Election Fraud or Strategic Voting? Can Second-digit Tests Tell the Difference?*

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Abstract

I simulate a mixture process that generates individual preferences that, when aggregated into precincts, have counts whose second significant digits approximately satisfy Benford's Law. By deriving sincere, strategic, gerrymandered and coerced votes from these preferences under a plurality voting rule, I find that tests based on the second digits of the precinct counts are sensitive to differences in how the counts are derived. The tests can sometimes distinguish coercion from strategic voting and gerrymanders. The tests may be able to distinguish strategic voting according to a party balancing logic from strategic voting due purely to wasted-vote logic, and strategic from nonstrategic voting. These simulation findings are supported by data from federal and state elections in the United States during the 1980s and 2000s.

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Introduction

Voting is complicated, and diagnosing whether something is wrong with the vote count in an election should take the complications into account. Among the primary complications any diagnostic scheme needs to acknowledge are strategic voting and gerrymandering. Strategic voting refers to the fact that when voters take the preferences, beliefs and likely behavior of other voters into account, many may cast votes that differ from what they would do if they acted based solely on their own preferences. Gerrymandering refers to the fact that often in drawing legislative districts imbalances are created so that one party has a systematic advantage. The term "gerrymandering" usually suggests intentional manipulation (Cox and Katz 2002), but imbalances may be created inadvertently, perhaps reflecting transient opinions rather than longstanding partisan divisions.

Under an assumption that most voters behave rationally, theory has been developed to describe the consequences of strategic behavior in many circumstances. The literature bearing on this topic is evidently too vast to be summarized here, but Cox (1994, 1996) discusses one of the ideas and demonstrates the existence of one of the phenomena of primary immediate interest. According to so-called "wasted-vote logic," some voters decide to vote not for their most preferred choice but instead for a lower ranked alternative in order to try to defeat an even lower ranked alternative that they believe is attracting more votes than their first choice is attracting. Cox (1994) developed this idea in connection with his $M + 1$ rule: if there is a single nontransferable vote (SNTV) system for M offices, then Duvergerian equilibria may exist in which no more than $M + 1$ candidates receive a positive proportion of the votes.

Wasted-vote logic can produce results that are surprising if one knows about voters' preferences but not about their beliefs or strategies. Some candidates may receive many more votes than preferences alone would indicate, while others surprisingly receive very small or even negligible shares of the vote. Allegations that there are irregularities in vote counts may seem plausible in such circumstances if the possibility that there was strategic voting is ignored.

Another kind of strategic behavior of interest that has been demonstrated to occur in American elections concerns split-ticket voting: some voters vote for a candidate for the U.S. House election in response to the outcome they expect in the presidential election; indeed, for many voters votes for president and for the House are connected in a large-scale equilibrium relationship (Fiorina 1992; Alesina and Rosenthal 1995; Mebane 2000; Mebane and Sekhon 2002).

Here I'm concerned with tests that purport to diagnose election irregularities in the absence of information even about preferences. Whether such diagnosis is possible at all is of course a question, but some claim that some preference-free diagnostic methods can detect problems (Pericchi and Torres 2004; Mebane 2006, 2008; Mebane and Kalinin 2009; Mebane 2010). The referent tests don't use any information about preferences, but instead look at patterns in the second significant digits of precinct vote counts. If the distribution of those digits differs significantly from the one implied by Benford's Law, then supposedly there is something wrong with the election; at least, investigation using much richer kinds of information is warranted. The issue here is whether this kind of test can distinguish irregularities from strategic voting and from gerrymandering. To put it a little more sharply, can the tests distinguish election fraud from normal politics?

Even though the tests proceed without having any information about preferences at all, a conceptual challenge expressed in terms of preferences may help to frame the issue in a clear way. Imagine two different scenarios for election day. In one, a voter arrives at the polls to find there a big man with a gun, who tells the voter the voter must vote the way he says or else he will return after the election and kill the voter's family and burn down the voter's village. Since the voter surmises that every other voter at that polling place is being similarly threatened, the voter complies and votes as instructed, different from the way the voter originally planned to vote. In the other scenario, there is no man with a gun, but while traveling to vote the voter hears a credible news report stating that preelection surveys suggest the election is very close between the top two parties, with the voter's most preferred party coming in a distant third. The voter decides to abandon his most preferred party and instead vote for the one of the top two parties that he likes the most.

In both scenarios, the voter's choice is determined not by the voter's own preferences but by someone else's preferences. One might argue that having one's vote determined by someone else is the core element of election fraud (cf. Lehoucq 2003). In the first scenario, the preferences represented by the man with a gun rule. No matter what the voter may think about the election, electorally irrelevant considerations such as not wanting his family murdered override what the voter was otherwise planning to do. In the second scenario, there is no coercion, but the voter responds to other voters' preferences and changes his vote. The voter's electorally relevant preferences play a role—citing Cox's theory we may assume the voter does not vote for his least preferred party—but still his choice depends on someone else's desires. But only the first scenario represents fraud.

Likewise different groupings of voters into constituencies—different gerrymanders—can produce different election outcomes even if individual voters' choices don't change under different ways of drawing district lines. But as the number of votes parties receive change we should expect the pattern of digits in the vote counts to change as well. The manipulations that produce such changes are also not fraud.

Can tests based on the second significant digits of vote counts distinguish the man with a gun from normal strategic voting and from routine gerrymanders? This paper takes up this question. For motivation there is the general conceptual puzzle just considered, but there is also a specific empirical challenge. Mebane (2008) concluded that "as measured by the [second-digit Benford's Law (2BL)] test, signs of election fraud in recent American presidential votes seem to be rare." As I will demonstrate below, this impression appears to be erroneous. A different form of test than was used in Mebane (2008) shows extensive and significant departures from the second-digit Benford's Law pattern in American elections during both the 1980s and the 2000s. The departures affect not only votes recorded for president but for other federal offices such as the U.S. House of Representatives. Election returns for state-level offices, such as votes for state legislative seats, similarly fail to follow the basic 2BL distribution. The patterns of departure from 2BL are similar across all these offices.

Since widespread fraud reaching across thousands of election contests and over several decades in the United States is not a likely possibility, I investigate whether another explanation holds, particularly the effect that strategic voting and gerrymandering have on 2BL tests. The answer is that it does.

After reviewing some basic definitions for 2BL test statistics, I start with two examples taken from American election data that help motivate the current analysis. Then I present a set of Monte Carlo simulation studies that illustrate the different effects strategic voting, gerrymandering and coercion have on the distribution of second digits in vote counts. Then I examine data from the aforementioned elections.

2BL Test Statistics

Benford's Law describes a distribution of digits in numbers that arises under a wide variety of conditions. Statistical distributions with long tails (like the log-normal) or that arise as mixtures of distributions have values with digits that often satisfy Benford's Law (Hill 1995; Janvresse and de la Rue 2004). Under Benford's Law, the relative frequency of each second significant digit $j = 0, 1, 2, \ldots, 9$ in a set of numbers is given by $r_j = \sum_{k=1}^{9} \log_{10}(1 + (10k + j)^{-1})$ or $(r_0, \ldots, r_9) = (.120, .114, .109, .104, .100, .097, .093, .090, .088, .085)$. Benford's Law has been used to look for fraud in finance data (Cho and Gaines 2007).

In general the digits in vote counts do not follow Benford's Law, but several examinations have found Benford's Law often approximately describes vote counts' second digits (e.g. Mebane 2006). It is best to think of vote counts as following not Benford's Law but rather distributions in families of Benford-like distributions. Vote counts are mixtures of several distinct kinds of processes: some that determine the number of eligible voters in each precinct; some for how many eligible voters actually vote; some for which candidate each voter chooses; some for how the voter's choice is recorded. Such mixtures can produce numbers that follow Benford-like distributions but not Benford's Law (Rodriguez 2004; Grendar, Judge, and Schechter 2007). While in previous work the following tests have been described as second-digit Benford's Law (2BL) tests, it may be more precise to refer to second-digit Benford-like tests.

Tests for the second digits of vote counts come in two forms. One uses a Pearson chi-squared statistic and is tied to Benford's Law: $X^2_{2BL} = \sum_{j=0}^{9}(n_j - Nr_j)^2/(Nr_j)$, where N is the number of vote counts of 10 or greater (so there is a second digit), n_j is the number having second digit j and r_j is given by the Benford's Law formula. If the counts whose digits are being tested are statistically independent, then this statistic should be compared to the chi-squared distribution with nine degrees of freedom.

The second statistic, inspired by Grendar et al. (2007), is the mean of the second digits, denoted $\hat{j}$. If the counts' second-digits follow Benford's Law, then the value expected for the second-digit mean is $\bar{j} = \sum_{j=0}^{9} jr_j = 4.187$.

American Election Examples

To illustrate the second-digit phenomena of interest, I consider precinct data from the presidential election of 2008 and the U.S. House elections of 1984.[1] For 2008 there are data for 41 states, and for 1984 the data include every state except California. Data are not available for every precinct in some states.

Consider the displays based on votes recorded for president and based on votes recorded for House elections in 1984, shown respectively in Figures 1 and 2. $\hat{j}$ is shown separately in four categories. Clockwise from the upper left in the display these are means for the

[1]Data from 2008 were collected by the author. 1984 data come from the Record of American Democracy (ROAD) (King, Palmquist, Adams, Altman, Benoit, Gay, Lewis, Mayer, and Reinhardt 1997) and from Office of the Clerk (2010).

Republican candidate in states where the Republican won, for the Republican candidate in states where the Democrat won, for the Democratic candidate in states where the Democrat won and for the Democratic candidate in states where the Republican won. In the display for the presidential election, states are placed along the x-axis at locations corresponding to the absolute margin between the Democratic and Republican candidates in each state.[2] Each plot shows a nonparametric regression curve (Bowman and Azzalini 1997) that indicates how the mean of the second digit of the vote counts for the candidate in each category varies with the state absolute margin. Use $\hat{\jmath}_x$ to denote this conditional mean. $\hat{\jmath}_x$ is shown surrounded by 95 percent confidence bounds. In the display for the legislative election the x-axis contains the absolute margin in each legislative district.[3] The question in all the plots is whether $\bar{\jmath}$, indicated by a horizontal dotted line in the plots, falls outside of the confidence bounds. In such cases I say $\hat{\jmath}_x$ differs significantly from $\bar{\jmath}$.

*** Figures 1 and 2 about here ***

If the second digits followed the pattern expected according to Benford's Law, then $\hat{\jmath}_x$ would not differ significantly from $\bar{\jmath}$, but evidently in Figure 1 it does differ in all states for the Democrat's votes where the Democrat won. The difference between $\hat{\jmath}_x$ and $\bar{\jmath}$ does not result simply from the fact that the Democrat got more votes in those places, because $\hat{\jmath}_x$ mostly does not differ significantly from $\bar{\jmath}$ for the Republican's votes in places where the Republican won. $\hat{\jmath}_x$ is about 4.27 for most of the distribution for the Democrat's votes where the Democrat won.

The second digits of 1984 U.S. House election vote counts also do not follow the pattern expected according to Benford's Law. In Figure 2, $\hat{\jmath}_x > \bar{\jmath}$ significantly over most of the distribution for Republican winners and over all of the distribution for Democratic winners. For losers of both parties $\hat{\jmath}_x > \bar{\jmath}$ significantly in close races but $\hat{\jmath}_x < \bar{\jmath}$ significantly in many races that are not so close. $\hat{\jmath}_x$ ranges from a high of about 4.4 for some winners to a low of

[2]In presidential races the absolute margin is the absolute difference between state vote proportions.

[3]In legislative races the absolute margin is the difference between shares of the district two-party vote.

about 4.0 for some losers, with both highs and lows significantly different from $\bar{j}$.

Similar patterns occur for many other American elections, as I'll illustrate further below. Mebane (2008) noted a few departures from Benford's Law expectations using X^2_{2BL}, but as illustrated here much more extensive discrepancies become apparent when $\hat{j}_x$ is computed. What's going on?

I will show that for the most part these deviations from Benford's Law expectations are produced in presidential elections by strategic voting and in U.S. House elections by gerrymandering and strategic voting. In many cases coercion would produce different patterns than we see, so to some extent that can be ruled out as systematically affecting these elections. To get evidence regarding coercion from the digit tests it is necessary both to define an appropriate covariate and to define the covariate's likely effects.

Simulating Strategic Voting, Gerrymandering and Coercion

I simulate a simple plurality election based on artificial preferences generated so that in the case of a preferentially balanced electorate nonstrategic votes approximately satisfy 2BL. For realism, to match in particular the findings of Mebane (2006), the first significant digits of the artificial votes do not satisfy Benford's Law. Then I simulate the effects of three kinds of manipulation: strategic voting according to wasted vote logic, where voters who most prefer a losing candidate switch their votes to one of the top two finishers; coercion, where some voters vote for a candidate regardless of their preferences; and gerrymandering, where the balance of support is skewed between two leading candidates. The idea is to presume a baseline 2BL distribution, as that is often observed, and then to see what effect the manipulations have on the simulated precinct vote counts' second digits. The simulation is constructed as a Monte Carlo exercise, so results reflect the average from hypothetically rerunning the election under the same conditions many times.[4] In real data such repetitions do not occur, of course, but often the repeated sampling methodology is

[4]All simulation conditions are replicated 500 times.

invoked to support studying observed statistics. We will see that the effects produced in simulation often appear in real data.

I simulate and then count votes by individuals in a set of 5,000 simulated precincts. Mebane (2006) and Mebane (2007) simulate precinct data that satisfy 2BL, and the approach taken here is prompted by ideas used in those simulations.

There are three simulations that represent variations of the same basic method. In the first the idea is to simulate precincts that contain individuals who have preferences for each of four candidates, preferences generated from a set of mixture distributions, where three of the candidates are on the ballot. It may help to think of precincts as having different concentrations of more or less intense partisans, even though of course there is no real political content to the numbers used in the simulation. In the second and third simulations there are respectively two and four candidates. Preferences are skewed in these simulations in a manner intended to represent gerrymandering. Only one election is simulated at a time, so these simulations do not represent all features of gerrymander. Indeed, they represent any factor that produces systematic deviations from an electoral situtation that is balanced between two candidates.

Each precinct has a basic offset selected using a uniform distribution on the interval $[-2-\nu_s, 2-\nu_s]$: $\mu \sim U(-2-\nu_s, 2-\nu_s)$, where the situation favors one of the candidates if $\nu_s \neq 0$. This determines the average "partisanship" of voters in the precinct. Setting $\nu_s = 0$ defines the balanced case. Gerrymanders are represented by setting $\nu_s \neq 0$.

There is a randomly generated number of voters in each precinct who have similarly generated preferences. Let $m_0 \sim P(\mathcal{M})$ denote an initial value for the number of eligible voters in the precinct, based on the Poisson distribution with mean $\mathcal{M}$. In the current simulations, $\mathcal{M} = 1300$. The number of different types of eligible voters in the precinct is an integer $K \sim I(2, 25)$ chosen at random with probability 1/24 from the set $\{2, \dots, 25\}$. The number of eligible voters of each type is a Poisson random variable $m_i \sim P(m_0/K)$, $i = 1, \dots, K$. Hence the total number of eligible voters in the precinct is $\tilde{m} = \sum_{i=1}^{K} m_i$, and the proportion of eligible voters of type i is $\phi_i = m_i/\tilde{m}$.

