Note on the presidential election in Iran, June 2009

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The presidential election that took place in Iran on June 12, 2009 has attracted considerable controversy. The incumbent, Mahmoud Ahmadinejad, was officially declared the winner, but the opposition candidates—Mir-Hossein Mousavi, Mohsen Rezaee and Mehdi Karroubi—have reportedly refused to accept the results. Widespread demonstrations are occurring as I write this.

Richard Bean[1] pointed me to district-level vote counts for 2009[2] This URL has a spreadsheet containing text in Persian, a language I'm unable to read, and numbers. I know nothing about the original source of the numbers. Dr. Bean supplied translations of the candidate names and of the provinces and town names. There are 366 observations of the district (town) vote counts for each of the four candidates. The total number of votes recorded in each district range from 3,488 to 4,114,384. Such counts are not particularly useful for several of the diagnostics I have been studying as ways to assess possible problems in vote counts. Ideally vote counts for each polling station would be available.

(added June 17) Shortly after I completed the original version of this report, Dr. Bean sent me a file, supposedly downloaded from the same source, containing district-level vote counts for the second round of the 2005 presidential election. The candidates in that contest were Mahmoud Ahmadinejad and Akbar Hashemi Rafsanjani. There are 325 observations of district (town) vote counts for each of the two candidates. I was able to match 320 districts with the same names across years. This allows the 2005 election results to be used to study what happened in 2009. Results from such analysis follow the text describing analysis completed for the original version of this report.

(added June 18) Most recently Dr. Bean has sent me a file containing the town-level vote counts from the first round of the 2005 election.[3] These data include vote counts for seven candidates: Mahmoud Ahmadinejad, Mohammad Bagher Ghalibaf, Mehdi Karroubi, Ali Larijani, Mohsen Mehralizadeh, Mostafa Moeen and Akbar Hashemi Rafsanjani. These data are the basis for the additional analysis newly reported in the current version of this report.

(added June 19) A new version of the 2009 data was posted[4] that contained slightly different counts in seven observations. I have rerun all the analysis using these updated values.

(added June 20) Polling station (or "ballot box") level data have started to become available.[5] I report second-digit Benford's Law test results for these data.

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[1] Richard Bean <richard.bean@gmail.com>, ROAM Consulting Pty Ltd, Toowong QLD 4066, Australia.

[2] At http://www.moi.ir/Portal/File/ShowFile.aspx?ID=0793459f-18c3-4077-81ef-b6ead48a5065.

[3] Source http://www.shora-gc.ir/portal/siteold/amar/reyast_jomhoriy/entekhabat_9/.

[4] http://www.moi.ir/ostan.xls.

[5] http://moi.ir/Portal/Home/ShowPage.aspx?Object=News&ID=3a120d23-ac85-4ce8-9312-74f62edc27e4&LayoutID=b05ef124-0db1-4d33-b0b6-90f50139044b&CategoryID=832a711b-95fe-4505-8aa3-38f5e17309c9.

With the town-level aggregates (for example, all of Tehran town, with 4,114,384 total votes, is included in one observation), tests such as tests of the distribution of the digits in the counts against the distribution expected according to Benford's Law are not particularly diagnostic: the large aggregates mix together such a heterogeneity of local events that conformity with Benford's Law is to be expected even if locally there are many problems. Conformity with Benford's Law is often treated as a marker for unproblematic results.

Nonetheless, pending the availability of less highly aggregated counts, it is easy to test the currently available data. A natural test is to check the distribution of the vote counts' second significant digits against the distribution expected by Benford's Law (Mebane 2008). Such a test for the full set of counts for each candidate shows no significant deviations from expectations. In the following table, a test statistic χ^2_{2BL} greater than 21.03 would indicate a deviation significant at the .05 test level (taking multiple testing—four candidates—into account; the critical value for the .10-level test would be 19.0).

Table 1: 2BL Test Statistics (Pearson chi-squared)

candidate	χ^2_{2BL}
Ahmadinejad	6.96
Rezaei	17.08
Karroubi	6.62
Mousavi	13.34

The single statistic value of $\chi^2_{2BL} = 17.08$ for Rezaei is significant at the .05 level if the fact that statistics for four candidates are being tested is ignored. So a statistically sharp approach to statistical testing—taking the multiple testing into account—fails to provide evidence against the hypothesis that the second digits are distributed according to Benford's Law. Tests based on the means of the second digits also fail to suggest any deviation from the second-digit Benford's Law distribution. But arguably, in view of the χ^2_{2BL} result for Rezaei, it's a bit of a close call. Given the large aggregates being analyzed, such a close result warrents further examination.

Another obvious analysis is to check for outliers. Having no observed variables other than the vote counts, the range of possible models is severely limited. I consider two kinds of analysis. First is simply to fit an overdispersed binomial regression model (Mebane and Sekhon 2004) to the vote counts for Ahmadinejad and Mousavi. The point is to see whether any outliers in the analysis correspond to places one would expect to be discrepant from a specification that implies the proportion of votes for Ahmadinejad is constant across all districts. The model gives the following estimates.

Table 2: Robust Overdispersed Binomial Regression Model, Ahmadinejad versus Mousavi

	coef.	SE
Constant	0.841	0.0267

Note: $\sigma_{\text{LQD}} = 62.0$. $\sigma_{\tanh} = 60.8$. Nine outliers.

Using the logistic function $\text{logistic}(x) = 1/(1+\exp(-x))$ we recover an average proportion for

Ahmadinejad of 0.700 = logistic(0.842). The nine outlier observations correspond to places not described by this constant-proportion specification. The province, town and observed-minus-expected difference for each of these nine outliers are as follows.

Table 3: Nine Outliers

observation	province	town	obs.−0.7
6	azerbaijan sharghi	tabriz	−0.1893817
87	tehran	tehran	−0.2433982
92	tehran	shemiranat	−0.3609246
95	tehran	karaj	−0.1366040
181	sistan va baluchistan	chabahar	−0.4456234
182	sistan va baluchistan	khash	−0.5262456
186	sistan va baluchistan	zahedan	−0.2352349
188	sistan va baluchistan	saravan	−0.4808019
366	yazd	yazd	−0.2239434

Numbers indicate the observation number in the original data (to facilitate double-checking the province and town transliterations). Someone who knows something about the political geography of Iran (which I do not) can say whether these exceptions are reasonable. Another 45 observations are downweighted by the estimation algorithm, which indicates the constant-proportion hypothesis works poorly for them as well (these observations are listed in the accompanying output file `mrob1.Rout`).

The last analysis adds the second significant digits of the candidates' vote counts as conditioning variables in the overdispersed binomial regression model. The digits from the Ahmadinejad counts are not significantly related to the support proportion, but the digits from the Mousavi counts are.

Table 4: Robust Overdispersed Binomial Regression Model with 2d Digits, Ahmadinejad versus Mousavi

	coef.	SE
Constant	0.9260	0.04770
Mousavi 2d Digit	−0.0195	0.00932

Note: $\sigma_{\text{LQD}} = 61.9$. $\sigma_{\tanh} = 61.3$. Eight outliers.

Such an association is another oddity that warrants further investigation once more data are available.

Further details and miscellaneous results are in the associated files `dbenf1.R`, `dbenf1.Rout`, `mrob1.R`, `mrob1.Rout`.

June 16 and June 17, 2009 updates

First I should emphasize a point about the analysis reported in Table 2 above. Not only the nine outliers listed in Table 4 but all 54 of the downweighted observations have observed vote

proportions for Ahmadinejad that are less than the robust average proportion of 0.700. Normally we might expect at least of few of the downweighted observations would be unusually large. But in 2009 that does not happen.

Contrast the results of a similar analysis performed using the data from the second stage of the 2005 election. A specification that implies the proportion of votes for Ahmadinejad is constant across all districts gives the following estimates.

Table 5: Robust Overdispersed Binomial Regression Model, 2005 Second Stage, Ahmadinejad versus Rafsanjani

	coef.	SE
Constant	0.623	0.0246

Note: $\sigma_{\text{LQD}} = 48.0$. $\sigma_{\tanh} = 43.6$. Six outliers.

Using the logistic function $\text{logistic}(x) = 1/(1 + \exp(-x))$ we recover an average proportion for Ahmadinejad of $0.651 = \text{logistic}(0.623)$. The six outlier observations all have negative observed-minus-expected differences, but in all there are 33 downweighted observations and eight of those have positive differences (for details see the associated file `m05091.Rout`). This result further highlights how unusual the 2009 results are.

