketch the graph of a function $f(x)$, that satisfies ALL of the following properties. Be sure to label your axes. [7 points]

$f(x)$ is continuous everywhere except $x = 5$

$f(x)$ is increasing and concave down on $(-\infty, 0)$

$f(0) = 2$

$f''(x) > 0$ on $(0, 5)$

$f(x)$ has a vertical asymptote at $x = 5$

$\displaystyle\lim_{x \to \infty} f(x) = 2$

$f'(x) < 0$ for $x > 5$
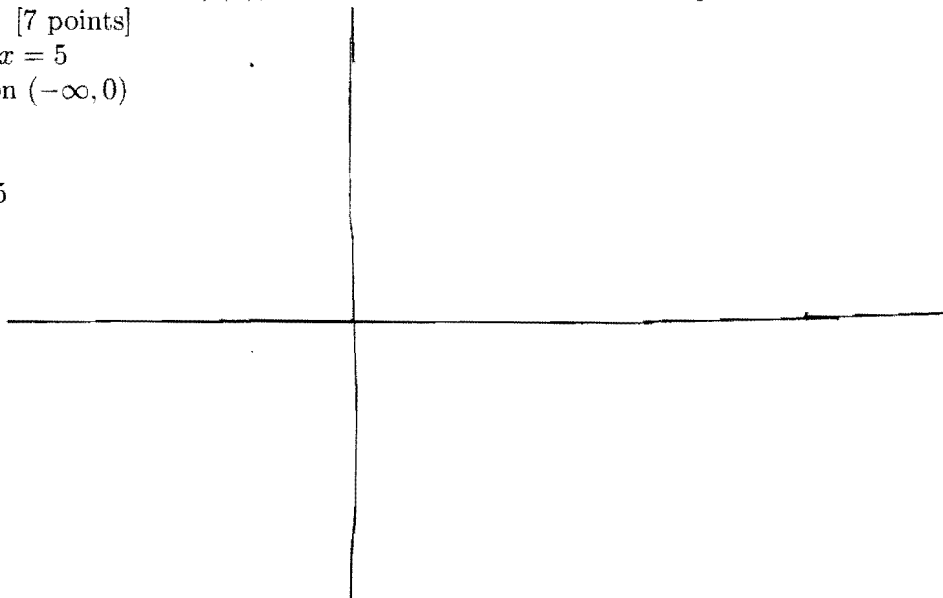
B) Using the graph of $g(x)$ below, compute the following limits. Write DNE if a limit does not exist. [6 points; 1 point each]

$\displaystyle\lim_{x \to -\infty} g(x) =$

$\displaystyle\lim_{x \to -2} g(x) =$

$\displaystyle\lim_{x \to 0} g(x) =$

$\displaystyle\lim_{x \to 3} g(x) =$

$\displaystyle\lim_{x \to 5^-} g(x) =$

$\displaystyle\lim_{x \to \infty} g(x) =$

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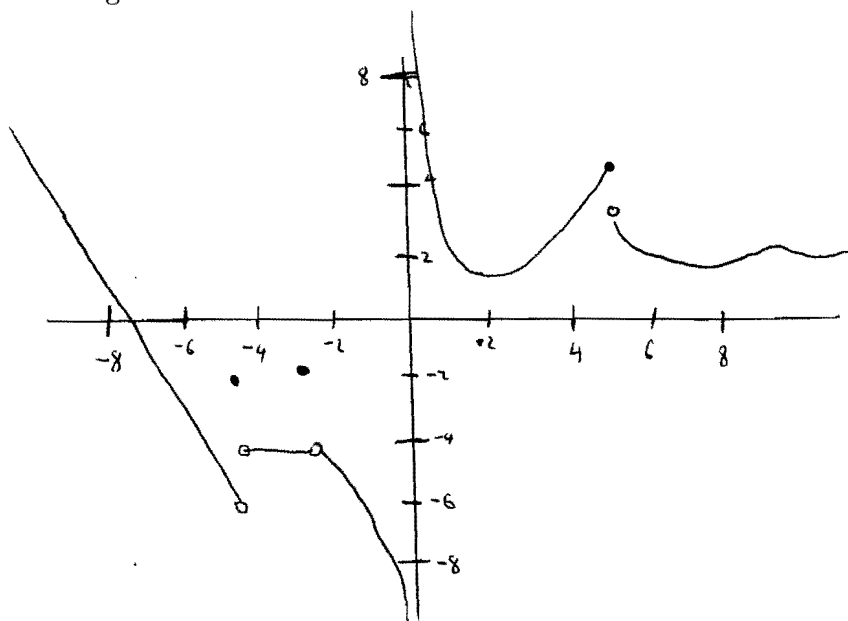
[2]For this just circle the correct answer. But for your own benefit, be able to justify your answers fully. For statements that are false, can you change the statement slightly to make it true?

4. The Detroit Tigers are hosting the New York Yankees in a baseball game Tuesday[3] night in Comerica Park. The game will start at 8pm. Let $N(h)$ be the number of fans, in thousands, in Comerica Park h hours before the start of the game. Assume $N(h)$ is invertible. [13 points]

A) Interpret $N(3) = 5$. [2 pt]

B) What is the sign of $N'(1)$? Justify your answer. [3 pts]

C) Interpret $N^{-1}(10) = 1$. [2 pts]

D) In addition to the fans coming to the stadium, there will be 5,219 players, coaches, security officers, and media writers are also in the stadium. In terms of N, write a function $P(m)$ which gives the number of total number of people in the stadium m minutes after noon on Sunday. [4 pts]

E) Suppose $P'(411) = 253$, where P is as in part D. Interpret this in practical terms. [2 pts]

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[3]Yesterday? Two days ago? I wrote this question on Monday.

5 ❧) A circular dartboard of diameter 20 inches is on a wall, with the bull's-eye (center) exactly 5 feet above the floor. Lucy the Ladybug and an Albert the Ant are sitting at the very top of the dartboard having a fascinating conversation about pesticide . Suddenly, Paul the Person decides to spin the dartboard (clockwise) at a rate of 10 roations per minutes (RPM's). [12 points]

A) How fast is the dartboard spinning, in radians per second? How long, in seconds, is one period of rotation? [2 points]

B) Because ladybugs have suction-like grip on their feet, Lucy does not fall off the dartboard. Instead she spins around with the edge of the dartboard, going in circles. Find a formula for $L(t)$ which gives Lucy's height, in inches, off the ground t seconds after Paul starts spinning the dartboard. [4 pts]

C) Paul is exactly 5'8" tall. When is the first time after Paul spins the dartboard that Lucy will be exactly even with the top of Paul's head? [3 points]

D) Plot $L(t)$ on the axes below. [3 points]
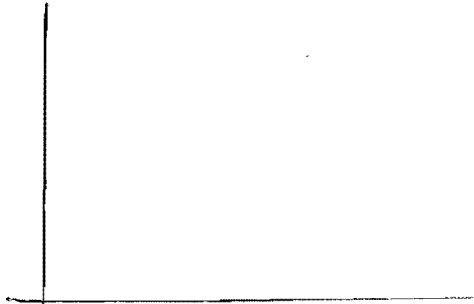
BONUS) Albert also has very good grip in his feet. However, instead of sitting in one place and riding the edge of the dartboard, Albert starts walking toward the bull's-eye at a rate of 1 inch per second. Once he gets to the center, he sits there. Find a formula for $A(t)$ which gives Albert's height, in inches, off the ground t seconds after Paul starts spinning the dartboard. Draw a sketch of the graph of $A(t)$. [4 points]