Each voter has a preference for each candidate that depends on the voter's type. The proportions ϕ_i are used to distribute the preferences types around the precinct offset μ. The mean type set proportion is $K^{-1}\sum_{i=1}^{K}\phi_i = K^{-1}$. Using the normal distribution with mean zero and variance σ, denoted $N(0,\sigma)$, define $\nu_{ji} \sim N(0,\sigma\sqrt{10})$ and generate base values for the preferences for choice j of the eligible voters of type i by

$$\mu_{1i} = \mu + \left(\phi_i - K^{-1}\right)\nu_{1i} \tag{1a}$$

$$\mu_{2i} = -\mu_{1i} \tag{1b}$$

$$\mu_{3i} = -0.1 + \mu + \left(\phi_i - K^{-1}\right)\nu_{3i} \tag{1c}$$

$$\mu_{4i} = -0.2 + \mu + \left(\phi_i - K^{-1}\right)\nu_{4i} \tag{1d}$$

These preference values are used for the first simulation where there are four candidates. Each normal variate is selected independently for each j and i. Hence, for example, the base value of preferences for candidate 1 held by eligible voters of type i is distributed normally with mean μ and variance $10\sigma^2\left(\phi_i - K^{-1}\right)^2$. The average base value for preferences among all eligible voters in the precinct is μ. If μ represents the basic "partisanship" of each precinct, then the $\left(\phi_i - K^{-1}\right)\nu_{ji}$ values represent effects different issues, performance judgments, social positions, campaign strategies and whatnot have on sets of voters.

A more positive number indicates a candidate is more preferred. Candidates 1 and 2 come from opposite "parties," while candidates 3 and 4 are typically positioned with values that have the same sign as but are slightly more negative than the values assigned to candidate 1. This structure implies that when candidate 1 is preferred to candidate 2 (i.e., when $\mu_{1i} > 0 > \mu_{2i}$), candidates 3 or 4 have some chance to be the most preferred candidate, but when $\mu_{2i} > 0 > \mu_{1i}$ candidates 3 and 4 are much less likely to be preferred over candidate 2. One might think of this as a situation in which there are two candidates that are ideologically similar to candidate 1 but usually less preferred than candidate 1.

The second simulation, with two candidates, uses only base preferences (1a) and (1b).

The third simulation, with four candidates, uses preference definitions (1a) and (1b) and a slightly different definition for μ_{3i} and μ_{4i}: using uniform variates $u_{ji} \sim U(0,1)$,

$$\mu_{3i} = \begin{cases} -0.1 - \mu + (\phi_i - K^{-1})\,\nu_{3i}, & \text{if } u_{3i} \leq .5 \\ -0.1 + \mu + (\phi_i - K^{-1})\,\nu_{3i}, & \text{if } u_{3i} > .5 \end{cases} \tag{2a}$$

$$\mu_{4i} = \begin{cases} -1.5 - \mu + (\phi_i - K^{-1})\,\nu_{4i}, & \text{if } u_{4i} \leq .5 \\ -1.5 + \mu + (\phi_i - K^{-1})\,\nu_{4i}, & \text{if } u_{4i} > .5 \end{cases} \tag{2b}$$

where each u_{ji} is drawn independently. In contrast with the first simulation, here candidates 3 and 4 are symmetrically positioned relative the first two candidates: in this case the values of candidates 3 and 4 at random have the same sign as either candidate 1 or candidate 2 instead of almost always having the same sign as candidate 1.

To get preferences for individuals, I add a type 1 extreme value (Gumbel) distributed component to each individual's base preference value. Let $\epsilon_{jik} \sim G(0,1)$ denote a type 1 extreme value variate with mode 0 and spread 1. For candidate $j \in \{1,2,3,4\}$ or $j \in \{1,2\}$, each of the m_i individuals k of type i has preference $z_{jik} = \mu_{ji} + \epsilon_{jik}$, with the extreme value variates being chosen independently for each candidate and individual. Hence each voter in the simulation has the same error structure for its preference as is implied if μ_{ji} is observed up to a set of unknown linear parameters which are estimated using a simple multinomial logit choice model (McFadden 1973).

To define the baseline of votes that are cast in the absence of strategic considerations, I define variables that measure for each individual which candidate is the first choice. This is the candidate for which the individual has the highest preference value. An individual does not vote unless the preferred candidate's value exceeds a threshold v. This represents the idea that not every eligible voter votes, perhaps due to the cost of voting.

Simulation 1: Strategic Voting and Coercion

The first simulation focuses on strategic voting and coercion. In this simulation there are four candidates but only candidates 1, 2 and 3 actually run. All voters with a first-place preference for candidate 4 are coerced to vote for candidate 1 regardless of their other preferences. So for each candidate j, first-place indicator y_{jik} is defined to be 1 if all the inequalities in the corresponding one of the following definitions are true, zero otherwise:[5]

$$y_{1ik} = z_{1ik} > v \wedge z_{1ik} > z_{2ik} \wedge z_{1ik} > z_{3ik} \wedge z_{1ik} > z_{4ik} \tag{3a}$$

$$y_{2ik} = z_{2ik} > v \wedge z_{2ik} > z_{1ik} \wedge z_{2ik} > z_{3ik} \wedge z_{2ik} > z_{4ik} \tag{3b}$$

$$y_{3ik} = z_{3ik} > v \wedge z_{3ik} > z_{1ik} \wedge z_{3ik} > z_{2ik} \wedge z_{3ik} > z_{4ik} \tag{3c}$$

$$y_{4ik} = z_{4ik} > v \wedge z_{4ik} > z_{1ik} \wedge z_{4ik} > z_{2ik} \wedge z_{4ik} > z_{3ik} \tag{3d}$$

Either zero or one of the y_{jik} values for each individual k will be nonzero. The total of these would-be votes for each candidate j is the sum of the y_{jik} values: $y_j = \sum_i \sum_k y_{jik}$.

The votes for candidates 1, 2 and 3 are subject to wasted-vote logic. I choose σ in equations (1a)–(1d) so that candidate 3 almost always has the smallest number of first-place finishes among candidates 1, 2 and 3. Hence some voters strategically abandon candidate 3 and vote for either candidate 1 or 2. The number of switches depends on both the relative valuations of the candidates and on whether the differences between candidates exceeds a threshold t: someone votes for their second-ranked candidate when their first-ranked candidate comes in last and the gaps between their choices are sufficiently large. Given that candidate 3 comes in last, the number of switched votes is

$$o_{312} = \sum_i \sum_k (z_{3ik} > v \wedge z_{3ik} > z_{1ik} + t \wedge z_{1ik} > z_{2ik} + t \wedge z_{3ik} > z_{4ik})$$

$$o_{321} = \sum_i \sum_k (z_{3ik} > v \wedge z_{3ik} > z_{2ik} + t \wedge z_{2ik} > z_{1ik} + t \wedge z_{3ik} > z_{4ik})$$

[5] $\wedge$ denotes logical 'and'.

The votes for each candidate after the strategic switching to second-ranked candidates are

$$w_1 = y_1 + o_{312} \tag{4a}$$

$$w_2 = y_2 + o_{321} \tag{4b}$$

$$w_3 = y_3 - (o_{312} + o_{321}) \tag{4c}$$

Notice that if $t = 0$, then $w_3 = 0$ and candidate 3 receives no votes.

Because voters who place candidate 4 first are coerced to vote for candidate 1, the total of votes for candidate 1 is $\tilde{w}_1 = w_1 + y_4$.

Table 1 reports the mean over the replications of χ^2_{2BL}, $\hat{j}$, the standard error of $\hat{j}$ and the total number of would-be votes in y and votes in w and $\tilde{w}$.

*** Table 1 about here ***

The results show the pattern of second digits to be sensitive to all the manipulations implemented in the simulation.[6] First, looking at the statistics for the would-be votes y_j, χ^2_{2BL} for y_1 shows no significant departure from the 2BL pattern, while $\hat{j}$ is slightly more than two standard errors greater than $\bar{j}$: $4.29 - 2(.04) > \bar{j}$. This excess above $\bar{j}$ is caused by the presence of the two other candidates, 3 and 4, competing for first place when μ_{1i} is positive. This is evident upon contrasting the statistics for y_2. Except for the presence of candidates 3 and 4, the preferences underlying y_2 are symmetrically opposite those underlying y_1. Solely due to the symmetry in the preference distribution, the statistics should be the same. Yet while χ^2_{2BL} again shows no significant departure from the 2BL pattern, $\hat{j} = 4.15$ for y_2 is less than but not significantly different from $\bar{j}$.[7] Considered on their own, the counts of would-be votes for candidates 3 and 4 do not have significantly discrepant χ^2_{2BL} values but do have $\hat{j}$ values significantly greater than $\bar{j}$.

[6]The simulation results themselves are stable within a range of variation of the model conditions. Using $v = 2$ produced similar results, but using $v = 1.5$ produced departures from 2BL in y_2 that were detectable by χ^2_{2BL}. For $\mathcal{M} \in \{1200, 1400, 1500\}$, $\hat{j}$ for y_2 remains not significantly different from $\bar{j}$, so that the other statistics can be considered relevant. In these cases the statistics for the other vote totals behave as described in the text. For $\mathcal{M} \in \{800, 900, 1000, 1100\}$, $\hat{j}$ for y_2 differs significantly from $\bar{j}$.

[7]Here I use "significantly different" to refer to means that differ by more than two standard errors.

Once wasted-vote logic is used to shift some votes away from candidate 3 and to candidates 1 and 2, the distribution of second digits changes noticeably. For w_1 and w_2, χ^2_{2BL} shows no significant departure from 2BL, but $\hat{j}$ is significantly greater than $\bar{j}$. These mean statistics however remain significantly smaller than the value of 4.5 that would occur if the second digits were distributed with equal frequencies (meaning, if each occurred with probability 1/10). For w_3, χ^2_{2BL} is very significantly different from what 2BL would imply, and $\hat{j}$ is substantially less than $\bar{j}$. Of course, having set $t = 0$ would have reduced w_3 to exactly zero, but setting other small values for t produces similar results.[8]

Finally, the effect of coercion is evident in the statistics for $\tilde{w}_1$. χ^2_{2BL} is very significantly different from what 2BL would imply, and $\hat{j}$ is substantially less than $\bar{j}$. Notably $\hat{j}$ here is significantly greater than $\hat{j}$ for the candidate that was abandoned for strategic reasons. The vote counts differ for the candidates, however—candidate 1 has more than 35 times the vote of candidate 3—so there should be little possibility of confusion between candidates whose statistics differ because of these respective mechanisms.

Most important for the prospect of detecting coercion is that the statistics for $\tilde{w}_1$ differ substantially from those for w_1 or even y_1. In this case, with two candidates having balanced support except a third candidate is more similar to one of the two major candidates, the second digits of vote counts of winning candidates allow fraud done by coercion to be distinguished from either strategic or nonstrategic normal politics.

Distinguishing strategic from nonstrategic normal politics is a less of a sure bet. χ^2_{2BL} seems not to be useful for this purpose at all, but $\hat{j}$ does tell us something. The digit mean statistic for y_2 differs significantly from that for w_2, but the difference between $\hat{j}$ for y_1 and for w_1 falls a bit short of statistical significance. Increasing the number of precincts to 15,000 or more would shrink the standard error of the mean and consequently produce a significant difference. Hence we might surmise that with a sufficiently large number of precincts, $\hat{j}$ could distinguish between situations where a candidate has no ideologically (or

[8] I found similar results for all the statistics reported here for $t \in \{.5, .45, .4, .35, .3, .25, .2, .15, .1, .05, .025\}$.

more generally, preferentially) similar competition due to voters having strategically abandoned all such candidates from the situation where such candidates never existed. The latter case might arise, for instance, where elites or processes (say primaries) act to keep the other candidates off the ballot and out of voters' considerations. A much larger number of precincts seem to be required to distinguish wasted-vote strategic voting from the situation where similar but less preferred candidates appear on the ballot in the absence of strategic voting. In both of these latter cases, significant deviations from 2BL in $\hat{j}$ can occur, but the mean appears to be slightly larger when there is strategic voting.

Simulation 2: Gerrymandering

The second simulation focuses on implications of gerrymandering. In this simulation there are two candidates. There is no strategic voting. The following inequalities determine votes:

$$y_{1ik} = z_{1ik} > v \wedge z_{1ik} > z_{2ik} \tag{5a}$$

$$y_{2ik} = z_{2ik} > v \wedge z_{2ik} > z_{1ik} \tag{5b}$$

The total votes for each candidate j is the sum of the y_{jik} values: $y_j = \sum_i \sum_k y_{jik}$.

In many cases, especially in plurality rule legislative elections that follow partisan primary elections, only two candidates are on the ballot, so strategic voting according to wasted-vote logic cannot happen. In such cases the two candidates often do not have balanced support, due to the drawing of legislative district lines and the effects of issues in the race, campaigns and other transient phenomena. I manipulate the value of ν_s to simulate the effect of such imbalances. I use $\nu_s \in \{0, .2, .4, .6\}$ so that in unbalanced cases it is candidate 2 who has the advantage.

A frequent corollary of gerrymanders due to districting decisions is decreased voter turnout: voters who support a party that is disadvantaged in the drawing of district lines may not vote in the legislative race, in the belief, perhaps, that their favored candidate has no chance of winning. I modify the turnout threshold parameter in order to represent this possibility. The turnout threshold is specified to increase as a function of the ratio between the first-place preferences for candidate 1 and the first-place preferences for candidate 2. Define a logistic function of the ratio between votes for the two candidates as follows:

$$f_j = 2\,/(1 + \exp\left[b_j\,(1 - y_1/y_2)\right]) \tag{6}$$

If $y_1 = y_2$, then $f_j = 1$, but given turnout factor $b_j < 0$ then $y_1 < y_2$ implies $f_j > 1$. I use f_j to modify the turnout threshold in the voting rule for candidate j. The modified votes are

$$y^*_{1ik} = z_{1ik} > f_1 v \wedge z_{1ik} > z_{2ik} \tag{7a}$$

$$y^*_{2ik} = z_{2ik} > f_2 v \wedge z_{2ik} > z_{1ik} \tag{7b}$$

As the gap between the votes for candidates 1 and 2 increases, an eligible voter who prefers candidate 1 has to have increasingly extreme preferences in order to motivate actually voting. Using y^*_{2ik} also allows voters for the advantaged party to vote less if they think the race will be lopsided. These votes total $y^*_j = \sum_i \sum_k y^*_{jik}$.

Some results from this simulation for $\nu_s \in \{0, .2, .4, .6\}$ appear in Figure 3. The first row of the figure shows $\hat{\jmath}$ computed from y^*_j and plotted against values of the turnout factor in the case $b_1 = b_2$.[9] $\hat{\jmath}$ almost never equals $\bar{\jmath}$, the second-digit mean expected according to Benford's Law.[10] As the advantage to candidate 2 increases, the Monte Carlo mean of $\hat{\jmath}$ increases and then decreases for candidate 1 but steadily decreases for candidate 2. At $\nu_s = 0$ and $b_1 = b_2 = 0$, on average $\hat{\jmath}$ is 4.20 for both candidates, but as the advantage increases through $\nu_s = .2$ to $\nu_s = .6$, even while holding $b_1 = b_2 = 0$, $\hat{\jmath}$ for candidate 1 first increases to 4.32 then decreases to 4.03. In the same case for candidate 2, $\hat{\jmath}$ decreases through 4.01 to 3.71. As turnout declines, $\hat{\jmath}$ declines for candidate 1 but rises

[9] The simulation was actually run for all combinations of values $b_1, b_2 \in \{0, -.5, -1, -1.5, -2, -2.5, -3\}$. Figure 3 uses the values produced when $b_1 = b_2$. Other values are interpolated.

[10] The standard error of $\hat{\jmath}$ is in the range .04 to .05.

for candidate 2. Depending on turnout, $\hat{j}$ for candidate 2 may be either below or above $\bar{j}$.