Conditioning the 2009 vote on two features of the 2005 election produces reasonable results. I use the second-stage 2005 data to construct two variables. First is the proportion of total votes received by Ahmadinejad. We expect towns that heavily supported Ahmadinejad in 2005 should continue to do so in 2009. I use the logit function ($\text{logit}(p) = \log(p/(1-p))$) to transform the proportion p, so that if the political environment were perfectly constant across the years the expected coefficient is 1.0. More generally we would expect the coefficient to be positive. The other variable is the ratio of the total number of votes in 2009 to the total number of votes in 2005. In 2005 some opposition politicians called for a boycott of the election. The surge in turnout in 2009 is widely interpreted as meaning that many who boycotted in 2005 decided to vote in 2009. Hence towns that have high ratios should have lower proportions of the vote for Ahmadinejad (the coefficient should be negative). Estimating a robust overdispersed binomial model for the vote counts for Ahmadinejad and Mousavi, using the observations that could be matched between 2005 and 2009, produces the following results.

Table 6: Robust Overdispersed Binomial Regression Model, Ahmadinejad versus Mousavi, Conditioning on 2005 Election Variables

	coef.	SE
Constant	1.140	0.0738
logit(2005 Ahmadinejad proportion)	0.417	0.0557
ratio(2009 total/2005 total)	−0.383	0.0314

Note: $\sigma_{\text{LQD}} = 50.3$. $\sigma_{\tanh} = 47.6$. Nine outliers.

Both coefficients have the signs we would expect if political processes like those that normally

prevail in election in other places were also at work in the Iranian election of 2009.

Many of the outliers are the same as in Table 4:

Table 7: Nine Outliers, Model that Conditions on 2005 Election Variables

observation	province	town	obs.$-\hat{p}_{i1}$
6	azerbaijan sharghi	tabriz	−0.1792774
37	ardabil	ardabil	−0.2028974
48	esfahan	esfahan	−0.1101559
87	tehran	tehran	−0.2434320
92	tehran	shemiranat	−0.2978053
95	tehran	karaj	−0.1381235
180	sistan va baluchistan	iranshahr	−0.3375769
181	sistan va baluchistan	chabahar	−0.3541439
188	sistan va baluchistan	saravan	−0.4752339

Twelve of the 46 downweighted observations have Ahmadinejad's observed vote share larger than predicted. The complete list of these observations follows in Tables 8 and 9.

Table 8: Downweighted Observations (Negative Residuals), Model that Conditions on 2005 Election Variables

observation	province	town	obs.$-\hat{p}_{i1}$ Ahmadinejad
5	azerbaijan sharghi	bonab	−0.17840989
6	azerbaijan sharghi	tabriz	−0.17927740
10	azerbaijan sharghi	shabestar	−0.21387544
12	azerbaijan sharghi	kalibar	−0.21017793
14	azerbaijan sharghi	marand	−0.20726288
20	azerbaijan gharbi	orumieh	−0.12456624
28	azerbaijan gharbi	khoy	−0.20981065
30	azerbaijan gharbi	salmas	−0.19616815
33	azerbaijan gharbi	maku	−0.41888373
35	azerbaijan gharbi	miandoab	−0.20921384
36	azerbaijan gharbi	naqade	−0.24039120
37	ardabil	ardabil	−0.20289744
39	ardabil	parsabad	−0.31005926
42	ardabil	garmi	−0.23636308
43	ardabil	meshkinshahr	−0.17145602
48	esfahan	esfahan	−0.11015594
76	bushehr	bushehr	−0.16025374
87	tehran	tehran	−0.24343200
90	tehran	rey	−0.08719435
92	tehran	shemiranat	−0.29780531
95	tehran	karaj	−0.13812345
153	khuzestan	behbahan	−0.18377898
180	sistan va baluchistan	iranshahr	−0.33757687
181	sistan va baluchistan	chabahar	−0.35414392
182	sistan va baluchistan	khash	−0.40070519
186	sistan va baluchistan	zahedan	−0.16465359
188	sistan va baluchistan	saravan	−0.47523388
189	sistan va baluchistan	sarbaz	−0.31376867
193	sistan va baluchistan	nikshahr	−0.31181106
275	golestan	turkman	−0.27476928
279	golestan	kalaleh	−0.24994806
281	golestan	gonbade kavus	−0.15988777
338	hormozgan	bastak	−0.28131202
358	yazd	ardekan	−0.29042651

Table 9: Downweighted Observations (Positive Residuals), Model that Conditions on 2005 Election Variables

observation	province	town	obs.$-\hat{p}_{i1}$ Ahmadinejad
159	khuzestan	shadegan	0.21975180
170	zanjan	khodabandeh	0.13292718
184	sistan va baluchistan	zabol	0.11246749
200	fars	jahrom	0.12196380
205	fars	zarrindasht	0.23260102
216	fars	marvdasht	0.12248798
238	kerman	bam	0.21686603
239	kerman	jiroft	0.20280172
247	kerman	anbar abad	0.30234651
304	lorestan	khoramabad	0.07687477
305	lorestan	delfan	0.09218499
348	hormozgan	minab	0.11964161

There have been many allegations about Lorestan. It may be especially remarkable that the two Lorestan observations that are downweighted have Ahmadinejad vote shares that are larger than predicted.

Further details and miscellaneous results are in the associated files `m05091.R`, `m05091.Rout`.

June 18 and June 19, 2009, updates

Using data from the first stage of the 2005 presidential election produces results that I think are much more illuminating than what could be uncovered before. I think the results give moderately strong support for a diagnosis that the 2009 election was affected by significant fraud.

To introduce the analysis, and to give those who know a lot about Iranian politics some basis for evaluating the credibility of the data, first consider a model that conditions the second-stage 2005 vote counts on the results from the first stage of the 2005 election. Once again I use a logit function to transform the set of first-stage vote proportions. In this case the logits are formed as the natural logarithm of the ratio of the proportion of votes for a candidate in each town to the proportion of votes for an arbitrarily chosen reference candidate. I use Mehralizadeh for this reference candidate, so the logits used here for candidate j is defined by

$$\text{logit}_M(p_j) = \log\left(\frac{p_j}{p_{\text{Mehralizadeh}}}\right).$$

Here $j \in \{\text{Ahmadinejad, Ghalibaf, Karroubi, Larijani, Mehralizadeh, Moeen, Rafsanjani}\}$.

Using these variables as regressors (except for the redundant variable $\text{logit}_M(\text{Mehralizadeh})$) gives the following estimates.

Table 10: Robust Overdispersed Binomial Regression Model, Ahmadinejad versus Rafsanjani, Conditioning on 2005 First-stage Election Variables

	coef.	SE
Constant	0.9760	0.0430
logit_M(2005 Ahmadinejad proportion)	0.3140	0.0165
logit_M(2005 Ghalibaf proportion)	0.0407	0.0212
logit_M(2005 Karroubi proportion)	−0.0627	0.0168
logit_M(2005 Larijani proportion)	0.0739	0.0141
logit_M(2005 Moeen, proportion)	−0.1420	0.0208
logit_M(2005 Rafsanjani proportion)	−0.3130	0.0340

Note: $\sigma_{\text{LQD}} = 29.0$. $\sigma_{\tanh} = 25.4$. Nine outliers.

All of the coefficient estimates are statistically significant (except marginally the estimated coefficient for Ghalibaf). At least two of the estimates are immediately reasonable: the largest positive coefficient is the one for Ahmadinejad and the largest negative coefficient is the one for Rafsanjani; in the second-stage contest against Rafsanjani, Ahmadinejad received the strongest support from towns that supported him strongly in the first stage and had the strongest opposition in towns that supported his second-stage opponent. The outliers (Table

Table 11: Nine Outliers, Model for 2005 Second Stage that Conditions on 2005 First-stage Variables

observation	province	town	obs.$-\hat{p}_{i1}$
82	bushehr	deylam	−0.11212621
87	tehran	tehran	−0.19412461
90	tehran	rey	−0.07518791
162	khuzestan	gotvand	−0.14845673
210	fars	fasa	−0.29382639
215	fars	lamerd	−0.11492087
228	kordestan	dehgelan	−0.11320949
268	kohgiluyeh va boyerahmad	boyerahmad	−0.12489951
272	golestan	azadshahr	−0.17898944

11) follow the pattern of Ahmadinejad receiving fewer votes than the model would predict. There are 17 additional downweighted observations (Table 12), five of which have positive residuals.

I use a version of the model of Table 6 in which the single variable logit(2005 Ahmadinejad proportion), which was based on the second-stage 2005 election results, is replaced with the set of variables that measure the first-stage 2005 results. On the way to that model it is convenient first to consider a slightly more complicated version of the model of Table 6 that additionally conditions on the product of the logit(2005 Ahmadinejad proportion) and ratio(2009 total/2005 total) variables. See Table 13. These results show that in places where turnout surged in 2009, strong support for Ahmadinejad in the second-stage of the 2005 election no longer indicated strong support in the 2009 election. Such a pattern represents a plausible refinement of the Table 6 results. The set of outliers (Table 14) overlaps with those associated with the Table 6 model: All of the outliers once again have negative residuals. The set of downweighted observations resembles those found using the Table 6 model. To see those values refer to the associated file `m05091c.Rout`.

