PS 787 Assignment 3 (Nov 25, 2008)

This describes the first part of an assignment that explores different ways of making inferences about parameters in the data and GEV models that were the focus of Assignment 2. This part concerns profile likelihood. A preliminary due date is December 8. The second part, to be distributed in a day or two, will focus on bootstrap.

Profile likelihood can produce confidence interval (CI) estimates substantially different from what Wald intervals would suggest (what I'm describing as the $100(1-\alpha)\%$ two-sided "Wald" interval for an estimator $\hat{\theta}$ of a scalar parameter θ is $\hat{\theta} \pm z_{1-\alpha/2}\mathrm{SE}(\hat{\theta})$). For instance, the profile likelihood CI may not be symmetric around $\hat{\theta}$. A two-dimensional Wald CI is elliptical by construction, but a profile likelihood CI for the same pair of parameters may have a very different shape.

The assignment is to compare Wald and profile likelihood CIs for some of the parameters in one of the GEV specifications. In particular, consider the GEV model based on the function

$$G_3 = \left(v_T^{1/(1-\sigma)} + v_O^{1/(1-\sigma)}\right)^{1-\sigma} + \left(v_R^{1/(1-\tau)} + v_D^{1/(1-\tau)}\right)^{1-\tau}$$

where as before we define

$$\begin{aligned} Z_T &= X_1 b_1 \qquad \text{(don't vote)} \\ Z_O &= X_2 b_2 \qquad \text{(rolloff)} \\ Z_R &= X_3 b_3 \qquad \text{(Republican vote)} \\ Z_D &= -X_3 b_3 \qquad \text{(Democratic vote)} \end{aligned}$$

with $v_T = \exp(Z_T)$, $v_O = \exp(Z_O)$, $v_R = \exp(Z_R)$ and $v_D = \exp(Z_D)$.

Compare 95% Wald and profile likelihood intervals for five sets of parameters:

1. a lower-tailed interval for σ (pertinent for testing whether $\sigma = 0$)
2. a two-sided interval for one of the coefficients in the vector b_3
3. a two-sided interval for the difference between the coefficients for the same variable in b_1 and b_2
4. a two-sided interval for the difference between the coefficients for the same variable in b_1 and b_3
5. (extra credit) a two-dimensional 95% confidence region for the joint distribution of $\hat{\sigma}$ and $\hat{\theta}$

Apparently there are packages and routines for various programming environments that can facilitate computations. I haven't used any of them. For **R**, try Googling "profile likelihood r."

The basic profile likelihood method for standard ML estimation of a regular parametric model is motivated directly from normal asymptotic theory, which I guess explains why there's nothing recent that talks about it at great length. Here's some suggested reading. A (terse) source I like is

O. E. Barndorff-Nielsen and D. R. Cox. 1994. *Inference and Asymptotics*. London: Chapman and Hall.

The theory appears to be explained in the following article, judging from the abstract, but no electronic version is available online so I am unable to check it (I can't recall whether I've ever read this article; I'll trek to the library to look it up by sometime next week).

W. M. Patefield. 1977. On the Maximized Likelihood Function. *Sankhya* Series B, 39 (1): 92–96.

A method that may simplify computing is described in the following article. I don't know whether any of the available profiling packages or functions use this method.

D. J. Venzon and S. H. Moolgavkar. 1988. A Method for Computing Profile-Likelihood-Based Confidence Intervals. *Applied Statistics* 37 (1): 87–94.

The following article briefly mentions the basic profile likelihood method and contrasts it to the Wald method (described as the "tangent method" in the article). The article then goes on to discuss some closely related methods in some detail.

Jian-Shen Chen and Robert I. Jennrich. 1996. The Signed Root Deviance Profile and Confidence Intervals in Maximum Likelihood Analysis. *Journal of the American Statistical Association* 91 (Sep.): 993–998.

The following article (with discussion) talks about profile likelihood with a class of semiparametric models (slightly beyond the scope of what we're dealing with).

S. A. Murphy and A. W. Van Der Vaart. 2000. On profile likelihood. *Journal of the American Statistical Association* 95 (June): 449–485.

The following piece you may have trouble reading, but it relates both to profile likelihood and the "Bartlett correction" piece on the syllabus. It has to do with higher-order asymptotics.

O. E. Barndorff-Nielsen. 1986. Inference on Full or Partial Parameters Based on the Standardized Signed Log Likelihood Ratio. *Biometrika* 73 (Aug.): 307–322.