*** Figure 3 about here ***

The second and third rows of Figure 3 provide some practical sense of the kinds of races the simulated conditions represent. The second row shows the margin of victory for candidate 2 over candidate 1, as a proportion. For each value of the advantage ν_s, the figure shows the relationship between the margin and the turnout decline factor applied three ways: when only votes for candidate 1 are affected ($b_2 = 0$); when only votes for candidate 2 are affected ($b_1 = 0$); and when both candidates are equally affected ($b_1 = b_2 < 0$). The margin increases as turnout for candidate 1 declines and decreases as turnout for candidate 2 declines, but it increases slightly as both candidates' turnout declines. The third row of the figure shows the proportion by which turnout decreases in each of the foregoing scenarios, taking the outcome when $\nu_s = b_1 = b_2 = 0$ as a baseline. Turnout decreases most when both candidates are affected and least when only candidate 1 is affected.

Figure 4 emphasizes the nonlinear effect candidate advantage has on $\hat{j}$ and how that effect depends on voter turnout. Each plot in the figure relates $\hat{j}$ to ν_s as the candidate advantage ν_s increases.[11] Plots are shown for turnout factors $b_1 = b_2 \in \{0, -2\}$. When $b_1 = b_2 = 0$, then as candidate 2's advantage increases a peak in $\hat{j}$ is evident for candidate 1 at $\nu_s = .2$ but $\hat{j}$ for candidate 2 decreases steadily. But when $b_1 = b_2 = -2$, the peak for candidate 1 in $\hat{j}$ occurs at a slightly smaller value of ν_s, and $\hat{j}$ for candidate 2 increases—with a peak at $\nu_s = .4$—before it decreases.

*** Figure 4 about here ***

Simulation 3: Gerrymandering, Strategic Voting and Coercion

The third simulation features gerrymandering, strategic voting and coercion. In this simulation there are again four candidates but only candidates 1, 2 and 3 actually run. Votes in this simulation reflect a combination of the logics used in the first two simulations.

[11]Figure 4 uses the values produced when $\nu_s \in \{0, .05, .1, .15, .2, .25, .3, .35, .4, .45, .5, .55, .6, .65, .7, .75, .8, .85\}$. Other values are interpolated.

All voters with a first-place preference for candidate 4 are coerced to vote for either candidate 1 or candidate 2 regardless of their other preferences. First-place indicator variables y_{jik} are defined by (3a)–(3d). Using f_j as defined in (6), vote thresholds for candidates 1 and 2 are modified according to (7). The threshold for candidate 4 is also modified by f_1: $y^*_{4ik} = z_{4ik} > f_1 v \wedge z_{4ik} > z_{1ik} \wedge z_{4ik} > z_{2ik} \wedge z_{4ik} > z_{3ik}$. The number of switched votes is now

$$o^*_{312} = \sum_i \sum_k (z_{3ik} > f_2 v \wedge z_{3ik} > z_{1ik} + t \wedge z_{1ik} > z_{2ik} + t \wedge z_{3ik} > z_{4ik})$$
$$o^*_{321} = \sum_i \sum_k (z_{3ik} > f_1 v \wedge z_{3ik} > z_{2ik} + t \wedge z_{2ik} > z_{1ik} + t \wedge z_{3ik} > z_{4ik})$$

The votes for candidates 1 and 2 after strategic switching to second-ranked candidates are

$$w^*_1 = y^*_1 + o^*_{312}$$
$$w^*_2 = y^*_2 + o^*_{321}$$

For the case $b_1 = b_2 = 0$, the main difference between the first simulation and the third is the symmetry in the generation of preferences for candidates 3 and 4 in the third simulation. Here these candidates are as likely to attract preferences with same sign as candidate 1 as they are candidate 2. The number of switched votes each candidate receives therefore differs—more for candidate 1 and fewer for candidate 2. While the rule for strategic vote switching according to wasted-vote logic is the same here as in the first simulation, I treat the first-place preferences for candidate 4 differently. Now I consider assigning all first-place finishes for candidate 4 either to candidate 1 or candidate 2, $\tilde{y}^*_j = y^*_j + y^*_4$ and $\tilde{w}^*_j = w^*_j + y^*_4$ for $j \in \{1, 2\}$.

Figure 5 shows the combined effects of strategic voting, gerrymandering and coercion in this case of symmetric third-party preferences, ignoring turnout effects. The figure plots $\hat{j}$ for candidates 1 and 2 against ν_s in four scenarios, two without strategic voting and two with.[12] The (a) and (b) plots show results for votes with no coercion, respectively y_j^* and w_j^*. The (c) and (d) plots show results for votes including coerced votes: $\tilde{y}_j^*$ and $\tilde{w}_j^*$. In almost all cases, the effect of strategic voting is to reduce $\hat{j}$: with strategic voting, $\hat{j}$ never exceeds $\bar{j}$ whereas without strategic voting it sometimes does. Without strategic voting, $\hat{j}$ is not significantly different from $\bar{j}$ for low levels of candidate advantage.[13] Adding coerced votes increases $\hat{j}$ for candidate 1 but has a negligible effect on $\hat{j}$ for candidate 2. The patterns seen with turnout effects enacted are similar to those seen in Figure 5.[14]

*** Figure 5 about here ***

Simulation Overview

The simulations suggest the second-digit means of precinct vote counts are sensitive to many kinds of manipulation. The second simulation shows that even without any kind of election fraud at all, normal politics in the form of gerrymandering can produce an array of distinctive patterns. The first and third simulations show that strategic voting can do so as well. When strategic voting is asymmetric, $\hat{j}$ can distinguish strategic voting from coercion much more effectively than when strategic voting is symmetric.

The idea of symmetry in strategic voting is relevent to the question of distinguishing two kinds of strategic voting. If one thinks in terms of a one-dimensional spatial model of politics, then one will probably observe that in presidential elections there are fringe parties on both the left and right, so it is not easy to see that occasions for strongly asymmetric wasted-vote actions, as in the first simulation, will routinely occur. But in the strategic theory of party balancing of Alesina and Rosenthal (1995), strategic switchers all go one way—only one party's presidential candidate and House candidates of the opposite party gain strategic votes—and substantial asymmetry emerges in the empirical estimates of Mebane (2000, 53). In terms of the pattern the simulation predicts for $\hat{j}$, in the case of

[12] Figure 5 uses the values produced when $\nu_s \in \{0, .05, .1, .15, .2, .4, .5, .6\}$. Other values are interpolated.
[13] The standard error for $\hat{j}$ is usually about .05.
[14] Specifically, the plots are very similar when $b_1 = b_2 = -2$.

asymmetric strategic switching as in the first simulation, strategic voting implies $\hat{j} > \bar{j}$ while the symmetric case of the third simulation implies $\hat{j} < \bar{j}$. I count evidence of asymmetry in strategic voting as evidence for strategic party balancing.

The margin in a race is an almost always measurable covariate with respect to which to array $\hat{j}$ values. If the second digits of precinct votes counts are available, then probably so are the counts themselves, so margins should be feasible to compute. Exceptions will occur when not all precincts are available and neither are constituency totals.

Turnout also evidently can be important in determining $\hat{j}$, but it is a fuzzier concept and one more difficult to measure than the margin of victory. The baseline of eligible voters can be tricky to define and impossible to obtain.

Nonetheless I consider here a concept of turnout in some U.S. House elections that is relevant for evaluating whether a principal feature of the simulation of gerrymander is appropriate. The second and third simulations particularly investigate the effects of turnout declining as a function of candidate advantage. Does it so decline?

Figure 6 shows that one measure of turnout seems to more or less decline with candidate advantage in the U.S. House elections of 1984–90 and 2006–08. I focus on these years because second-digit data from them are examined elsewhere in this paper. In the figure, "House Turnout" is defined as the ratio of the sum of votes cast for either the Democrat or Republican candidate in each race divided by the voting age population. I exclude votes cast for third parties especially to assess the appropriateness of the second simulation which includes exactly two parties. The figure shows the results of nonparametric regressions for each year's data on the margin between the Democrat and Republican in each race. "Margin" is the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using election returns data from Office of the Clerk (2010). On the Democratic-winner side of each graph, where Margin is positive, Turnout clearly declines in every case except 1986 and 2008. In 1986 there appears to be a slight hitch upward just above Margin = 0, after which Turnout declines, while Turnout is flat for much of Margin above zero in 2008. These patterns match the simulations. On the Republican-winner side things are more complicated. Turnout declines right at Margin = 0 in 1986, but in 1984 and 1990 there is a slight hitch up after which Turnout declines, in 1984 Turnout increases for quite some time as Margin decreases, and in 2008 Turnout is flat for Margin down to about $-.2$ before declining.

*** Figure 6 about here ***

Another measure of House election turnout also provides some support for the simulation design. Figure 7 uses self-report data from the American National Election Studies (ANES) from years 1984–90 to measure whether a person voted in the House election.[15] The measure of "Margin" again is computed as in Figure 6, using election returns data from Office of the Clerk (2010). Compensating for the lack of geographic coverage in the ANES data—the ANES sample includes responses from only a subset of congressional districts—is the ability to separate voters by self-described partisanship. Figure 7 shows nonparametric regressions for turnout plotted against Margin for each level of party identification.[16] Turnout always eventually declines as Margin moves away from zero, but immediately near Margin = 0 there is a slight increase among Democracts and among Independents in midterm election years.

*** Figure 7 about here ***

The simulations, while perhaps complicated, are not particularly realistic. Precinct sizes, for instance, do not generally follow a mixed Poisson distribution.[17] Other features of the simulations also are admittedly artificial. The least one can say is that real data represent mixtures that are much more complicated and irregular than the simulations.

[15] A person is counted as having voted in the House election if the response was "yes" to the question, "How about the election for the House of Representatives in Washington. Did you vote for a candidate for the U.S. House of Representatives?" and was not validated as having not voted. Someone who said "yes" but was validated as not voting is coded as not having voted in the House election (Miller and the National Election Studies 1982, 1986, 1989; Miller, Rosenstone, and the National Election Studies 1993).

[16] Strong and weak "Democrats" and "Republicans" are counted as respectively Democrats and Republicans, and all kinds of "Independents" are counted as Independents.

[17] Nor do precinct sizes follow a negative binomial distribution as was used in the "calibration" effort of Mebane (2007).

Rather than attempt to make the simulations much more realistic, I turn instead to their qualitative correspondence with real data from some actual elections.

Recent Elections in the United States

I consider precinct data from several kinds of elections conducted in the United States of America during the 1980s, 1990 and the 2000s.[18] For several years, I have vote totals reported for both federal and state offices.[19] For the 1980s and 1990 the data include every state except California. For the other years data were obtained for most but not all states (including DC): 36 states in 2000; 44 states in 2004; 33 states in 2006; 41 states in 2008. Data are not available for every precinct in some states.

First consider how the simulations bear on the two real data examples introduced above. Considering Figure 1, $\hat{j}_x$ persistently having a value of about 4.3 for the Democratic candidate in states where the Democrat won while $\hat{j}_x$ is not significantly different from $\bar{j}$ for the Republican in those same states matches the pattern from the first simulation that diagnoses strategic voting. Similar values of $\hat{j}_x$ are not observed for the Republican candidate in states where the Republican won, while $\hat{j}_x$ is not significantly different from $\bar{j}$ for the Democrat in those states. There is evidence in favor of strategic voting only for one of the candidates in this election: asymmetric strategic voting.

Considering Figure 2, $\hat{j}_x$ has values of about 4.3 for the whole distribution of Democratic candidates in districts where the Democrat won, but for Republican candidates in districts where the Republican won $\hat{j}_x$ is not significantly distinguishable from $\bar{j}$ for margins near zero, rises as the margin rises and then declines. The latter pattern closely resembles the pattern observed for winners with gerrymandering and turnout decline in the second simulation (Figure 4), but the former resembles the pattern for strategic voting

[18] The 1980s and 1990 precinct data come from ROAD (King et al. 1997). Data from 2000 and 2004 come from the Atlas of U.S. Presidential Elections (Leip 2004) and from collections done by the author. Data from 2006 and 2008 were collected by the author. U.S. House and president margin data are computed from Office of the Clerk (2010)

[19] I have data for federal and state elections for the 1980s, 1990, 2006 and 2008. For 2000 and 2004 I have only presidential election data.

observed in the first simulation. For both sets of losers in Figure 2, the pattern in $\hat{j}_x$ resembles the pattern observed for losers with gerrymandering and turnout decline in the second simulation (Figure 4): for margin near zero, $\hat{j}_x > \bar{j}$, and for high margins $\hat{j}_x < \bar{j}$.

The difference between Democratic winners and Republican winners in Figure 2 can be explained by considering Figure 8, which shows second-digit mean results for the presidential election of 1984. Values near 4.3 are evident for the Republican candidate in states where the Republican won. $\hat{j}_x$ is significantly greater than $\bar{j}$ for the Democrat in states where the Republican won, for margin values up to about 0.1. States where the Democrat won are too few to allow $\hat{j}_x$ to be estimated reliably. The pattern for the Democrat in states where he lost resembles the pattern for sincere preferences in the third simulation (Figure 5), while the pattern for the Republican in states where he won resembles the pattern from the first simulation that diagnoses strategic voting. If asymmetric strategic voting is diagnosed for both the winning Republican presidential candidate and for Democratic winnners in House races from the same year, then the overall pattern is close to what we should expect if there is strategic party balancing as described by Alesina and Rosenthal (1995) and Mebane (2000).

*** Figure 8 about here ***

A similar paired pattern may be observed for 1988. Figure 9 shows that in 1988 $\hat{j}_x > \bar{j}$ over most of the distribution for the Republican presidential candidate in states where the Republican won. In states where the Democrat won, $\hat{j}_x < \bar{j}$ for Margin $< .06$ and $\hat{j}_x > \bar{j}$ only where Margin $> .06$, unlike any of the simulations. There is evidence in favor of strategic voting only for one of the two presidential candidates. The pattern for the Democrat in states where he lost again resembles the pattern for sincere preferences in the third simulation (Figure 5). In Figure 10, $\hat{j}_x$ for Democratic House winners resembles the pattern for strategic voting observed in the first simulation while $\hat{j}_x$ for Republican winners again resembles the pattern observed for winners with gerrymandering and turnout decline in the second simulation. Again the overall pattern is close to what we should expect when there is strategic party balancing.

*** Figures 9 and 10 about here ***

The strategic party balancing theory of Alesina and Rosenthal (1995) implies there is no strategic vote switching in midterm House elections, and looking at data from 1986 and 1990 that is what we find. Figure 11, which displays results for House elections in 1986, shows no departures of $\hat{j}_x$ from $\bar{j}$ that cannot be explained as a result of gerrymandering and turnout decline: $\hat{j}_x$ for Republicans is not significantly different from $\bar{j}$ for Margin $= 0$, then rises to be significantly greater than $\bar{j}$ as Margin increases, then falls back to not be distinct from $\bar{j}$ for high values of Margin; for losers $\hat{j}_x$ is not significantly different from $\bar{j}$ for low values of the absolute margin but is significantly below $\bar{j}$ at high values; and for Democratic winners $\hat{j}_x$ is not significantly different from $\bar{j}$. Similar patterns are observed for 1990, in Figure 12, except for Republican winners $\hat{j}_x$ is never significantly different from $\bar{j}$.

*** Figures 11 and 12 about here ***

Unfortunately precinct data are not available for House elections in 2004, but they are available for the 2004 presidential election, and the second-digit mean results in that case follow a pattern different from the one seen in 1984 and 1988. In Figure 13 it is evidently the Democrat in states where the Democrat won whose pattern of $\hat{j}_x$ values most closely match the values for strategically switched votes obtained in the first simulation. $\hat{j}_x$ for both the Republican and the Democrat in states where the Republican won resemble if anything the pattern for nonstrategic votes observed in the third simulation (Figure 5), but the value of $\hat{j}_x$ for margin equal to zero is too high to match that pattern. The pattern of $\hat{j}_x$ for Republican losers resembles the pattern for losers in the second simulation. Turnout was very high in 2004, so it is possible that the simulations here, which focus on declines in turnout associated with gerrymanders, do not apply.