In addition to replacing logit(2005 Ahmadinejad proportion) as a regressor with the set of variables from the model of Table 10 that measure the first-stage 2005 results, I expand the analysis to include all four of the 2009 candidates. In this case we estimate coefficients for three of the candidates, treating the fourth candidate (Mousavi) as the reference category that has coefficients normalized to zero. Estimates from this model are in Table 15.

Table 12: Downweighted Observations (Negative Residuals), Model that Conditions on 2005 Election Variables

observation	province	town	obs.$-\hat{p}_{i1}$ Ahmadinejad
31	azerbaijan gharbi	shahindezh	−0.23673656
45	ardabil	nir	−0.04771238
65	esfahan	nain	−0.10157632
88	tehran	damavand	−0.05100643
98	chahar mahal va bakhtiari	ardal	0.11629402
105	khorasan junubi	boshruyeh	0.14925666
132	khorasan razavi	kashmar	−0.04512956
158	khuzestan	ramhormoz	0.12683604
159	khuzestan	shadegan	−0.15973883
160	khuzestan	shush	−0.16477504
178	semnan	garmsar	−0.07221767
194	fars	abadeh	0.05409334
207	fars	sarvestan	−0.15164320
211	fars	firuzabad	−0.15239005
304	lorestan	khoramabad	0.11297023
305	lorestan	delfan	−0.19213209
325	mazandaran	noshahr	−0.05340309

Table 13: Robust Overdispersed Binomial Regression Model, Ahmadinejad versus Mousavi, Conditioning on 2005 Election Variables with Interaction Term

	coef.	SE
Constant	1.0800	0.0858
logit(2005 Ahmadinejad proportion)	0.5530	0.1170
ratio(2009 total/2005 total)	−0.3610	0.0375
logit × ratio	−0.0649	0.0555

Note: $\sigma_{\text{LQD}} = 50.2$. $\sigma_{\tanh} = 47.8$. Eight outliers.

Table 14: Eight Outliers, Model that Conditions on 2005 Election Variables with Interaction Term

observation	province	town	obs.$-\hat{p}_{i1}$
6	azerbaijan sharghi	tabriz	−0.1769599
37	ardabil	ardabil	−0.2005185
48	esfahan	esfahan	−0.1112957
87	tehran	tehran	−0.2400696
92	tehran	shemiranat	−0.2898809
95	tehran	karaj	−0.1352988
180	sistan va baluchistan	iranshahr	−0.3378426
188	sistan va baluchistan	saravan	−0.4639516

Table 15: Robust Overdispersed Multinomial Regression Model, All Four Candidates, Conditioning on 2005 First-stage Election Variables with Interaction Terms

	Ahmadinejad		Rezaei		Karroubi	
	coef.	SE	coef.	SE	coef.	SE
Constant	1.40000	0.2290	−3.63000	0.3150	−4.4200	0.3560
ratio(2009 total/2005 total)	−0.60200	0.1590	0.07160	0.2160	0.2720	0.2340
logit_M(2005 Ahmadinejad prop)	0.23200	0.0517	0.49400	0.1110	0.5970	0.0977
logit_M(2005 Ghalibaf prop)	0.19400	0.0888	−0.10900	0.1490	−0.3760	0.1720
logit_M(2005 Karroubi prop)	0.11100	0.0900	0.44700	0.1390	−0.0359	0.1410
logit_M(2005 Larijani prop)	−0.01090	0.0792	0.00296	0.1160	−0.3120	0.1440
logit_M(2005 Moeen, prop)	−0.80400	0.0978	−0.46300	0.1630	0.5910	0.1800
logit_M(2005 Rafsanjani prop)	0.26300	0.1500	−0.26800	0.2470	−0.1950	0.2740
ratio × Ahmadinejad	−0.10400	0.0366	−0.06820	0.0778	−0.3970	0.0629
ratio × Ghalibaf	0.05890	0.0576	0.15200	0.1050	0.2250	0.1170
ratio × Karroubi	0.00672	0.0663	−0.24200	0.0990	0.3570	0.0916
ratio × Larijani	0.02780	0.0579	−0.00318	0.0814	0.2440	0.1000
ratio × Moeen	0.24400	0.0726	0.14200	0.1220	−0.4820	0.1320
ratio × Rafsanjani	−0.17900	0.1080	0.08070	0.1730	−0.0535	0.1850

Note: $\sigma_{\text{LQD}} = 17.9$. $\sigma_{\text{tanh}} = 15.8$. 81 outliers.

Evidently the model is complicated, but it is easy to see that several aspects of the results seem natural. Places that strongly supported Ahmadinejad in the first stage of the 2005 election tended to support him in 2009 (the coefficient equal to 0.232), but in places where turnout surged in 2009, strong support for Ahmadinejad in the second-stage of the 2005 election no longer indicated strong support in the 2009 election (the coefficient equal to -0.104). Places that strongly supported Karroubi in 2005 tended strongly to support him in 2009, especially so as 2009 turnout surged above 2005 levels.

What is most striking about these results, however, is the large number of outliers. One might expect that given the increased political resolution provided by having measures of the first-stage candidates' support, combined with the turnout ratio variable interactions, the model would do a good job capturing more of the variations in the 2009 vote. In fact, however, 60 of the 81 outliers[6] represent vote counts for Ahmadinejad that the model wholly fails to describe (for details, see Table 16). Among those outliers 35 observations have positive residuals—Ahmadinejad did much better than the model predicts—and 25 have negative residuals. There are 21 observations with outliers tied to the votes for Rezaei and Karroubi, all with positive residuals. In all the estimation algorithm downweights 189 observations, and 172 of those involve poorly modeled vote counts for Ahmadinejad (see Table 17). Of the downweighted observations, 115 have positive residuals for Ahmadinejad. There are 44 observations where downweighting is tied to the votes for Rezaei or Karroubi, and all but onetwo of those instances involves a positive residual for the affected candidate.

More than half of the 320 towns included in this part of the analysis exhibit vote totals for Ahmadinejad that are not well described by the natural political processes the model of Table 15 represents. These departures from the model much more often represent additions than declines in the votes reported for Ahmadinejad. Correspondingly the poorly modeled observations much more often represent declines than additions in the votes reported for Mousavi.

Further details and miscellaneous results are in the associated files `m05.R`, `m05.Rout`, `m05091c.R`, `m05091c.Rout`, `m05R1091b.R` and `m05R1091b.Rout`.

[6]More than one outlier may occur for each observation because for each observation there are three independent proportions. The 81 outliers are spread over 73 different observations.

Table 16: 81 Outliers, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
4	azerbaijan sharghi	bostan abad	0.2197	—	—
13	azerbaijan sharghi	maraqeh	0.1680	—	—
16	azerbaijan sharghi	mianeh	0.1960	—	—
33	azerbaijan gharbi	maku	−0.2640	—	—
35	azerbaijan gharbi	miandoab	—	—	0.0227
37	ardabil	ardabil	−0.1523	—	—
39	ardabil	parsabad	−0.1772	—	—
42	ardabil	garmi	−0.2137	—	—
50	esfahan	tiran va karvan	0.1922	—	—
52	esfahan	khomeinishahr	0.1075	—	—
60	esfahan	falavarjan	0.1653	—	—
64	esfahan	mobarakeh	0.1548	—	—
66	esfahan	najafabad	0.1145	—	—
68	ilam	abdanan	—	—	0.0559
87	tehran	tehran	−0.1614	—	0.0063
88	tehran	damavand	—	—	0.0250
90	tehran	rey	−0.0679	—	—
92	tehran	shemiranat	−0.2162	—	0.0113
95	tehran	karaj	−0.0857	—	—
102	chahar mahal va bakhtiari	koohrang	—	0.2847	—
116	khorasan razavi	taybad	0.1486	—	—
119	khorasan razavi	torbat e heydarieh	−0.1112	—	—
128	khorasan razavi	sabzevar	−0.1092	—	—
129	khorasan razavi	sarakhs	0.1721	—	—
135	khorasan razavi	mashhad	−0.0695	—	—
148	khuzestan	andimeshk	−0.1170	0.0357	—
149	khuzestan	ahvaz	—	0.0265	—
150	khuzestan	izeh	−0.1578	0.2952	—
151	khuzestan	baqe malek	—	0.0378	—
153	khuzestan	behbahan	−0.1390	—	—
155	khuzestan	dezful	—	0.0569	—
159	khuzestan	shadegan	0.1862	—	—
161	khuzestan	shushtar	−0.1294	0.1262	—
163	khuzestan	lali	−0.3836	0.6176	—
164	khuzestan	masjed soleiman	−0.2279	0.3240	—
170	zanjan	khodabandeh	0.2171	—	—
172	zanjan	zanjan	0.0725	—	—
174	zanjan	mahneshan	0.2526	—	—
178	semnan	garmsar	0.1829	—	—

Table 16 (cont'd): 81 Outliers, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
184	sistan va baluchistan	zabol	0.1670	—	—
200	fars	jahrom	0.0951	—	—
208	fars	shiraz	−0.0498	—	—
211	fars	firuzabad	0.1896	—	—
216	fars	marvdasht	0.1028	—	—
217	fars	mamasani	−0.1807	—	—
223	qazvin	takestan	0.1141	—	—
233	kordestan	ghorveh	0.1595	—	—
236	kerman	baft	0.1192	—	—
241	kerman	rafsanjan	−0.1618	—	—
247	kerman	anbar abad	0.1121	—	—
249	kerman	kerman	−0.0644	—	—
263	kermanshah	kermanshah	−0.0776	—	—
295	gilan	tavalesh	0.2067	—	—
300	lorestan	azna	—	—	0.0355
301	lorestan	aligodarz	−0.1502	—	0.1971
304	lorestan	khoramabad	—	—	0.0148
305	lorestan	delfan	—	—	0.0649
308	lorestan	koohdasht	—	—	0.0549
309	mazandaran	amol	0.0914	—	—
310	mazandaran	babol	0.0679	—	—
333	markazi	saveh	0.1042	—	—
337	hormozgan	abu musa	—	0.1659	—
340	hormozgan	bandar abbas	0.1166	—	—
345	hormozgan	rudan	0.2301	—	—
348	hormozgan	minab	0.3206	—	—
350	hamedan	bahar	0.1596	—	—
351	hamedan	toyserkan	—	—	0.0302
352	hamedan	razan	0.1763	—	—
353	hamedan	kabudrahang	0.1265	—	—
354	hamedan	malayer	0.0852	—	—
358	yazd	ardekan	−0.1762	—	—
364	yazd	mehriz	−0.1728	—	—
366	yazd	yazd	0.3465	—	—

Table 17: Downweighted Observations, Model that Conditions on 2005 First-stage Election Variables

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
1	azerbaijan sharghi	azar shahr	0.1345	—	—
3	azerbaijan sharghi	ahar	0.0942	—	—
4	azerbaijan sharghi	bostan abad	0.2197	—	—
7	azerbaijan sharghi	jolfa	0.1415	—	—
9	azerbaijan sharghi	sarab	0.1244	—	—
11	azerbaijan sharghi	ajabshir	0.1154	—	—
13	azerbaijan sharghi	maraqeh	0.1680	—	—
14	azerbaijan sharghi	marand	0.0691	—	—
16	azerbaijan sharghi	mianeh	0.1960	—	—
19	azerbaijan sharghi	varzaghan	0.1034	—	—
28	azerbaijan gharbi	khoy	−0.0578	—	—
30	azerbaijan gharbi	salmas	−0.1114	—	—
33	azerbaijan gharbi	maku	−0.2640	—	—
35	azerbaijan gharbi	miandoab	−0.0490	—	0.0227
36	azerbaijan gharbi	naqade	−0.1335	—	—
37	ardabil	ardabil	−0.1523	—	—
38	ardabil	bilesavar	−0.1389	—	—
39	ardabil	parsabad	−0.1772	—	—
40	ardabil	khalkhal	0.0834	—	—
41	ardabil	kowsar	0.1529	—	—
42	ardabil	garmi	−0.2137	—	—
46	esfahan	aran va bidgol	−0.0636	—	—
48	esfahan	esfahan	−0.0195	—	—
50	esfahan	tiran va karvan	0.1922	—	—
52	esfahan	khomeinishahr	0.1075	—	—
55	esfahan	samirom	0.0764	—	—
60	esfahan	falavarjan	0.1653	—	—
63	esfahan	lanjan	0.0484	0.0227	—
64	esfahan	mobarakeh	0.1548	—	—
66	esfahan	najafabad	0.1145	—	—
68	ilam	abdanan	—	—	0.0559
71	ilam	darrehshahr	—	—	0.0243
72	ilam	dehloran	−0.1167	—	0.0244
76	bushehr	bushehr	−0.0763	—	—
79	bushehr	dashtestan	−0.0701	—	—
87	tehran	tehran	−0.1614	—	0.0063
88	tehran	damavand	−0.1000	—	0.0250
90	tehran	rey	−0.0679	—	—
92	tehran	shemiranat	−0.2162	—	0.0113
95	tehran	karaj	−0.0857	—	0.0037
96	tehran	nazarabad	0.0665	—	—

Table 17 (cont'd): Downweighted Observations, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
98	chahar mahal va bakhtiari	ardal	0.1186	0.0329	—
100	chahar mahal va bakhtiari	shahrekord	—	0.0117	0.0074
101	chahar mahal va bakhtiari	farsan	—	0.0336	—
102	chahar mahal va bakhtiari	koohrang	—	0.2847	—
104	chahar mahal va bakhtiari	lordegan	0.0600	0.0228	—
109	khorasan junubi	sarbisheh	0.1086	—	—
111	khorasan junubi	qaenat	0.0964	—	—
112	khorasan junubi	nehbandan	0.1202	—	—
116	khorasan razavi	taybad	0.1486	—	—
118	khorasan razavi	torbat e jam	0.1000	—	—
119	khorasan razavi	torbat e heydarieh	−0.1113	—	—
126	khorasan razavi	roshtkhar	0.1121	—	—
128	khorasan razavi	sabzevar	−0.1093	—	—
129	khorasan razavi	sarakhs	0.1721	—	—
130	khorasan razavi	fariman	0.1014	—	—
132	khorasan razavi	kashmar	−0.0433	—	—
133	khorasan razavi	kalat	0.1925	—	—
134	khorasan razavi	gonabad	—	—	0.0147
135	khorasan razavi	mashhad	−0.0695	—	—
141	khorasan shomali	shirvan	0.0645	—	—
142	khorasan shomali	faruj	0.1199	—	—
148	khuzestan	andimeshk	−0.1170	0.0357	—
149	khuzestan	ahvaz	−0.0251	0.0265	—
150	khuzestan	izeh	−0.1578	0.2952	—
151	khuzestan	baqe malek	0.0998	0.0378	—
152	khuzestan	bandar mahshahr	—	0.0155	—
153	khuzestan	behbahan	−0.1390	—	—
155	khuzestan	dezful	−0.0516	0.0569	—
158	khuzestan	ramhormoz	—	0.0200	—
159	khuzestan	shadegan	0.1862	—	—
161	khuzestan	shushtar	−0.1294	0.1262	—
163	khuzestan	lali	−0.3836	0.6176	—
164	khuzestan	masjed soleiman	−0.2279	0.3240	—
168	zanjan	abhar	0.0996	—	—
169	zanjan	ijrood	0.2008	—	—
170	zanjan	khodabandeh	0.2171	—	—
171	zanjan	khoramdareh	0.0864	—	—
172	zanjan	zanjan	0.0725	—	—
173	zanjan	tarom	0.1759	—	—
174	zanjan	mahneshan	0.2526	—	—

Table 17 (cont'd): Downweighted Observations, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
177	semnan	shahrood	0.0495	—	—
178	semnan	garmsar	0.1829	—	—
180	sistan va baluchistan	iranshahr	−0.0776	—	—
181	sistan va baluchistan	chabahar	−0.0722	—	0.0135
182	sistan va baluchistan	khash	−0.1343	—	—
184	sistan va baluchistan	zabol	0.1670	—	—
188	sistan va baluchistan	saravan	−0.0583	—	—
194	fars	abadeh	0.1114	—	—
195	fars	arsanjan	0.1130	—	—
198	fars	bavanat	0.0978	—	—
199	fars	pasargad	0.1963	—	—
200	fars	jahrom	0.0951	—	—
203	fars	darab	0.0572	—	—
205	fars	zarrindasht	0.1414	—	—
208	fars	shiraz	−0.0498	−0.0020	—
209	fars	farashband	0.1727	—	—
211	fars	firuzabad	0.1896	—	—
212	fars	ghirokarzin	0.0870	—	—
213	fars	kazerun	0.0760	—	—
214	fars	larestan	0.0497	—	—
215	fars	lamerd	0.1442	—	—
216	fars	marvdasht	0.1028	—	—
217	fars	mamasani	−0.1807	—	—
218	fars	mehr	0.1240	—	—
219	fars	neyriz	0.0772	—	—
220	qazvin	abyek	0.0889	—	—
221	qazvin	alborz	0.0726	—	—
222	qazvin	buin zahra	0.0849	—	—
223	qazvin	takestan	0.1141	—	—
224	qazvin	qazvin	0.0419	—	—
225	qom	qom	—	0.0098	—
227	kordestan	bijar	0.0941	—	—
229	kordestan	diwandarreh	0.1402	—	—
231	kordestan	saghz	−0.0940	—	—
232	kordestan	sanandaj	0.0451	—	—
233	kordestan	ghorveh	0.1595	—	—
236	kerman	baft	0.1192	—	—
239	kerman	jiroft	0.0887	—	—
241	kerman	rafsanjan	−0.1618	—	—
244	kerman	zarand	−0.0894	—	—
247	kerman	anbar abad	0.1121	—	—
249	kerman	kerman	−0.0644	—	—