*** Figure 13 about here ***

I also lack precinct data for House elections in 2000, but precinct data for the presidential election that year are available. In Figure 14 the pattern in $\hat{j}_x$ suggests strategic vote switching in favor of the Democratic presidential candidate and not the Republican. Of course, the Democrat received more votes in that election even though it was the Republican who took office (Wand, Shotts, Sekhon, Mebane, Herron, and Brady 2001; Mebane 2004). The pattern in $\hat{j}_x$ for the Democrat in 2000 differs from that observed for the presidential winners in 1984, 1988 and even 2008, however, in that $\hat{j}_x$ is not significantly different from $\bar{j}$ for Margin very near zero.[20]

*** Figure 14 about here ***

The patterns in $\hat{j}_x$ for the 2008 presidential election, in Figure 1, clearly reflect asymmetric strategic voting in favor of the Democrat: $\hat{j}_x$ is always significantly greater than $\bar{j}$ for the Democratic winner, rarely is so for the Republican in states where he won, and the patterns for losers in the data match the pattern for losers who did not receive any strategic votes. In Figure 15, which shows results for House elections that year, it is Democratic winners and not Republican winners who seem to benefit from asymmetric strategic voting, although $\hat{j}_x$ is not significantly greater than $\bar{j}$ for all Democratic candidates. House losers of both parties have $\hat{j}_x$ values matching those for losers who received no strategic votes. This asymmetric pattern, which suggests strategic voting for Democrats both for House and President, does not match Alesina and Rosenthal's theory. Patterns that suggest asymmetric strategic voting favoring Democrats in House elections in 2006, apparent in Figure 16, also do not match the theory.

*** Figures 15 and 16 about here ***

The evidence for the strategic party balancing theory does not imply that strategic voting according to wasted-vote logic does not occur. There is evidence in favor of strategic

[20] The state with Margin very near zero is New Mexico.

voting in presidential elections from a test of the "bimodality" hypothesis introduced in Cox (1994): if there is a Duvergerian equilibrium so that the $M + 1$ rule holds, then the ratio of the second loser's vote total to the first loser's vote total should be approximately zero. In presidential elections in the 1980s, 2004 and 2008 this relationship holds in all states, and in 2000 it holds in all states except Alaska. In 1992 and other years with a prominent third-party presidential candidate, of course, bimodality test results do not support the predictions of the $M + 1$ rule.

With one big difference, patterns for state legislative election data resemble those observed for U.S. House elections. Fiorina (1992) suggests that party balancing also has implications for state elections. A quick look at the data suggests that during the 1980s this does not happen with the kinds of strategic adjustments that would tie state legislatures to the president as happens with the federal legislature and the president in the theory of Alesina and Rosenthal (1995). Figure 17, which shows $\hat{j}_x$ for state house and state senate data pooled over years 1984, 1986, 1988 and 1990, strongly resembles the pattern for what happens under gerrymandering with turnout decline and no strategic voting. It should be emphasized that Fiorina's theory does not depend on strategic behavior. But the entanglement across levels of government that would support a strategic version of Fiorina's argument about federalism may be true during the late 2000s. Figure 18, which shows $\hat{j}_x$ for 2006 and 2008 state house and state senate elections, suggests asymmetric strategic voting in favor of the Democrats.

*** Figures 17 and 18 about here ***

Discussion

This paper begins with a mixture process that generates individual preferences that, when aggregated into precincts, approximately satisfy 2BL. By deriving sincere and then strategic and gerrymandered and then coerced votes from these preferences, I find that tests based on the second significant digits of the precinct counts are sensitive to differences in how the counts are derived, at least in plurality elections for a single office. The tests can sometimes distinguish the effects of coercion—where votes are cast regardless of preferences—from the effects of strategic voting and gerrymanders. To some extent the tests may be able to distinguish strategic voting according to a party balancing logic from strategic voting due purely to wasted-vote logic, and strategic from nonstrategic voting. These findings based on simulations support plausible interpretations of real data. In some years, however, such as 2004, 2006 and 2008, voter behavior going well beyond the kinds of strategic voting considered here appears to affect the results. And in elections where the simulations are informative, the explanation they provide for the second-digit patterns is qualitative: not every variation in $\hat{j}_x$ is accounted for. The mechanisms considered here matter, but evidently they are not all that's going on.

Except in the vaguest way, by citing work such as Rodriguez (2004) and Grendar et al. (2007), this paper does not explain why precinct-level vote counts so often satisfy 2BL, which, empirically, they very often do. Nonetheless, given that starting point, the evidence is strong that departures from 2BL, which also occur frequently, are related both to normal political phenomena and to serious election anomalies.

Using digit tests to understand the consequences of strategic voting and gerrymandering and to diagnose possible fraud depends on the availability of suitable covariates. In the American case the margin in each jurisdiction and the party of the apparent winner are the covariates used to identify $\hat{j}_x$. Digits alone are about as minimal a foundation for drawing inferences about what happened in elections as might be imagined. If all one has are vote counts and consequently their digits, then there is no information about preferences, strategies, campaigns or anything else that one would normally use to try to understand what went on in an election. Given appropriate covariates, tests based on vote counts' digits can do a lot to give strong suggestions about what happened.

References

Alesina, Alberto and Howard Rosenthal. 1995. *Partisan Politics, Divided Government, and the Economy*. New York: Cambridge University Press.

Bowman, Adrian W. and Adelchi Azzalini. 1997. *Applied Smoothing Techniques for Data Analysis: The Kernel Approach with S-Plus Illustrations*. Oxford: Clarendon Press.

Cho, Wendy K. Tam and Brian J. Gaines. 2007. "Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance." *The American Statistician* 61 (Aug.): 218–.

Cox, Gary W. 1994. "Strategic Voting Equilibria Under the Single Nontransferable Vote." *American Political Science Review* 88 (Sep.): 608–621.

Cox, Gary W. 1996. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. New York: Cambridge University Press.

Cox, Gary W. and Jonathan N. Katz. 2002. *Elbridge Gerry's Salamander: The Electoral Consequences of the Reapportionment Revolution*. New York: Cambridge University Press.

Fiorina, Morris P. 1992. *Divided Government*. New York: Macmillan.

Grendar, Marian, George Judge, and Laura Schechter. 2007. "An Empirical Non-Parametric Likelihood Family of Data-Based Benford-Like Distributions." *Physica A* 380 (1): 429–438.

Hill, Theodore P. 1995. "A Statistical Derivation of the Significant-digit Law." *Statistical Science* 10: 354–363.

Janvresse, Élise and Thierry de la Rue. 2004. "From Uniform Distributions to Benford's Law." *Journal of Applied Probability* 41: 1203–1210.

King, Gary, Bradley Palmquist, Greg Adams, Micah Altman, Kenneth Benoit, Claudine Gay, Jeffrey B. Lewis, Russ Mayer, and Eric Reinhardt. 1997. "The Record of American Democracy, 1984–1990." Harvard University, Cambridge, MA [producer], Ann Arbor, MI: ICPSR [distributor].

Lehoucq, Fabrice. 2003. "Electoral Fraud: Causes, Types, and Consequences." *Annual Review of Political Science* 6 (June): 233–256.

Leip, Dave. 2004. "Atlas of U.S. Presidential Elections." URL http://uselectionatlas.org/.

McFadden, Daniel. 1973. "Conditional Logit Analysis of Qualitative Choice Behavior." In Paul Zarembka, editor, *Frontiers in Econometrics*, .New York: Academic Press. Pages 105–142.

Mebane, Walter R., Jr. 2000. "Coordination, Moderation, and Institutional Balancing in American Presidential and House Elections." *American Political Science Review* 94 (Mar.): 37–57.

Mebane, Walter R., Jr. 2004. "The Wrong Man is President! Overvotes in the 2000 Presidential Election in Florida." *Perspectives on Politics* 2 (Sep.): 525–535.

Mebane, Walter R., Jr. 2006. "Election Forensics: Vote Counts and Benford's Law." Paper prepared for the 2006 Summer Meeting of the Political Methodology Society, UC-Davis, July 20–22.

Mebane, Walter R., Jr. 2007. "Statistics for Digits." Paper presented at the 2007 Summer Meeting of the Political Methodology Society, Pennsylvania State University, July 18–21.

Mebane, Walter R., Jr. 2008. "Election Forensics: The Second-digit Benford's Law Test and Recent American Presidential Elections." In R. Michael Alvarez, Thad E. Hall, and Susan D. Hyde, editors, *The Art and Science of Studying Election Fraud: Detection, Prevention, and Consequences*, .Washington, DC: Brookings Institution.

Mebane, Walter R., Jr. 2010. "Fraud in the 2009 Presidential Election in Iran?" *Chance* 23 (Mar.): 6–15.

Mebane, Walter R., Jr. and Kirill Kalinin. 2009. "Electoral Falsification in Russia: Complex Diagnostics Selections 2003-2004, 2007-2008 (in Russian)." *Electoral Policy* REO 2/09: 57–70.

Mebane, Walter R., Jr. and Jasjeet S. Sekhon. 2002. "Coordination and Policy Moderation at Midterm." *American Political Science Review* 96 (Mar.): 141–157.

Miller, Warren E., Steven J. Rosenstone, and the National Election Studies. 1993. "American National Election Study, 1992: Pre- and Post-Election Survey." [enhanced with 1990 and

1991 data]. Study # 6067. Ann Arbor: University of Michigan, Center for Political Studies, and Inter-University Consortium for Political and Social Research.

Miller, Warren E. and the National Election Studies. 1982. "American National Election Study, 1980: Pre- and Post-Election Survey." Study # 7763. Ann Arbor: Center for Political Studies, University of Michigan. 2d ICPSR ed. Ann Arbor, MI: Inter-University Consortium for Political and Social Research.

Miller, Warren E. and the National Election Studies. 1986. "American National Election Study, 1984: Pre- and Post-Election Survey." Study # 8298. Ann Arbor: Center for Political Studies, University of Michigan. 2d ICPSR ed. Ann Arbor, MI: Inter-University Consortium for Political and Social Research.

Miller, Warren E. and the National Election Studies. 1989. "American National Election Study, 1988: Pre- and Post-Election Survey." Study # 9196. Ann Arbor: Center for Political Studies, University of Michigan. 2d ICPSR ed. Ann Arbor, MI: Inter-University Consortium for Political and Social Research.

Office of the Clerk, U.S. House of Representatives. 2010. "Election Information: Election Statistics." Office of the Clerk, U.S. House of Representatives. URL http://clerk.house.gov/member_info/electionInfo/index.html.

Pericchi, Luis Raúl and David Torres. 2004. "La Ley de Newcomb-Benford y sus aplicaciones al Referendum Revocatorio en Venezuela." Reporte Técnico no-definitivo 2a. versión: Octubre 01,2004.

Rodriguez, Ricardo J. 2004. "First Significant Digit Patterns From Mixtures of Uniform Distributions." *American Statistician* 58 (Feb.): 64–71.

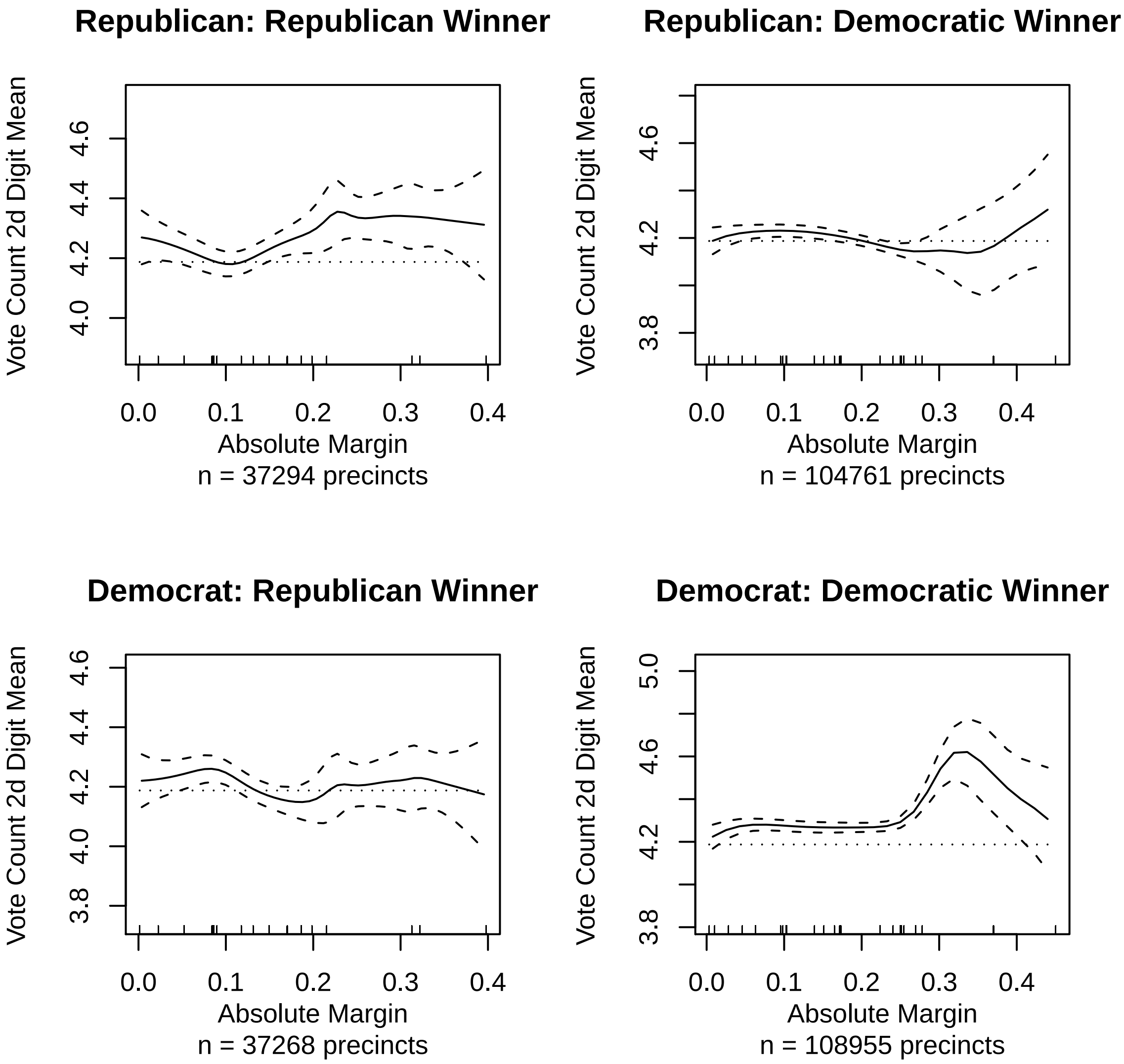
Wand, Jonathan, Kenneth Shotts, Jasjeet S. Sekhon, Walter R. Mebane, Jr., Michael Herron, and Henry E. Brady. 2001. "The Butterfly Did It: The Aberrant Vote for Buchanan in Palm Beach County, Florida." *American Political Science Review* 95 (Dec.): 793–810.