Table 17 (cont'd): Downweighted Observations, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
253	kermanshah	eslamabad gharb	−0.0874	—	—
256	kermanshah	javanrood	−0.1042	—	—
258	kermanshah	ravansar	−0.1239	—	—
260	kermanshah	songhor	0.0648	—	—
263	kermanshah	kermanshah	−0.0776	—	−0.0026
264	kermanshah	kangavar	—	—	0.0183
268	kohgiluyeh va boyerahmad	boyerahmad	−0.0498	—	—
269	kohgiluyeh va boyerahmad	dena	0.0819	—	—
270	kohgiluyeh va boyerahmad	kohgiluyeh	—	0.0220	—
272	golestan	azadshahr	0.0688	—	—
274	golestan	bandar gaz	0.0950	—	—
275	golestan	turkman	−0.1298	—	0.0292
276	golestan	ramiyan	0.1220	—	—
279	golestan	kalaleh	−0.1059	—	—
280	golestan	gorgan	−0.0414	—	—
281	golestan	gonbade kavus	−0.0930	—	—
283	golestan	minoodasht	0.0955	—	—
289	gilan	rezvanshahr	0.1437	—	—
292	gilan	siahkal	0.0975	—	—
293	gilan	shaft	0.1201	—	—
294	gilan	somehsara	0.1082	—	—
295	gilan	tavalesh	0.2067	—	—
296	gilan	fuman	0.0979	—	—
299	gilan	masal	0.1851	—	—
300	lorestan	azna	—	—	0.0356
301	lorestan	aligodarz	−0.1502	—	0.1971
303	lorestan	pol dokhtar	—	—	0.0205
304	lorestan	khoramabad	—	—	0.0148
305	lorestan	delfan	0.0593	—	0.0649
306	lorestan	doroud	—	—	0.0137
307	lorestan	selseleh	0.1386	—	—
308	lorestan	koohdasht	−0.0728	—	0.0549

Table 17 (cont'd): Downweighted Observations, Model that Conditions on 2005 First-stage Election Variables with Interaction Term

obs#	province	town	Ahmadinejad obs.$-\hat{p}_{i1}$	Rezaei obs.$-\hat{p}_{i2}$	Karoubi obs.$-\hat{p}_{i3}$
309	mazandaran	amol	0.0914	—	—
310	mazandaran	babol	0.0679	—	—
313	mazandaran	tonekabon	−0.0617	—	—
314	mazandaran	jooybar	0.0821	—	—
315	mazandaran	chalus	—	—	0.0095
317	mazandaran	sari	0.0612	—	—
318	mazandaran	savad kooh	0.1322	—	—
320	mazandaran	qaem shahr	0.0677	—	—
323	mazandaran	neka	0.0975	—	—
328	markazi	tafresh	0.0829	—	—
329	markazi	khomein	0.0608	—	—
332	markazi	zarandieh	0.1188	—	—
333	markazi	saveh	0.1042	—	—
334	markazi	sarband	0.1015	—	—
335	markazi	komijan	0.1405	—	—
337	hormozgan	abu musa	—	0.1659	—
340	hormozgan	bandar abbas	0.1166	—	—
343	hormozgan	hajiabad	0.1067	—	—
345	hormozgan	rudan	0.2301	—	—
348	hormozgan	minab	0.3206	—	—
349	hamedan	asadabad	0.0652	—	0.0150
350	hamedan	bahar	0.1596	—	—
351	hamedan	toyserkan	0.1316	—	0.0302
352	hamedan	razan	0.1763	—	—
353	hamedan	kabudrahang	0.1265	—	—
354	hamedan	malayer	0.0852	—	—
355	hamedan	nahavand	0.0984	—	0.0094
356	hamedan	hamedan	0.0382	—	—
358	yazd	ardekan	−0.1762	—	—
360	yazd	taft	−0.1283	—	—
362	yazd	sadugh	−0.2150	—	—
363	yazd	tabas	0.0827	—	—
364	yazd	mehriz	−0.1728	—	—
366	yazd	yazd	0.3465	—	—

June 20, June 22, June 23 and June 24 2009, updates

All of the ballot box data have been released (not all of it yet in final form). The data give very strong support for a diagnosis that the 2009 election was affected by significant fraud.

Several people have given me ballot box data. I currently have data for all 30 provinces:

```
> pnames;
 [1] "Qazvin"                    "Guilan"
 [3] "Isfahan"                   "Khorasan Razavi"
 [5] "Sistan"                    "Zanjan"
 [7] "ardabil"                   "azarbaijan gharbi"
 [9] "azarbaijan sharghi"        "bushehr"
[11] "chahar mahal va bakhtiari" "fars"
[13] "ghom"                      "golestan"
[15] "hamedan"                   "hormozgan"
[17] "ilam"                      "kerman"
[19] "kermanshah"                "khorasan jonoobi"
[21] "khorasan shomali"          "khuzestan"
[23] "kohgilooye va boyerahmad"  "kordestan"
[25] "lorestan"                  "markazi"
[27] "mazandaran"                "semnan"
[29] "tehran"                    "yazd"
```

I ran the second-digit Benford's Law (2BL) tests described in Mebane (2006). That paper also gives a rationale for ignoring the first digits. Additionally, since writing that paper I've seen many cases where the first digits are way off but the second digits are okay.

There are at least two ways to consider the results: (1) pooling all candidates together; and (2) treating the candidates separately. In addition one can look at each province separately or consider all the provinces at once. The choices would depend on which hypotheses one wished to test.

I won't get into possible reasons for choosing different test sets, but I will point out, again, that conventional statistical theory requires that one take the number of tests being done into account. Pooling over all provinces, by using (1) there is one test and by using (2) there are four tests. The following table lays out a few relevant critical values for each of these cases.

Table 18: Critical Values to Use with Second-digit Benford's Law (2BL) Tests

Confidence level:	99%	95%	90%	80%
one test	21.66599	16.91898	14.68366	12.24215
four tests	25.46248	21.03407	19.02277	16.91898

2BL test results for the ballot-box data are in `benf5all.Rout`. Pulling out the results when pooling all provinces, we have for (1)

```
chiB = 85.22637 (significant with 99% confidence)
nobs = 118229
```

and for (2):

```
              nobs      chiB
Mousavi      44721   9.868244
Karoubi       9441 315.130703
Rezaei       18437 155.298108
Ahmadinejad  45630  57.704875
```

`chiB` denotes the test statistic and `nobs` denotes the number of observations with a vote count of ten or greater (so there is a second digit). The tests are insignificant for Mousavi and highly significant—well beyond 99 percent confidence—for the other three candidates. The results for both Karroubi and Rezaei reflect the very small proportions of the votes each received. This might be caused by either (a) inherently low levels of support, (b) voters strategically abandoning the candidates, or (c) fraudulent counts. If there is good reason to believe either (a) or (b), then (c) is less likely.

Clearly the counts for Karoubi and Rezaei are contributing heavily to the single-test result. Above I report there is a huge number of outliers only when all four candidates are included (in the model of Table 15). So much seems to be riding on what one thinks about what's going on with Karoubi and Rezaei.