Table 1: Second-digit χ^2_{2BL} statistics, means, standard errors and "vote" totals: asymmetric four-candidate simulation

	y_1	y_2	y_3	y_4	w_1	w_2	w_3	$\tilde{w}_1$
χ^2_{2BL}	10.7	12.6	11.9	12.6	12.3	12.2	951.1	58.0
$\hat{j}$	4.29	4.15	4.32	4.32	4.35	4.35	2.68	3.75
s.e.	.040	.041	.041	.040	.041	.041	.043	.042
votes	$200,284$	$271,628$	$181,172$	$163,970$	$329,043$	$310,300$	$13,741$	$493,013$

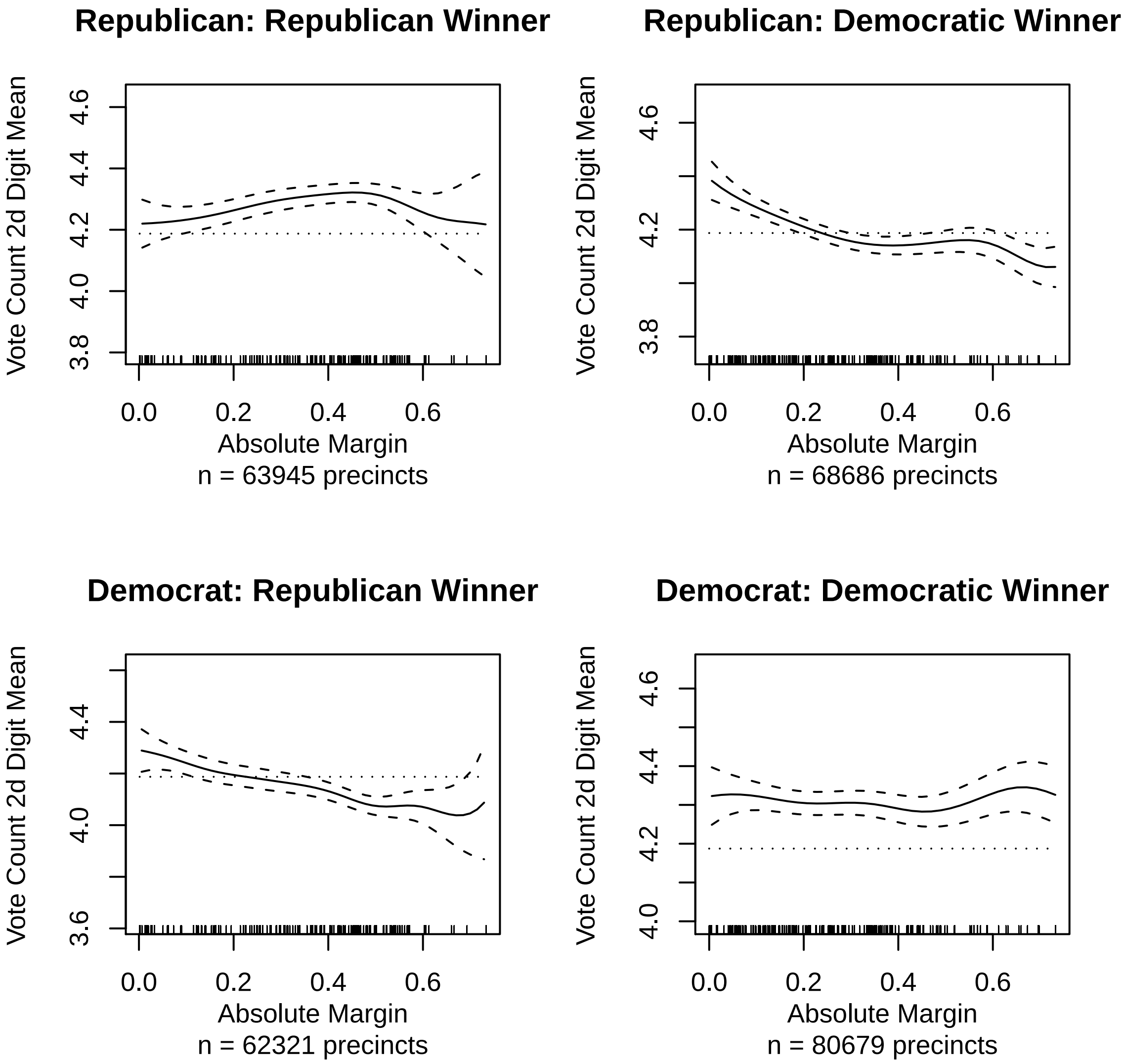
Note: $n = 5000$ precincts. $\mathcal{M} = 1300$, $\sigma = 1$, $v = 1.75$, $t = 0.15$, 500 replications.

Figure 1: Vote Counts for President, 2008



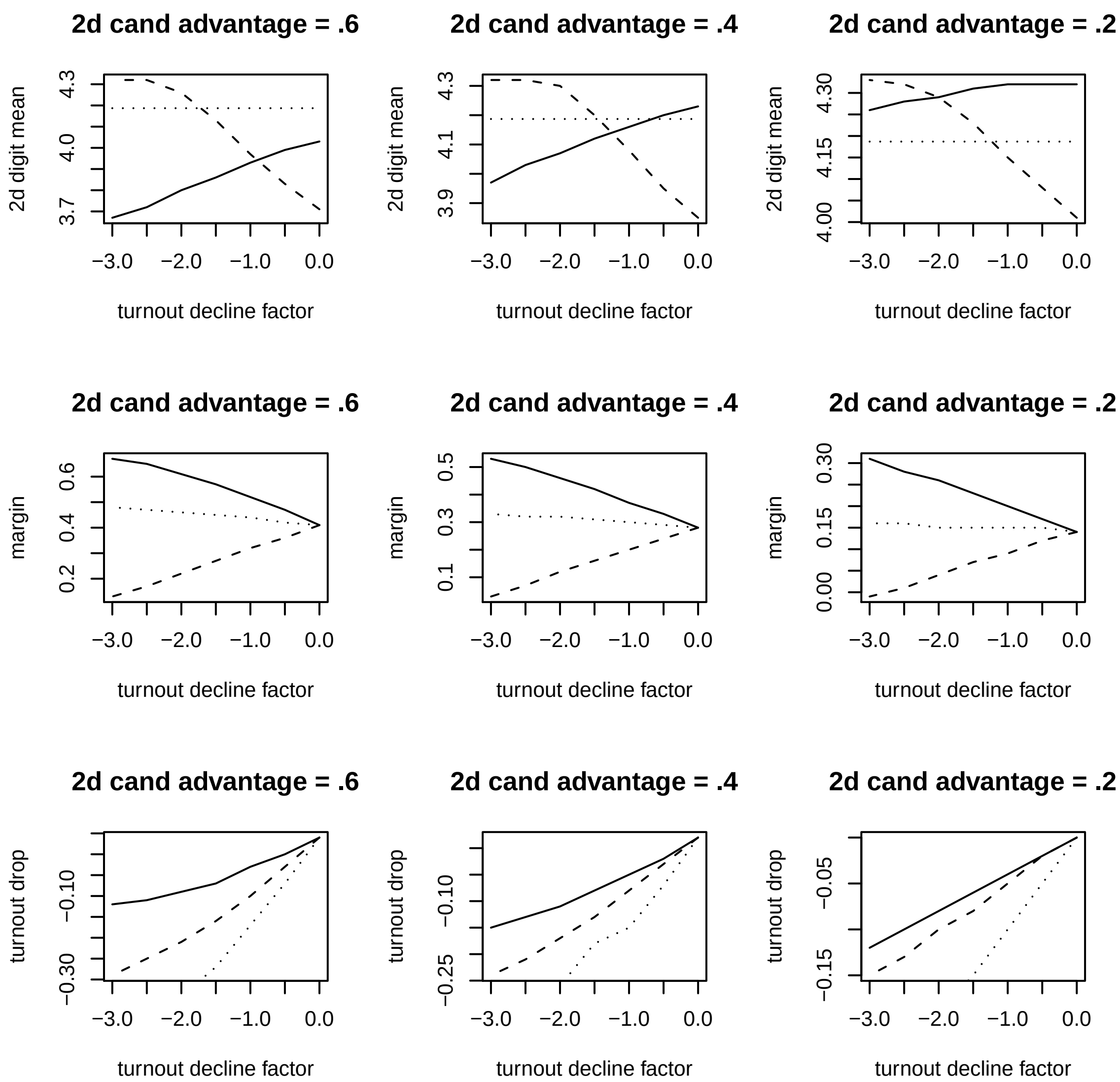
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 2: Vote Counts for United States Representative, 1984



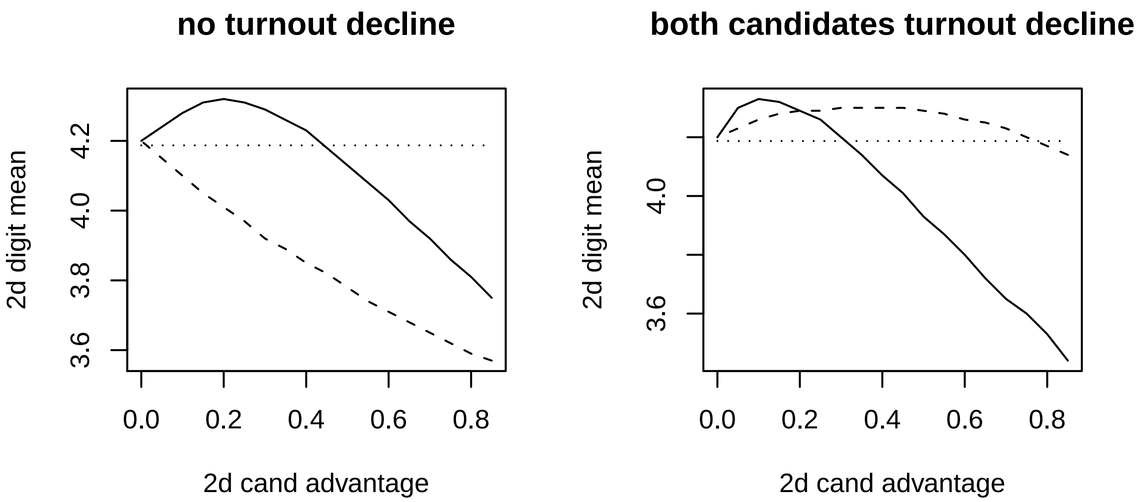
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 3: Second-digit means, margins and turnout drop proportions by turnout decline: two-candidate simulation



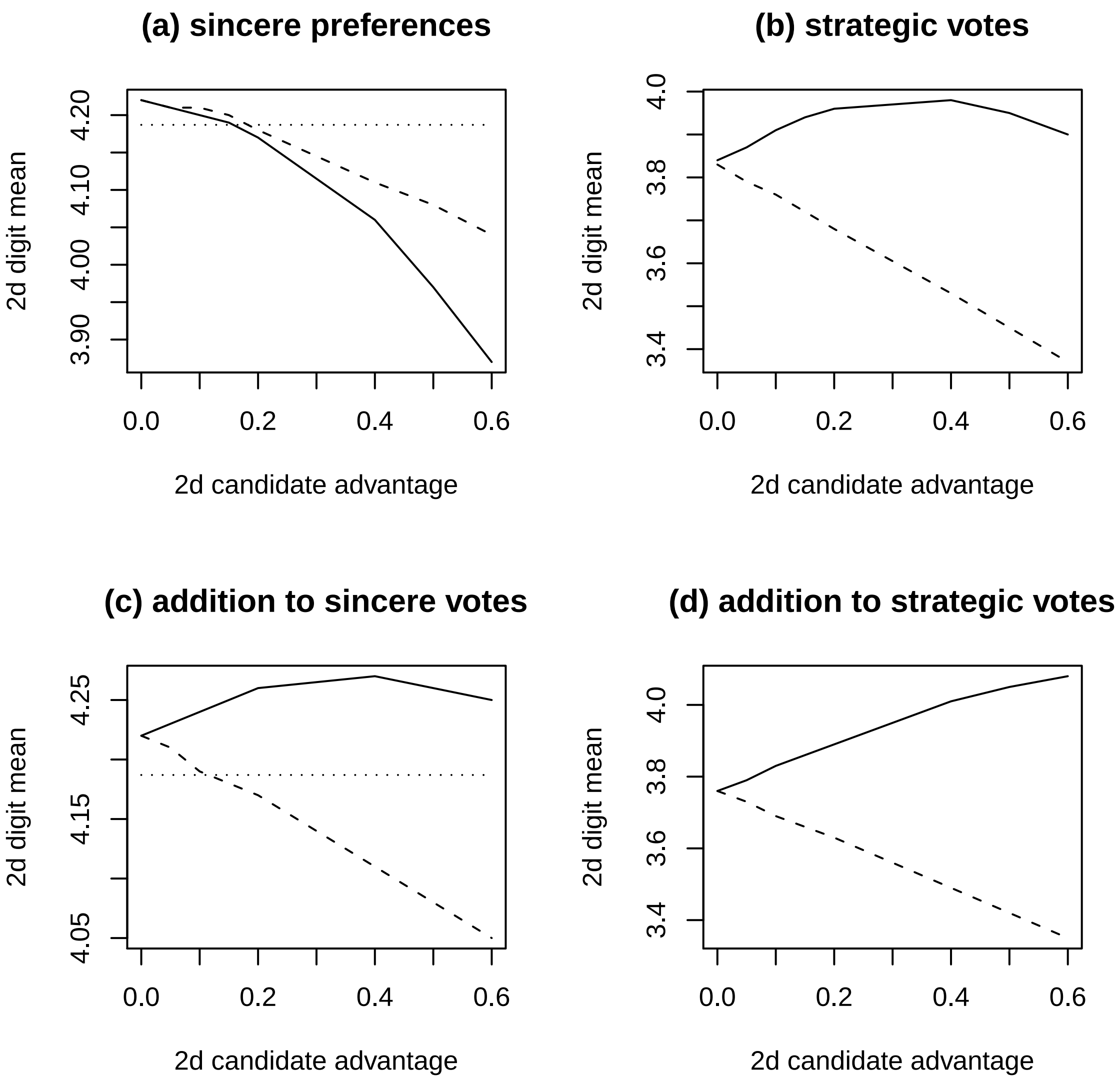
Note: In second-digit mean plots, solid line is first candidate, dashed line is second candidate and dotted line is mean expected under Benford's Law. In margin and turnout drop plots, solid line is margin or turnout drop as only first candidate's turnout factor increases, dashed line is margin or turnout drop as only second candidate's turnout factor increases and dotted line is margin or turnout drop as both candidates' turnout factor increases.