Some evidence bearing on this point appears in Figure 1. Allegations regarding the election include assertions that many ballot boxes were sealed before they could be inspected by opposition candidates' representatives (Erdbrink and Branigin 2009). This opens the possibility of ballot box stuffing or other vote tampering. Figure 1 shows how the proportion of ballots that are deemed invalid (or "void") relates to the second digits of the ballot-box vote counts. The solid line shows a nonparametric regression curve, surrounded by dashed lines representing 95% confidence bounds.[7] The dotted horizontal lines show the mean value expected for the the second digits according to Benford's Law. In the Figure values of the proportion invalid greater than .04 are not shown.[8]

The patterns in Figure 1 strongly suggest there was ballot box stuffing. The average of the second digits in the vote counts for Karoubi and Rezaei fall significantly below the expected mean value for invalid vote proportions less than about 0.03. The average of the second digits in Ahmadinejad's vote counts fall significantly above the expected mean value for invalid vote proportions less than about 0.003.[9] Mousavi's vote counts have second digits significantly greater than the expected mean value for invalid proportions ranging between about 0.02 and 0.025. Outside of these intervals, all the candidates' vote counts have second digits that do not differ significantly from the mean value according to Benford's Law. Considering that overall Ahmadinejad's vote counts have second digits that differ very significantly from the Benford's Law expectations, the small range of departures for Ahmadinejad's votes indicates that the ballot boxes that have very few invalid ballots have great influence on the overall

[7]To estimate the nonparametric regressions I use the package `sm` (Bowman and Azzalini 1997) for the statistical programming environment **R** (R Development Core Team 2005).

[8]The confidence bounds expand greatly and always include the expected mean value for higher invalid proportion values.

[9]Added June 26, 2009: Note that the proportion of invalid votes has first quartile 0.003850, median 0.010240 and third quartile 0.014140. The minimum is zero and the maximum is 0.5.

results. The simplest interpretation is that in many ballot boxes the votes for Karoubi and Rezaei were thrown out while in many ballot boxes extra votes were added for Ahmadinejad.

If we accept the conclusion that such ballot box stuffing occurred—some combination of vote tossing, vote switching and vote adding—then the range of ballot boxes that were most heavily involved in the fraud are those where the proportion of invalid votes is less than about 0.03. That is roughly the threshold below which the digits in the vote counts for Karoubi and Rezaei fall significantly below the mean value expected according to Benford's Law. It is worthwhile, then, to see how the proportions of votes for each candidate line up against the invalid vote proportions. A noticeable change in the candidates' support in the vicinity of this invalid vote proportion threshold might help quantify how many votes were shifted as part of the fraud.

Under this theory, Figure 2 shows that the proportion of votes for each of the candidates does change in ways that suggest that without fraudulent activity the election would have had a very different outcome. The figure shows the nonparametric regression curve of each candidate's vote proportion against the proportion of invalid votes, surrounded by dashed lines representing 95% confidence bounds. Plainly the vote proportions for Mousavi and Karoubi increase as the invalid vote proportion increases into the vicinity of 0.03 to 0.04, after which the curves more or less flatten out. The vote proportions for Rezaei increase for invalid vote proportions up to about 0.02. The vote proportions for Ahmadinejad decrease sharply for invalid vote proportions up to about 0.04, after which the curve flattens out. In the region where their vote curves are relatively flat, Mousavi has vote proportions averaging about 0.45, roughly the same as the average vote proportions for Ahmadinejad. The ballot boxes that typically have substantial margins for Ahmadinejad over Mousavi are all in the range of invalid vote proportions that, based on Figure 1, the Karoubi and Rezaei second digits strongly suggest are affected by fraud. Without the ballot-box stuffing fraud, this analysis suggests, the election outcome should have been at least a runoff between Ahmadinejad and Mousavi. Other kinds of fraud that may have also affected the results are of course not to be ruled out.

June 29 2009, update

Modified conclusion: In general, combining the first-stage 2005 and 2009 data conveys the impression that while natural political processes significantly contributed to the election outcome, outcomes in many towns were produced by very different processes. The natural processes in 2009 have Ahmadinejad tending to do best in towns where his support in 2005 was highest and tending to do worst in towns where turnout surged the most. But in more than half of the towns where comparisons to the first-stage 2005 results are feasible, Ahmadinejad's vote counts are not at all or only poorly described by the naturalistic model. Much more often than not, these poorly modeled observations have vote counts for Ahmadinejad that are greater than the naturalistic model would imply. While it is not possible given only the current data to say for sure whether this reflects natural complexity in the political processes or artificial manipulations, the numerous outliers comport more with the idea that there was widespread fraud than with the idea that all the departures from the model are benign. Additional information of various kinds can help sort out the question. The polling station data show evidence of significant distortions in the vote counts not only

for Karroubi and Rezaei but also for Ahmadinejad. No significant distortions are apparent for Mousavi's vote counts. A key to interpreting these results is understanding why the vote counts for Karroubi and Rezaei are typically so small. Is it (a) inherently low levels of support, (b) voters strategically abandoning the candidates, or (c) fraudulent counts? If there is good reason to believe either (a) or (b), then (c) is less likely. The appearance of an association between invalid ballots and the ballot-box second digits that strongly suggests extensive ballot box stuffing on Ahmadinejad's behalf clearly argues for (c). Checking the proportion of the votes for each candidate suggests that without the ballot-box stuffing fraud the election outcome would have been at least a runoff between Ahmadinejad and Mousavi. The significant results for Ahmadinejad are not direct proof that Ahmadinejad's votes are fraudulent.

Remaining is the need to check all ballot-box data to verify they match the earlier town-level results. More transparency about how the election was conducted would also be useful. Especially information is needed regarding how the ballots were distributed and protected (chain-of-custody) before being counted.

Further details and miscellaneous results are in the associated files `benf5all.R`, `benf5all.Rout`, `invalida.R`, `invalida.Rout`, `invalid1.R` and `invalid1.Rout`.

Caveat: It is important to be clear that none of the estimates or test results in this report are proof that substantial fraud affected the 2009 Iranian election. The results suggest very strongly that there was widespread fraud in which the vote counts for Ahmadinejad were substantially augmented by artificial means. But it is possible that Ahmadinejad actually won, supported by many who might have voted for Karroubi or Rezaei instead voting for Ahmadinejad. The likelihood of such votes being cast needs to be assessed based on information beyond what can be extracted from the 2005 and 2009 election returns alone. To support the benign interpretation, the additional evidence needs to explain how the strong support for Ahmadinejad happens to line up so strongly with the proportion of invalid votes in the ballot-box vote counts.

To emphasize the ambiguity of these assessments, note that similar patterns of both outliers and significant 2BL test statistics can be obtained using data from the 2008 U.S. presidential election. For instance, consider the 2008 votes for the Democratic, Republican and Libertarian candidates and Nader. A county-level robust overdispersed multinomial model regressing the 2008 vote counts on 2004 vote proportions, using data from fourteen states (AR, HI, ID, MD, MN, MT, NM, ND, OH, RI, TN, VT, WI, WY), produces a large number of outliers involving the Democratic candidate. This set of states includes a number of precincts comparable to the number of polling stations in Iran. In contrast to Iran 2009, however, in this set of states neither the Democrat nor the Republican has precinct vote counts with a significant 2BL test statistic (but the third-party candidates do).[10] One way to get a highly significant 2BL test for the major party candidates is to include precincts from CA (as in Mebane 2008). Nothing in the 2008 U.S. data resembles Figures 1 or 2, however.

Tests such as those considered in this paper can in general only identify places where there

[10]The Democratic and Republican precinct vote counts have significant departures from the 2BL expectations in the context of tests on four candidates if false discovery rate corrections (see Mebane 2006) are used instead of only a single Bonferroni adjustment.

may be problems with the votes. In some places the suggestions may be extremely strong (e.g., for recent Russian elections, see Mebane and Kalinin 2009). In general the tests' best use is for screening election results, not confirming or refuting claims of fraud. A significant finding should prompt investigations using administrative records, witness testimony and other facts to try to determine what happened. The problem with the 2009 Iranian election is that the serious questions that have been raised are unlikely to receive satisfactory answers. Transparency is utterly lacking in this case. There is little reason to believe the official results announced in that election accurately reflect the intentions of the voters who went to the polls.

Following are some additional remarks, prompted mainly by comments, questions and challenges I have received.

Remark 1: In earlier versions of this report I mentioned that "fraud is certainly a reasonable inference in light of reports that 'Iran's Guardian Council has admitted that the number of votes collected in 50 cities surpass the number of those eligible to cast ballot in those areas' (Press TV, 2009)." Whether that finding by the Guardian Council is evidence of fraud is disputed. The Guardian Council itself reportedly found nothing suspicious in their investigation, citing the fact that voters in Iran can vote wherever they choose. Such mobility of voters has many implications for the analysis. Details about the mobility sufficient to allow these implications to be investigated are not available (e.g., lists of which voters voted where both in 2009 and 2005, with individual voters matched across time, would be useful). As it is, the purported great mobility of voters helps obscure what happened during the election.

Remark 2: By "ballot box stuffing" I do not mean to exclude the possibility that vote counts reported for some ballot boxes have been faked without any effort to generate corresponding paper ballots. This was implicit in my original discussion, but based on what some have written me I did not make the point clear.

Remark 3: To emphasize the point that additional evidence intended to validate the official results needs to explain how the strong support for Ahmadinejad lines up so strongly with the proportion of invalid votes in the ballot-box vote counts, consider Figure 3. The figure shows the same kinds of nonparametric regression analysis as in Figure 1, except now for invalid vote proportions less than the first quartile value of 0.003850. Rug plots are added to the figure to show where the observed values of the invalid vote proportion actually occur. The increase in Ahmadinejad's average vote proportion as the invalid vote proportion decreases in this interval is very steep. Near the first quartile value (invalid= 0.00385), Ahmadinejad's average vote proportion is about 0.73 while near invalid= 0.0028 it is about 0.76. Roughly a 0.1 percent change in the invalid vote share goes with nearly a three percent change in Ahmadinejad's average vote share. As the invalid vote proportion falls from 0.00385 down to zero, the average Ahmadinejad vote proportion increases steadily up to a value of about 0.78 (at about invalid= 0.0018). So a 0.2 percent change in the invalid vote share goes with nearly a five percent change in Ahmadinejad's average vote share. Below that the average Ahmadinejad vote proportion flattens out if the confidence bounds are taken into account. Such a steep relationship makes it implausible to argue that the relationship between invalid vote proportions and the respective shifts in votes for Ahmadinejad or Mousavi reflects changes in protest votes, i.e., in blank or spoiled ballots cast by people who liked none of the candidates.

Remark 4: The distribution of the values listed as the Total ballots in the ballot box data shows an unusual pattern that at the very least seems to indicate that election administration was not uniform throughout the country. It is not clear what these Total values are supposed to measure (at least, I have not seen a description). If they are intended to show the number of ballots available at each location, then they should all be a multiple of 50: ballots, I've been told, were distributed in bundles of 50. Taking the integer remainder of each Total count after it is divided by 50,[11] we find that about 4.7 percent of the ballot box Total counts end in either 00 or 50, and another 4.8 percent end in either 49, 99, 48 or 98. Table 19 shows these percentages (as proportions), along with statistics that show for each digit combination how much the proportion deviates from an equal-occurrence hypothesized value of 0.02.[12] To make the implications of the table easier to see, the remainder value of zero is shown as 50. With false discovery rate adjustments (see Mebane 2006), the deviations are significant at test level .05 for remainders 50 (i.e., 0), 49, 28 and 48. Does the excess of Total counts that equal or nearly equal a multiple of 50 signal problems with the corresponding ballot boxes, or does the fact that nearly 90 percent of the ballot boxes do not have Total counts near such a multiple signal problems? One allegation about the election is that many of the ballots produced for the election remain unaccounted for.

Remark 5: Checks of the ballot box data against the town-level vote counts so far have revealed a number of minor discrepancies. Until all the data are released in final form, it may be premature to put much emphasis on such discrepancies. I do not know which if any of the data are considered to be the official, certified results according to the Iranian authorities.

Remark 6: By now (June 29, 2009) there are a number of other reports that statistically analyze various features of the 2009 Iran election. I have decided to defer citing or commenting on those, although I know several make more definite claims about the election result than I am currently prepared to do. These reports have had no impact on the analysis reported here.

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[11] The operation is Total mod 50.

[12] The deviation statistics are the signed square roots of the components of a Pearson chi-squared statistic: let p_j be the observed frequency of digit combination j and let N be the number of ballot boxes; the signed square root statistic is $\text{sign}(p_j - 0.02)[N(p_j - 0.02)^2/0.02]^{1/2}$.

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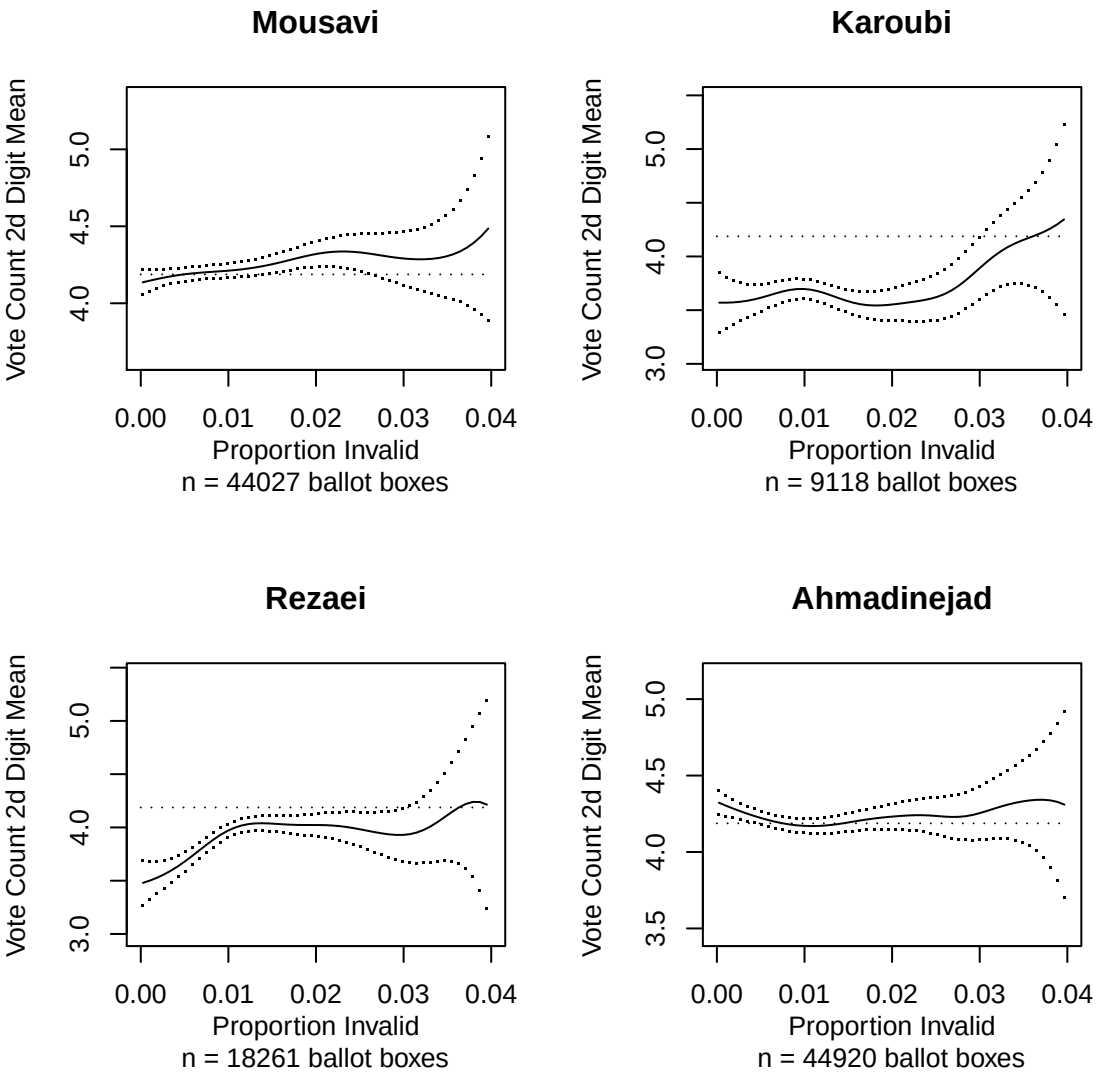

Figure 1: Second Digits of Ballot-Box Vote Counts for President by Proportion of Invalid Votes, 2009