Figure 4: Second-digit means by candidate advantage: two-candidate simulation



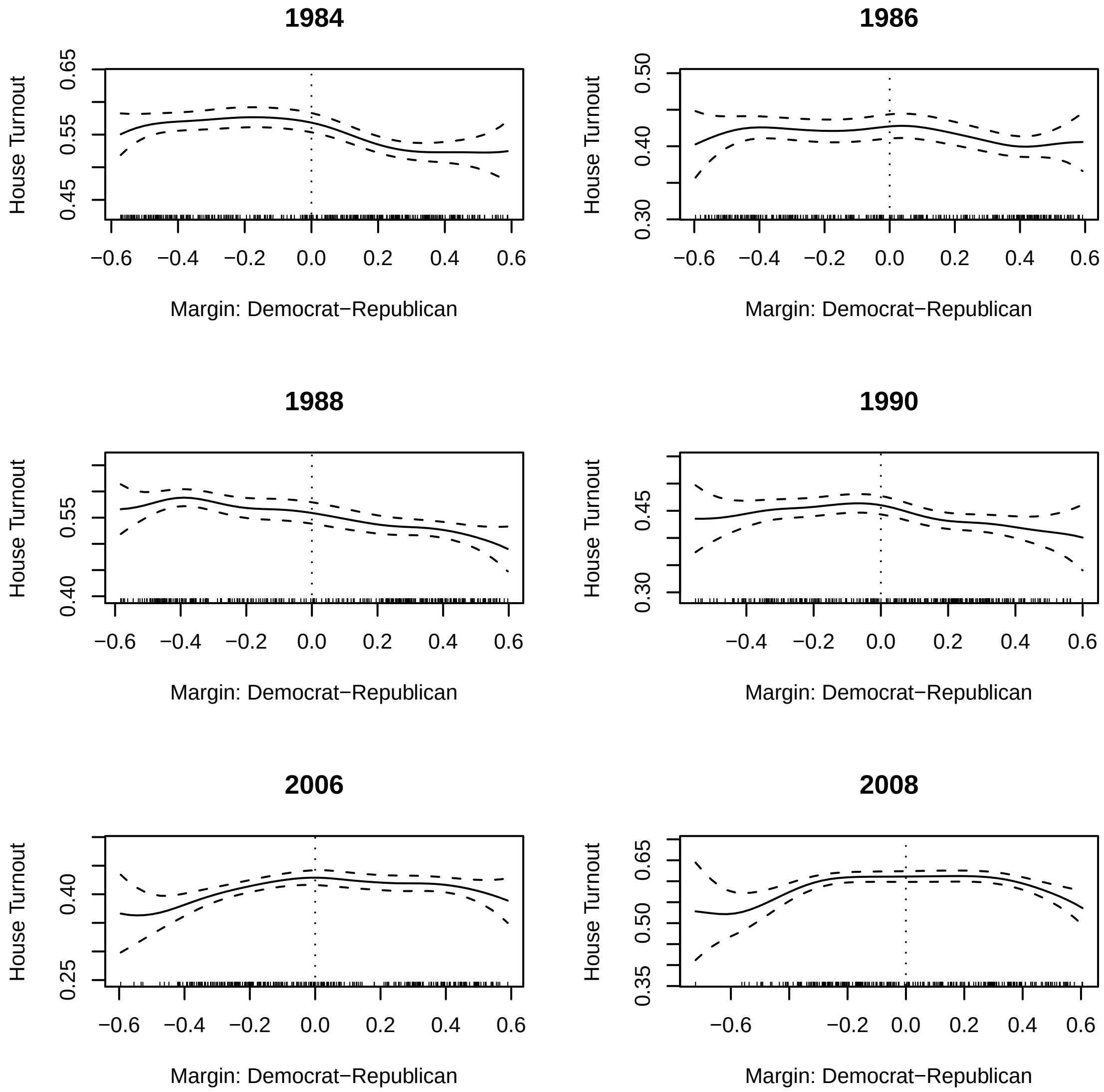
Note: In rightmost graph, turnout decline factor $= -2$. Solid line is first candidate. Dashed line is second candidate. Dotted line is mean expected under Benford's Law.

Figure 5: Second-digit means by candidate advantage (0 turnout decline factor): symmetric four-candidate simulation



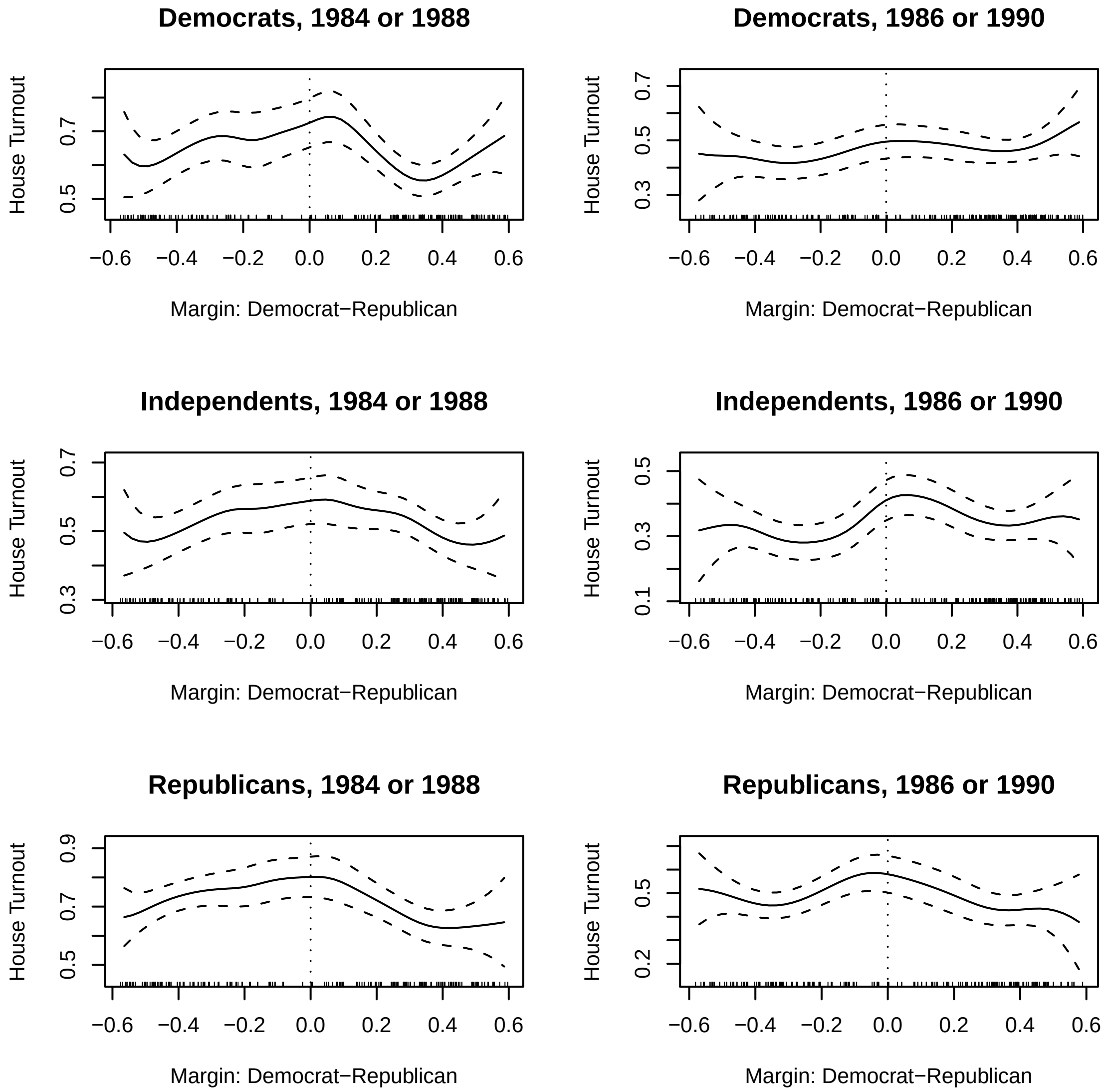
Note: Solid line is first candidate. Dashed line is second candidate. Dotted line is mean expected under Benford's Law.

Figure 6: House Turnout by Margin, 1984–90 and 2006–08



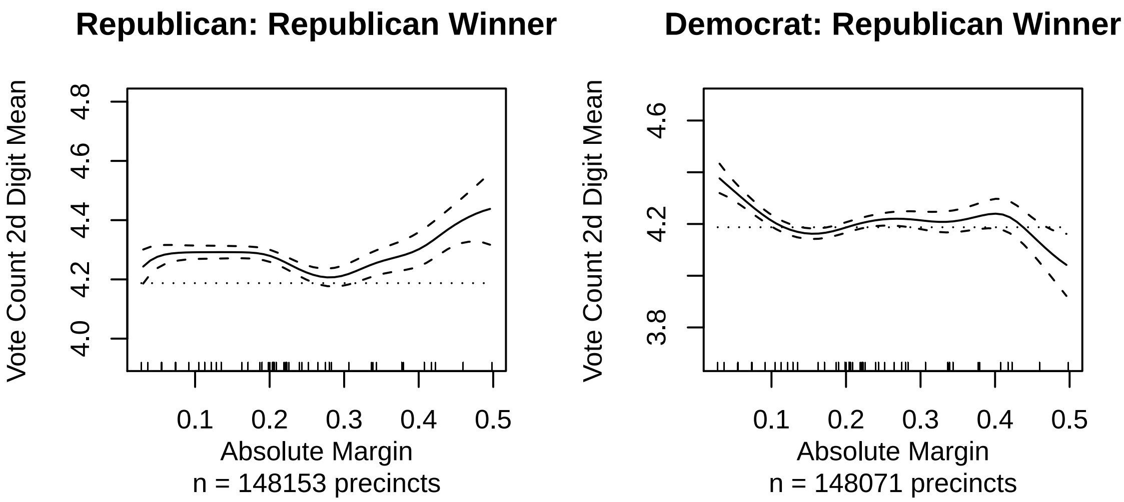
Note: "House turnout" is the total number of votes cast for either the Democratic or Republican candidate divided by the voting-age population in each district (voting-age citizen population in 2006 and 2008). "Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district margins.

Figure 7: House Turnout by Margin by Party Identification, 1984–90



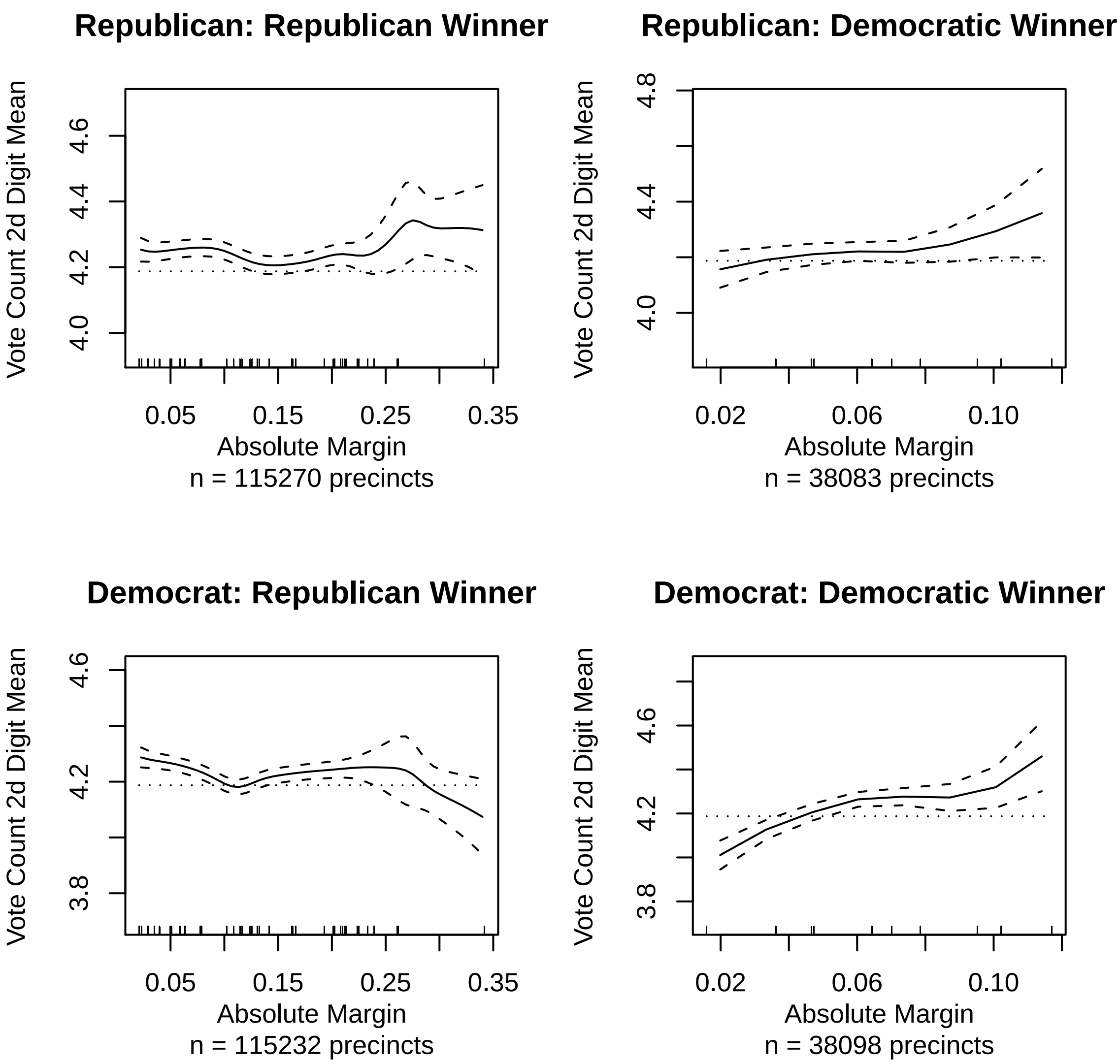
Note: "House turnout" is based on American National Election Studies data using"yes" responses to the question "Did you vote for a candidate for the U.S. House of Representatives?" with those who were validated as not having voted being counted "no." "Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district margins.

Figure 8: Vote Counts for President, 1984



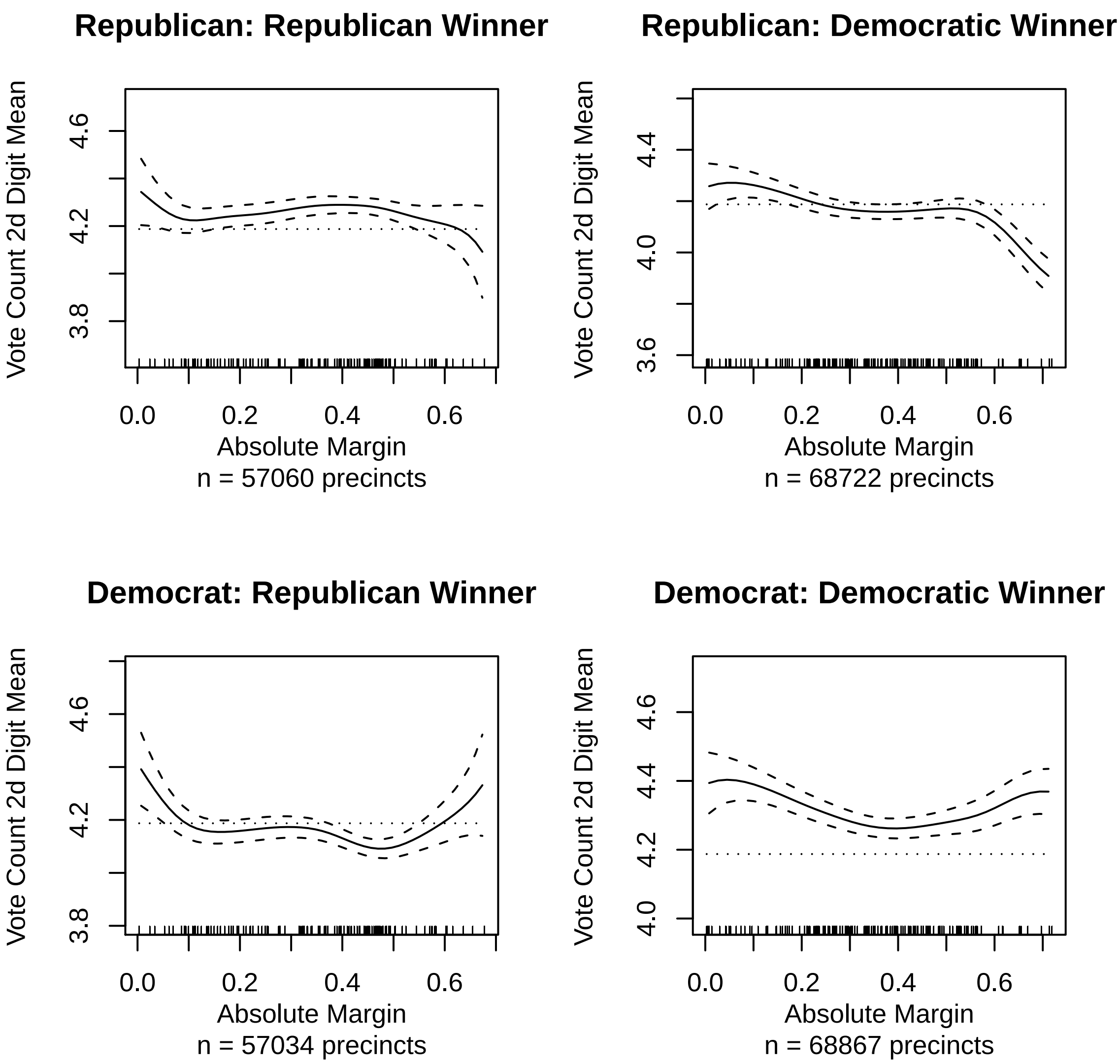
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 9: Vote Counts for President, 1988



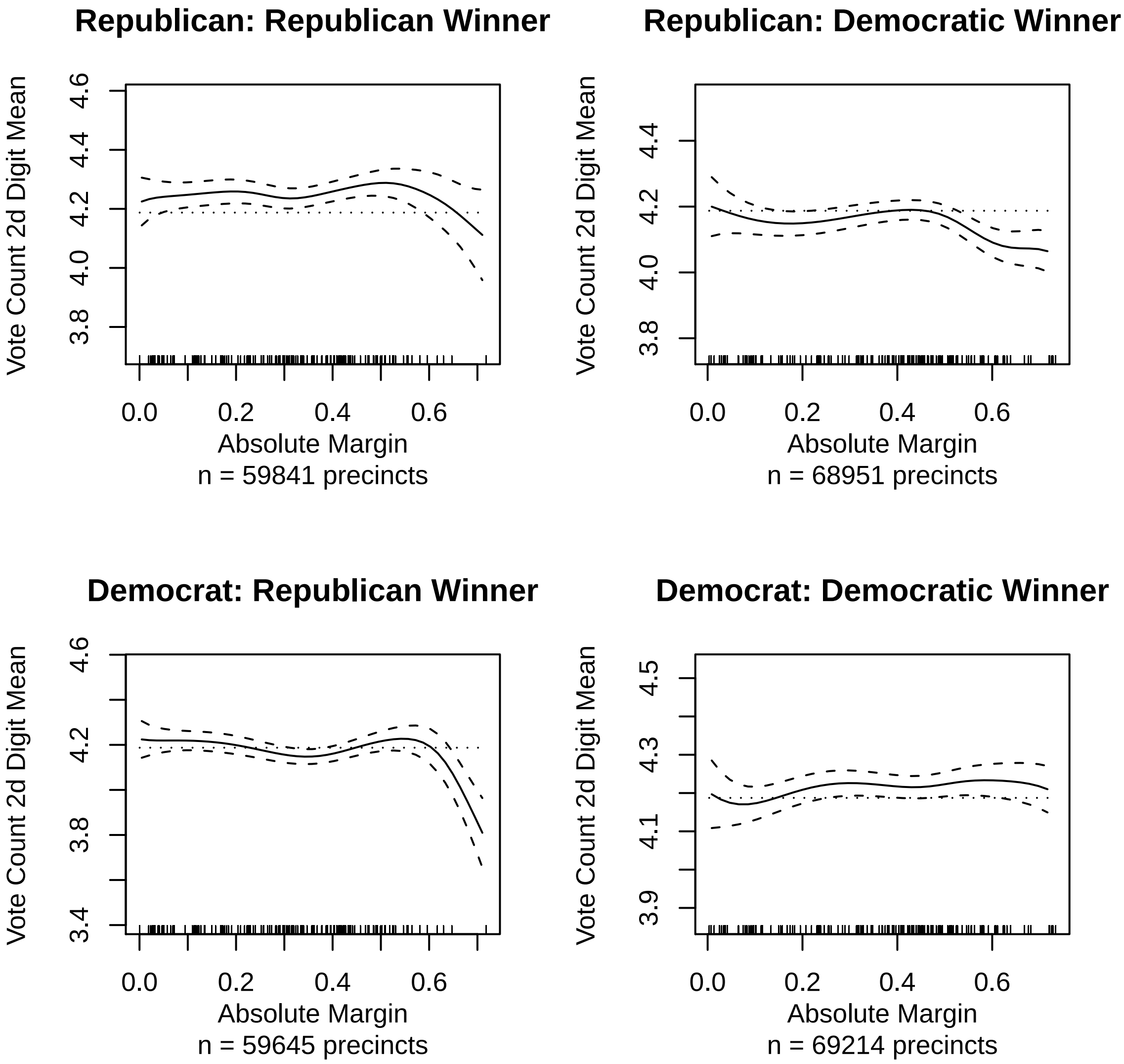
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 10: Vote Counts for United States Representative, 1988



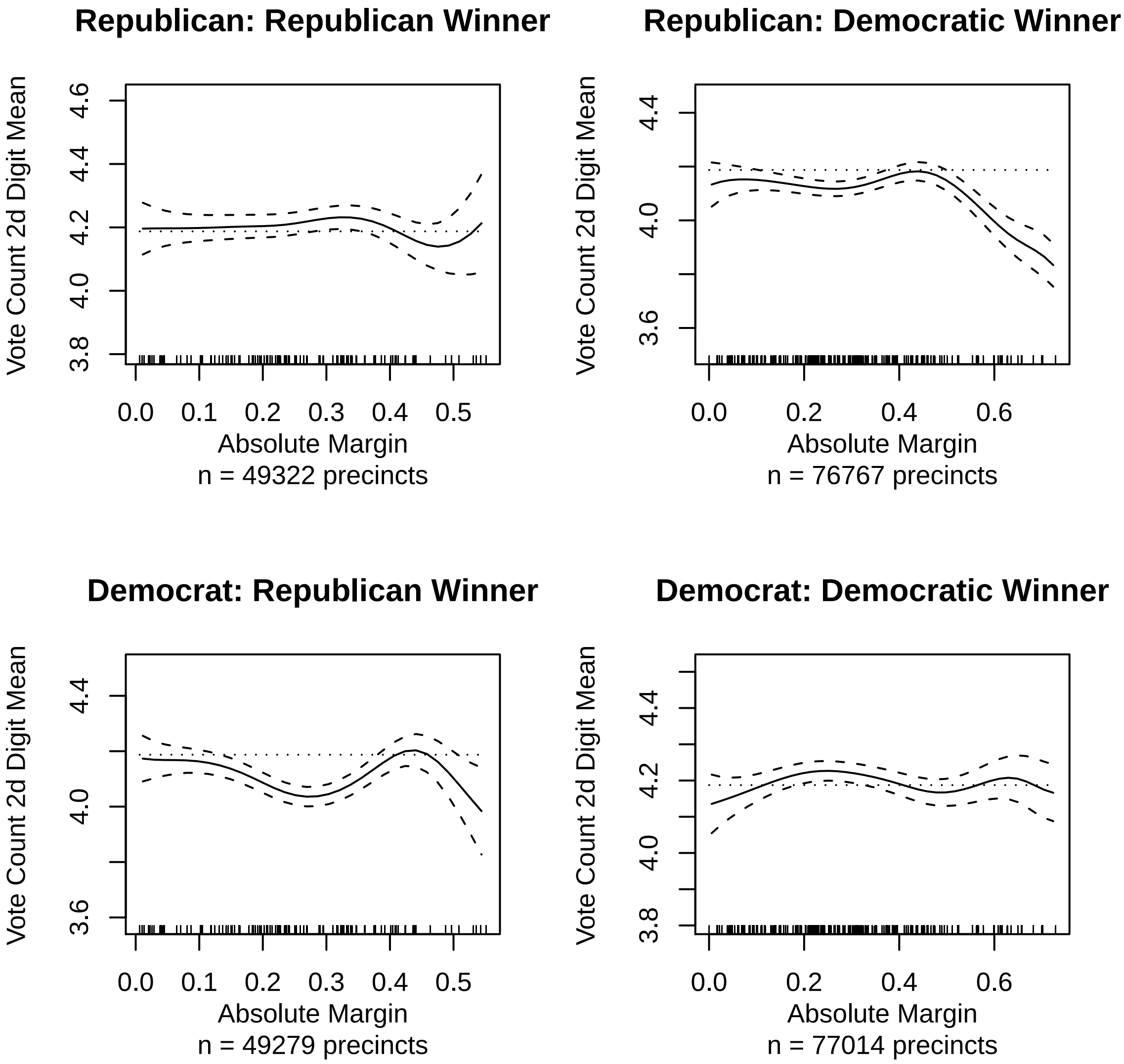
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 11: Vote Counts for United States Representative, 1986



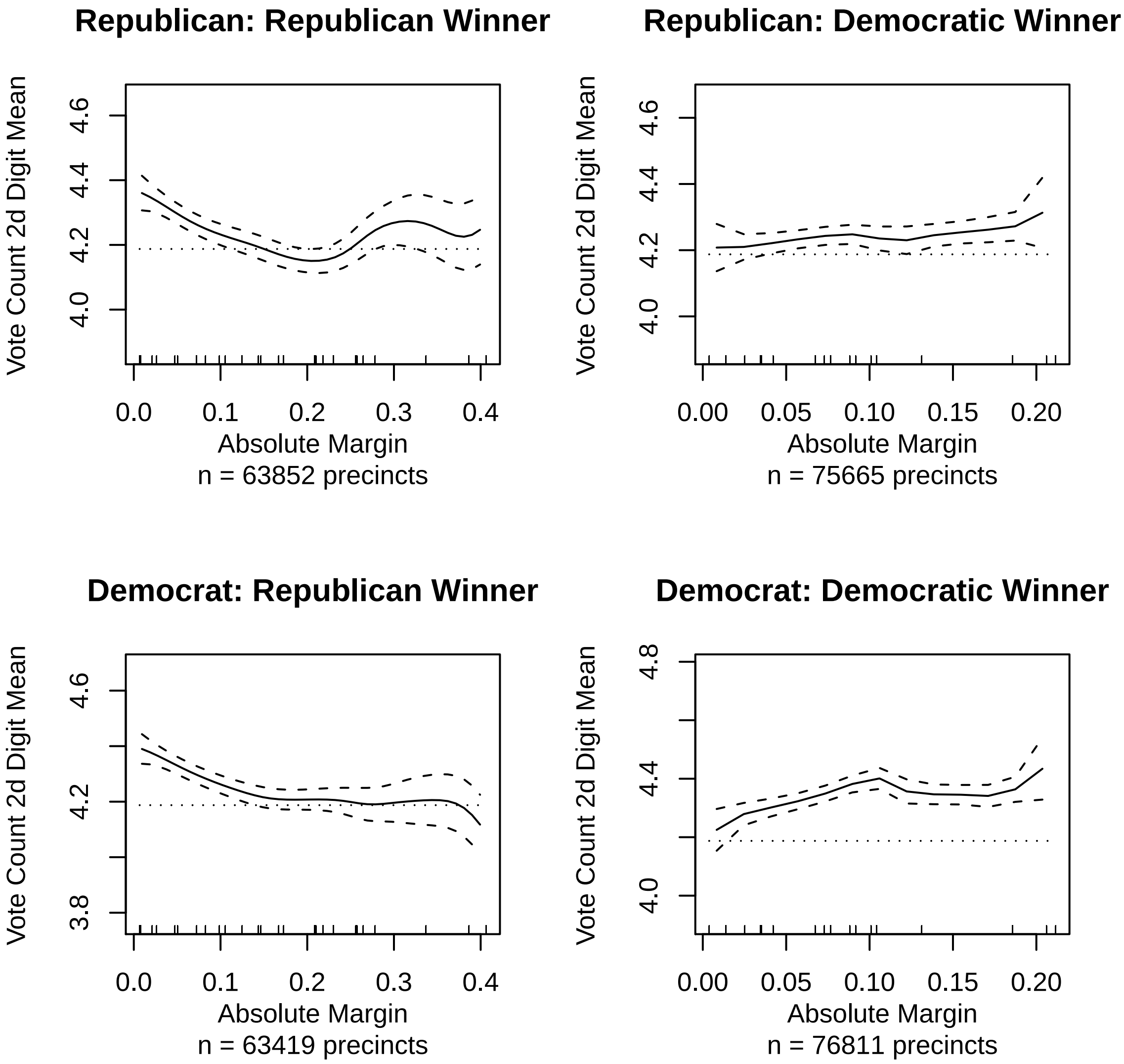
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 12: Vote Counts for United States Representative, 1990



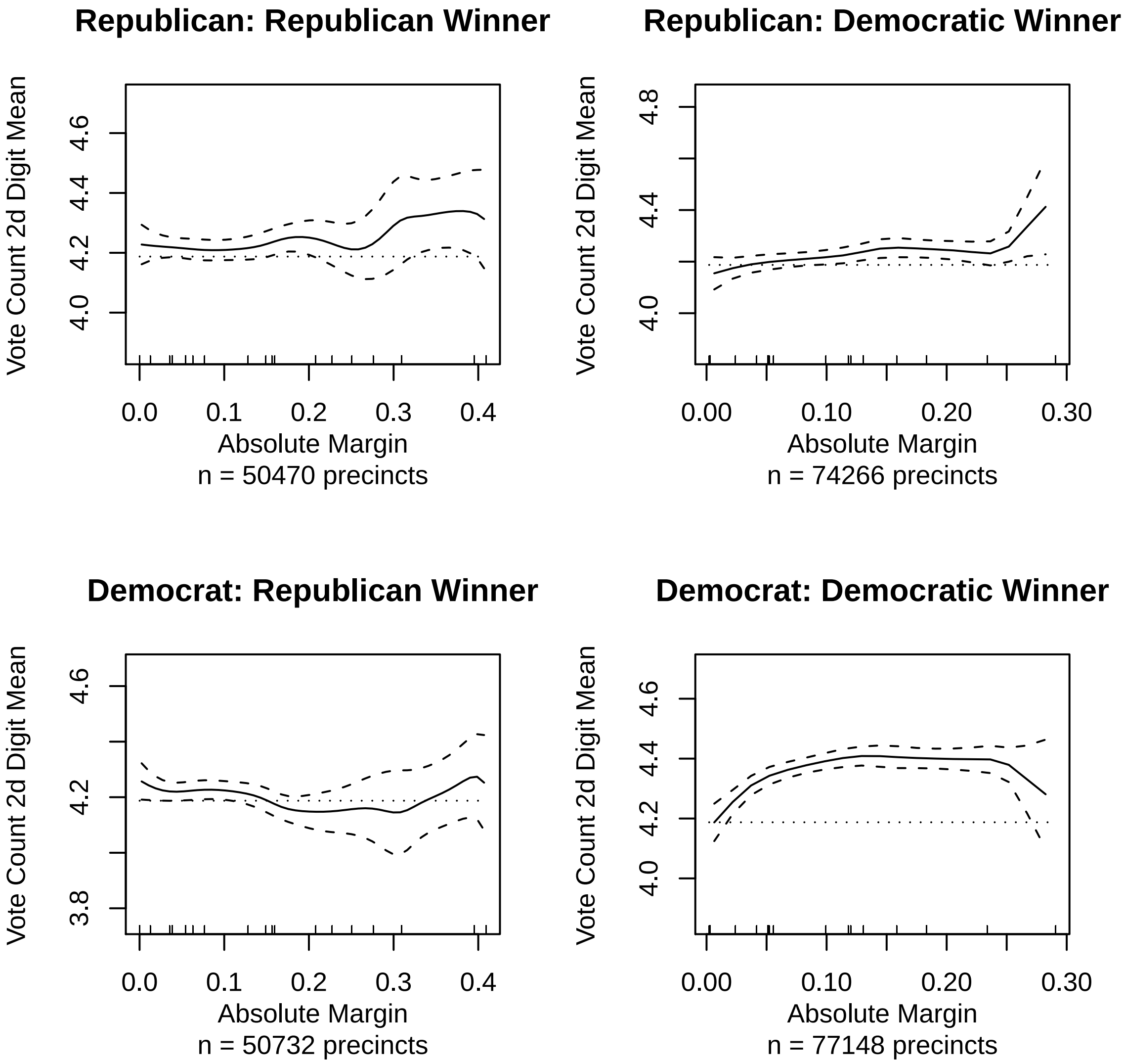
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 13: Vote Counts for President, 2004