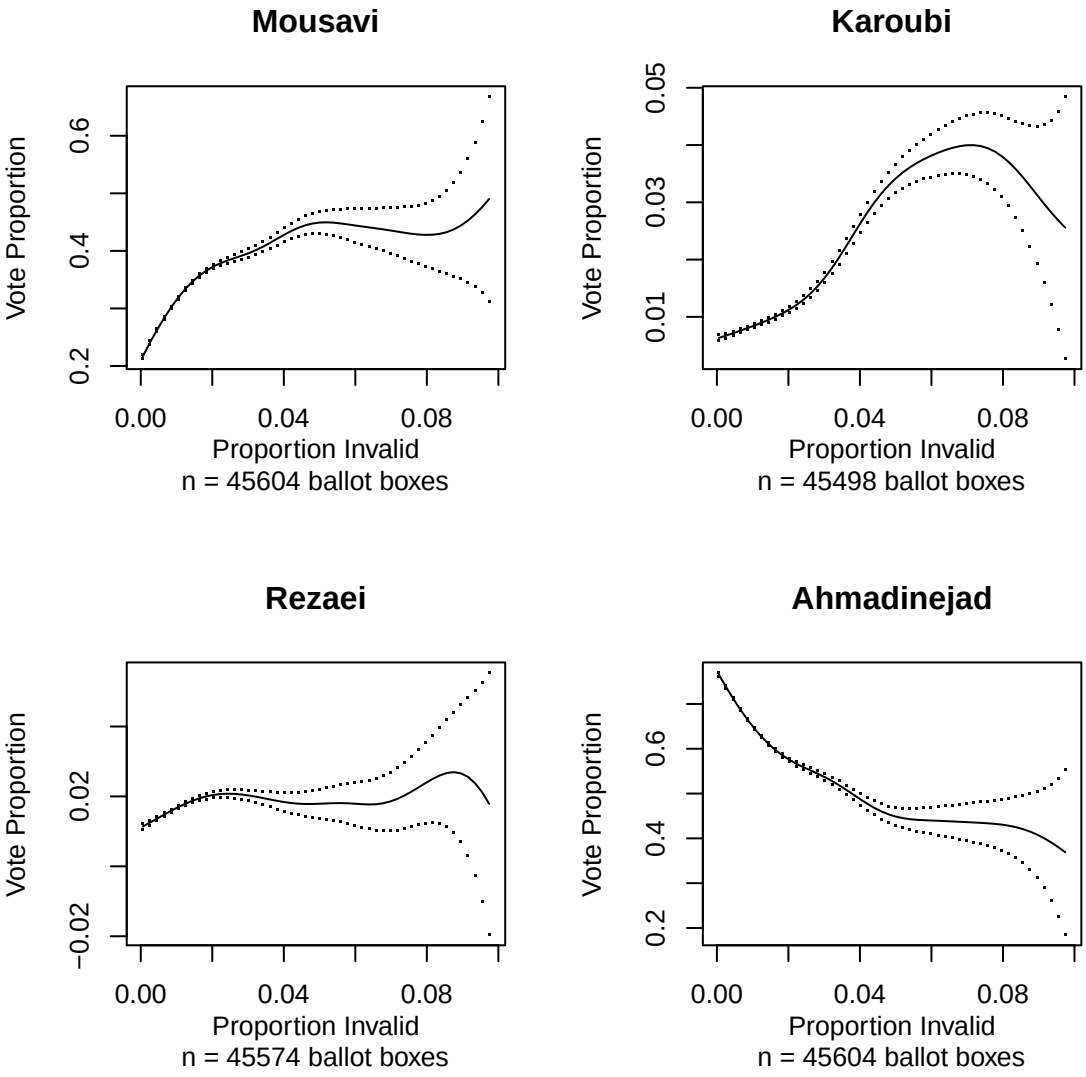

Figure 2: Vote Proportions for President by Proportion of Invalid Votes, 2009

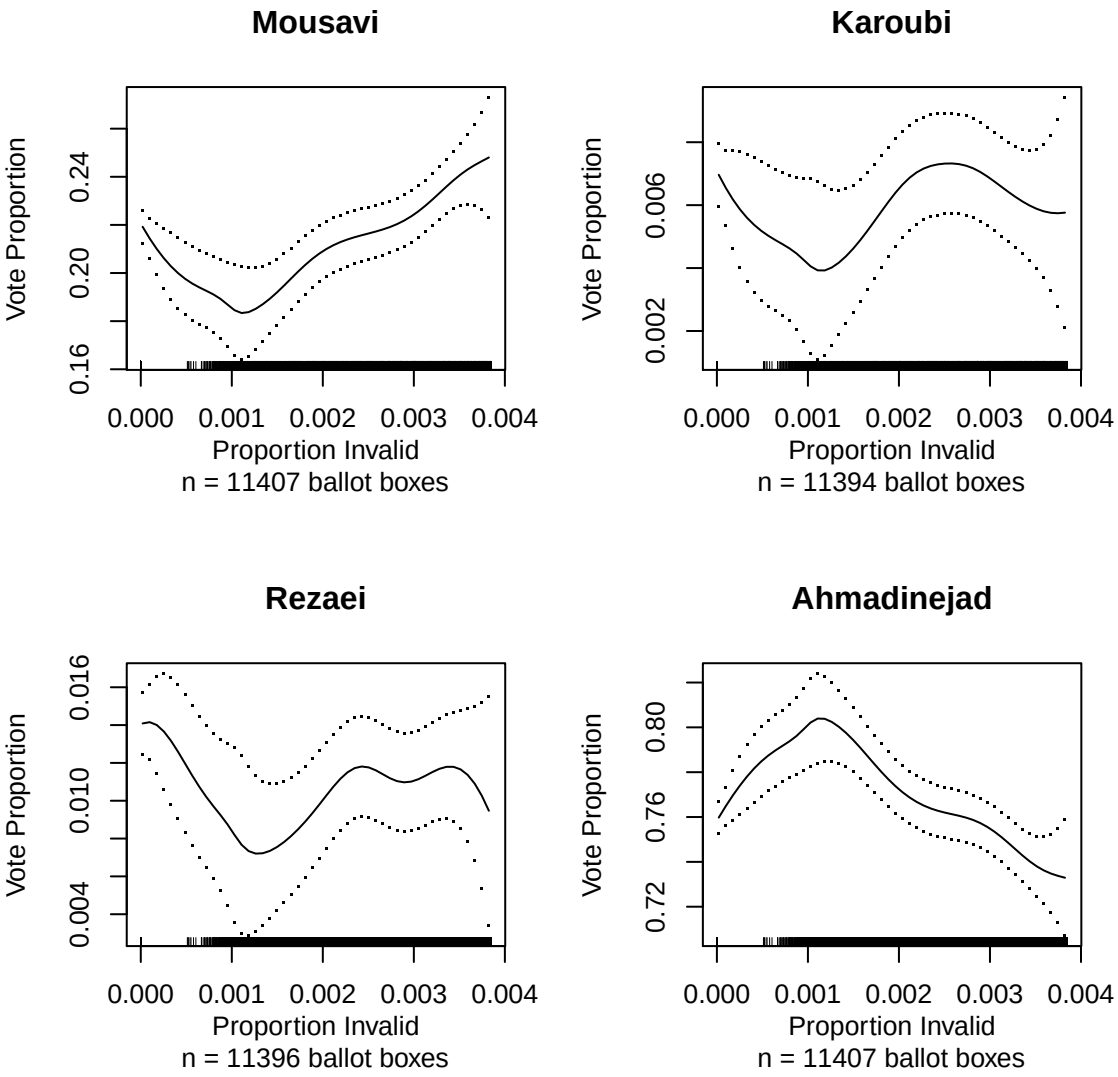

Figure 3: Vote Proportions for President by Proportion of Invalid Votes, 2009, for Invalid Vote Proportions Below First Quartile

Table 19: Distribution of Total Votes' Last Two Digits mod 50

	1	2	3	4	5
prop.	0.01991639	0.01853757	0.01823116	0.01825305	0.01960999
dev.	−0.12636681	−2.21042682	−2.67355127	−2.64047095	−0.58949126
	6	7	8	9	10
prop.	0.01836248	0.01980696	0.01866888	0.01851568	0.01941301
dev.	−2.47506936	−0.29176840	−2.01194492	−2.24350714	−0.88721412
	11	12	13	14	15
prop.	0.01919415	0.01866888	0.01893152	0.01978508	0.02024469
dev.	−1.21801729	−2.01194492	−1.61498111	−0.32484872	0.36983795
	16	17	18	19	20
prop.	0.01941301	0.01912849	0.01836248	0.01952244	0.01976319
dev.	−0.88721412	−1.31725825	−2.47506936	−0.72181253	−0.35792904
	21	22	23	24	25
prop.	0.01904095	0.01950056	0.01840625	0.01840625	0.02098882
dev.	−1.44957952	−0.75489285	−2.40890873	−2.40890873	1.49456875
	26	27	28	29	30
prop.	0.02002583	0.01945679	0.01796853	0.01980696	0.02070430
dev.	0.03903477	−0.82105348	−3.07051508	−0.29176840	1.06452462
	31	32	33	34	35
prop.	0.01958810	0.01939113	0.01989451	0.01991639	0.01831871
dev.	−0.62257158	−0.92029444	−0.15944713	−0.12636681	−2.54123000
	36	37	38	39	40
prop.	0.01993828	0.01947867	0.01976319	0.01849380	0.02079184
dev.	−0.09328650	−0.78797317	−0.35792904	−2.27658746	1.19684589
	41	42	43	44	45
prop.	0.01849380	0.01864700	0.01814362	0.01947867	0.01952244
dev.	−2.27658746	−2.04502523	−2.80587254	−0.78797317	−0.72181253
	46	47	48	49	50
prop.	0.01943490	0.01969753	0.02201747	0.02580377	0.04655184
dev.	−0.85413380	−0.45716999	3.04934368	8.77223862	40.13237971