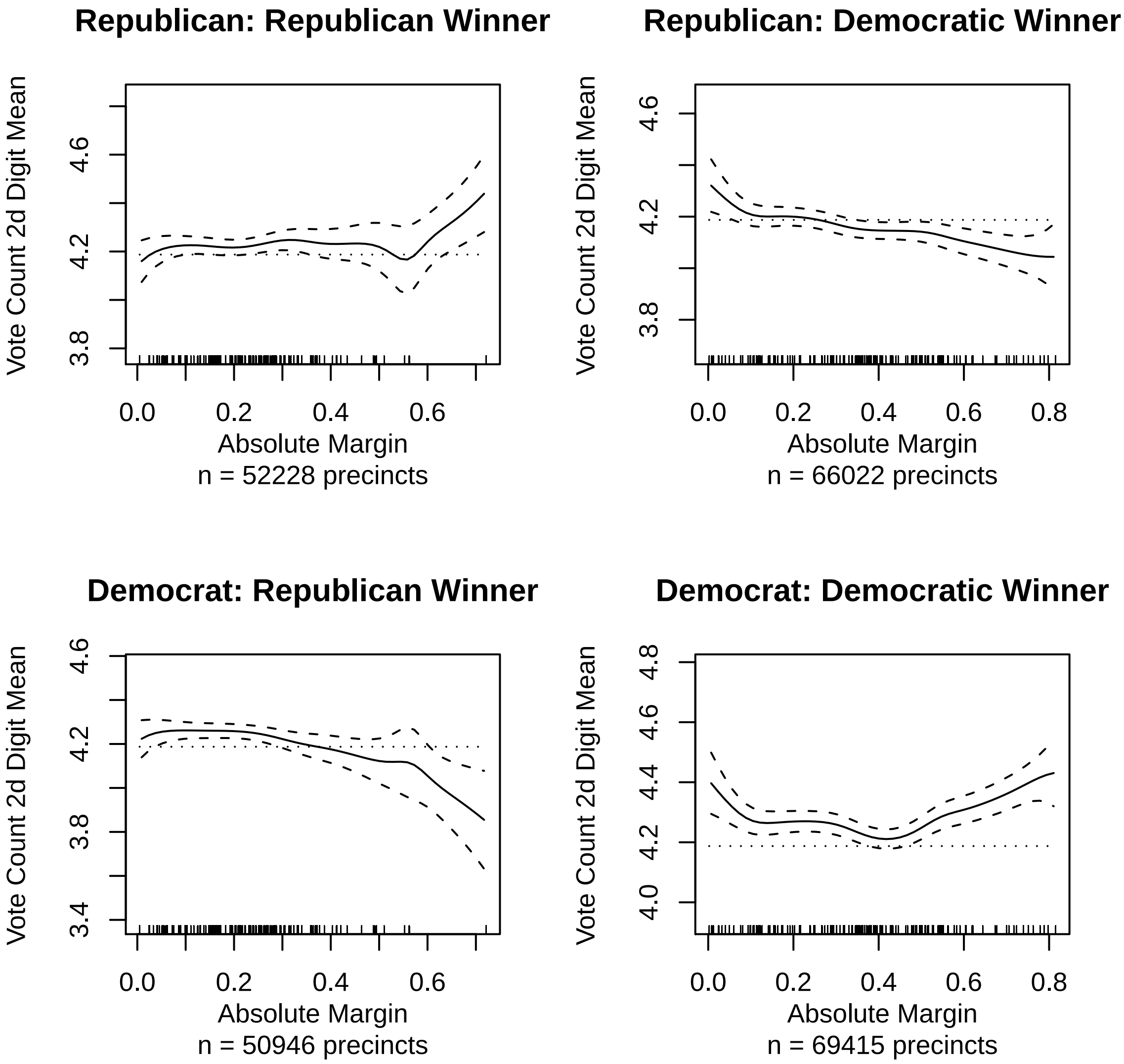
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 14: Vote Counts for President, 2000



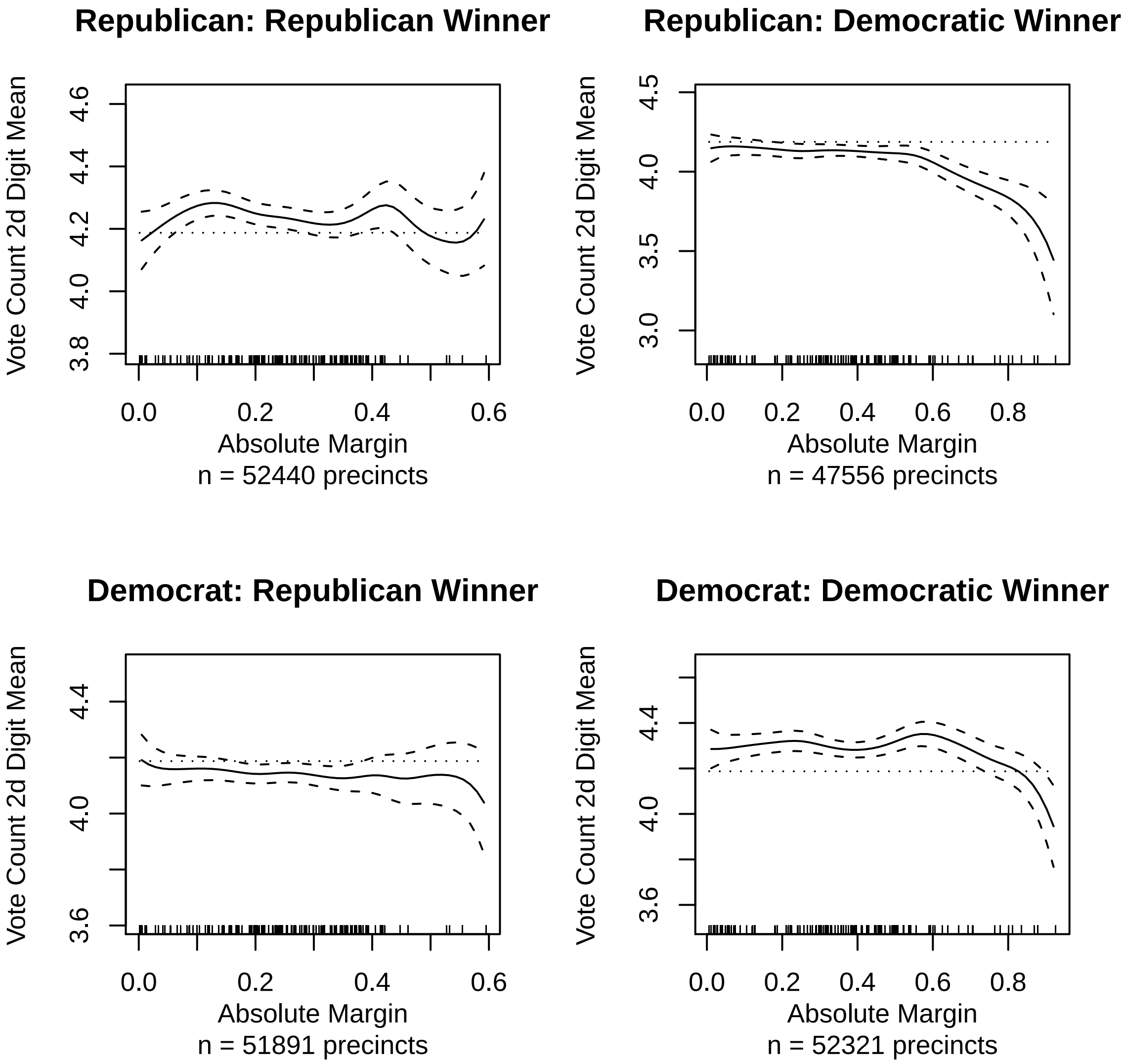
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 15: Vote Counts for United States Representative, 2008



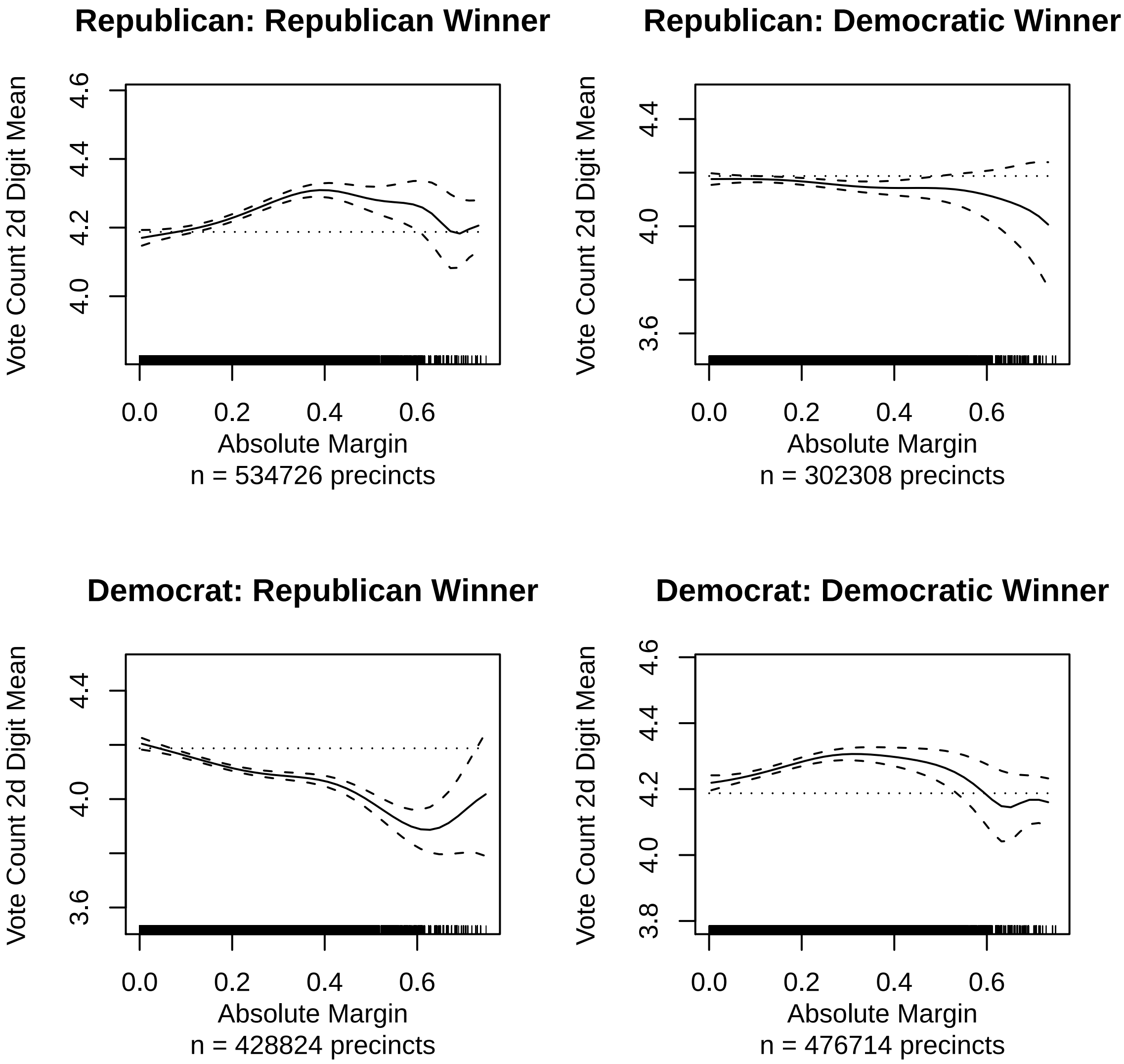
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 16: Vote Counts for United States Representative, 2006



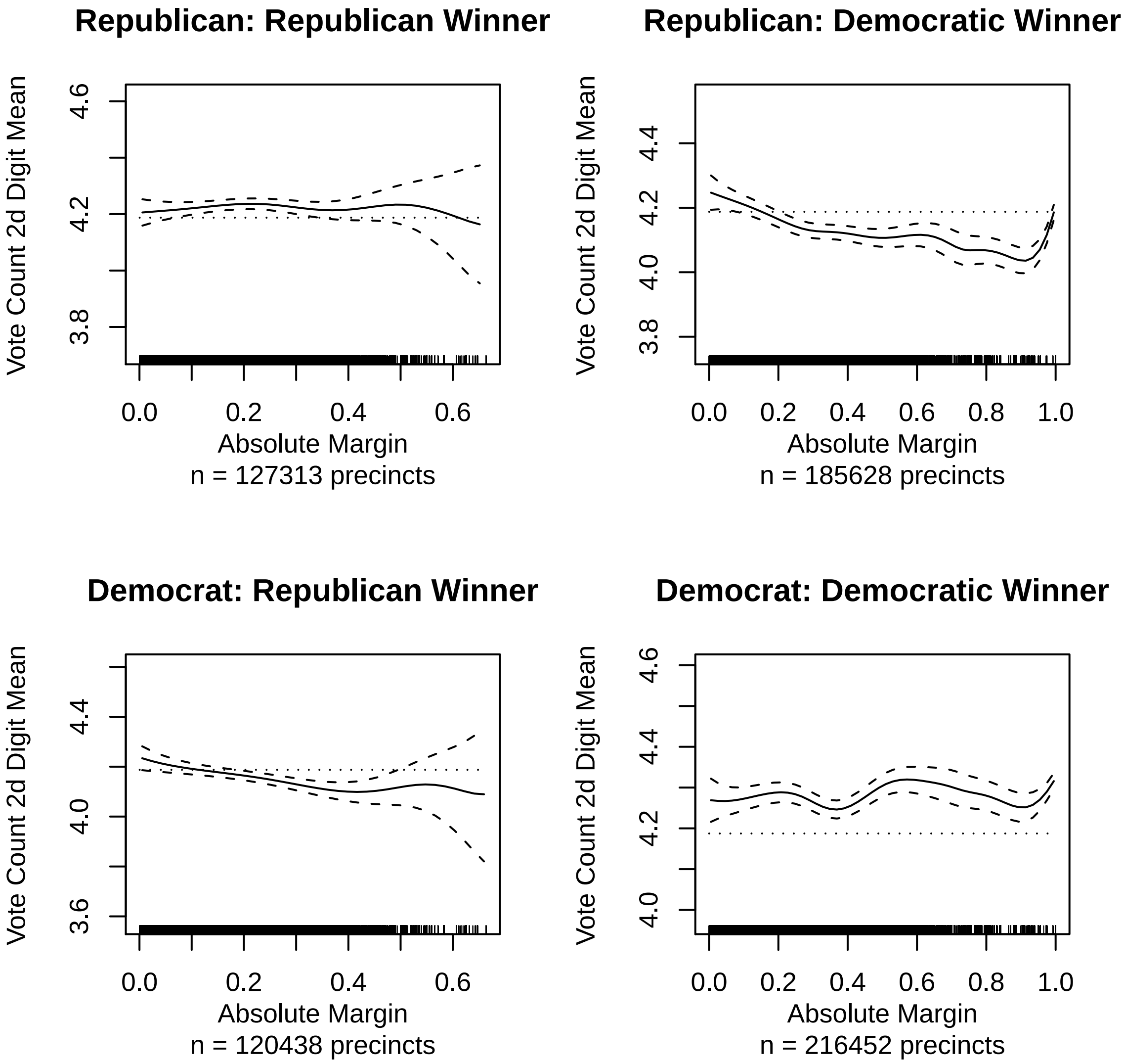
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 17: Vote Counts for State House and Senate, 1984–90



Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes based on district totals computed from the precinct data. Rug plots show the locations of district absolute margins.

Figure 18: Vote Counts for State House and Senate, 2006–08



Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes based on district totals computed from the precinct data. Rug plots show the locations of district absolute margins.