Comparative Election Fraud Detection*

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Abstract

Elections in Russia are widely believed to be fraudulent in various ways, a claim some support especially by looking at voter turnout, others by looking at vote counts’ digits. We use polling station level data from the Russian Duma elections of 2003 and 2007 and presidential elections of 2004 and 2008 to examine how several methods for diagnosing election fraud complement one another. The methods include estimating the distribution of turnout, measuring the relationship between turnout and party support and testing for vote counts’ second digits following the distribution implied by Benford’s Law. Anomalies the methods detect are worse by the end of the period under study than at the beginning. The digit test detects anomalies beyond those suggested by a simple idea that turnout in many places was fraudulently inflated.

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Introduction

The least one can say about national elections in Russia over the most recent election cycles is that they have become less competitive, with fewer political parties presenting candidates for both the Duma (federal parliament) and the presidency. The United Russia [UR] party, associated with Vladimir Putin, has unquestionably become increasingly dominant. But many observers go further and argue that during the recent period Russian elections have become increasingly unfree and unfair. Myagkov and Ordeshook (2008) argue that during the past 15 years “falsifications in the form of stuffed ballot boxes and artificially augmented election counts” have become prevalent throughout the country (see also Myagkov, Ordeshook, and Shaikin 2008, 2009).

Since 2000 there have been increases in institutional barriers that suppress electoral competition between parties and candidates: tougher registration requirements; the rise of a parliamentary threshold for parties; cancellation of protest voting; and cancellation of the minimum electoral threshold. In the electoral campaign there has been excessive positive informational and financial support of candidates favored by the Kremlin along with negative campaigning against alternative candidates and parties. Administrative changes decreased the transparency of elections. Electoral commission activities became more closed off from the public, with independent public observation being canceled and the rights of the legal observers being frequently violated. The OSCE identified serious problems in the 2004 election (OSCE Office for Democratic Institutions and Human Rights 2004), and by 2008 problems had become so severe that international observer groups declined to observe the election (OSCE Office for Democratic Institutions and Human Rights 2008).

Lubarev, Buzin, and Kynev (2007) argue that administrative changes since 2000 increased the extent to which officials from all levels of the government participated in election administration. Since 2003 both federal and regional administrative resources as well as the mass media have been deployed in favor of UR, which thereby received substantial informational advantages, and against its main competitor the KPRF (Communist party) (Buzin and Lubarev 2008). The Kremlin now controls the appointments of regional executives (each governor can be fired for
“loss of trust”), which has made them responsible for delivering “recommended” electoral figures to the Kremlin. After gubernatorial elections were canceled in December 2004, by the spring of 2007, 70 of 85 governors had announced they were participating in the party of power (Gel’man 2007). Thus, it appears that the entire regional state apparatus is now at the service of the party of power, making it one large electoral “political machine.” This operation is characterized by control over the mass media, administrative pressure on both opposition and voters, and possibly falsifications as well. In fact, state officials have excessive control over all levels of electoral commissions, including precincts (UIKs), territorial (TIKs) and regional commissions.

Allegations point to a wide variety of methods used to distort reported votes (Kalinin 2008). Many of these methods relate to voter turnout and so as markers for fraud may appear ambiguous to the extent they resemble efforts to boost genuine electoral support. In the 2004 presidential election, due to the effectiveness of administrative resources and the popularity of Putin, the outcome of presidential elections was essentially predetermined. As a result, neither of the opposition parties was promoting its leader as their candidate (Buzin and Lubarev 2008, 26). Nonetheless a wide variety of methods was used to increase turnout, including forced voting by absentee certificates. According to Buzin and Lubarev, in 214 territories the 2004 election was the first election where the turnout rate and the share of votes for the winner simultaneously exceeded 90 percent (Buzin and Lubarev 2008, 26). This phenomenon occurred frequently in republics as well as in Rostovskaya, Tumenskaya, Chukotskii and Yamalo-Nenetskii AO.

Voters in Russia do not personally register to vote, but all eligible voters are assigned to specific UIKs depending on where they live. There is a permanent gap between the number of real voters and the “listed voters”—the average number of unaccounted voters in Russia is 2–5 million people—and on election days there are always extensive corrections of the voter lists (Arbatskaya 2004). The large-scale character of these corrections depends on the specific ways the voter lists are formed, methods that differ in different territories. Arbatskaya argues that this correction of voter lists phenomenon opens the door for administrative tyranny, violating the democratic rights of citizens Arbatskaya (2004, 224–226).
The federal elections of 2007 and 2008 took place under different conditions. The president’s popularity remained high, and the “party of power” controlled not only the Duma but also many federal legislatures, encompassing the majority of regional heads, mayors of big cities, and other representatives of political, administrative and economic elites. Putin had promised not to change the Constitution to allow himself to stand for reelection, so he was preparing for his successor. Between 2003 and 2007, the Duma election was changed from a mixed system to a system based entirely on proportional representation. The UR party list was headed by Putin, and it also included a majority of the governors. In the absence of any viable political competitors and Putin’s unique position in UR’s list (he was the only one in its federal list), the 2007 federal elections were labeled as referendum for the all-national Leader. The lack of competition and the absence of the “against all” (Protiv vseh) option on the ballot—this protest voting option was prohibited after the 2004 election—produced a danger of low turnout. According to Buzin and Lubarev, the main task of federal authorities in 2007 and 2008 was twofold: to provide the victory of Kremlin candidates, and to provide high turnout (Buzin and Lubarev 2008, 184, 257-258).

Therefore Buzin and Lubarev claim that vote falsifications were not solely about shifting votes from one candidate to another, but rather about simultaneously increasing the number of votes and the number of voters. These goals can be implemented by “stuffing” the ballot boxes (vbros) or “adding figures to protocols” (pripiska) (Buzin and Lubarev 2008, 184).

Like Myagkov et al. (2009), Lubarev et al. (2007) and Buzin and Lubarev (2008) argue that direct falsifications played a much larger role in the federal elections of 2007 and 2008 than they had in the federal elections and regional elections of the 1990s and early 2000s. Buzin and Lubarev (2008) present electoral data, observer reports and multiple stories from observers and ordinary voters that illustrate the growth of crude falsifications and their widespread character, a pattern they refer to as “mass administrational electoral technology.” Buzin and Lubarev (2008) conclude that compared to all other elections, the elections of 2007 and 2008 showed that direct falsifications started to affect the results of elections, by affecting the distribution of votes between the candidates.
Myagkov et al. (2009), Lubarev et al. (2007) and Buzin and Lubarev (2008) all emphasize what they claim is fraudulent voter turnout. Buzin and Lubarev (2008) state that along with stuffed ballot boxes, the easiest and the most popular technique is to change figures in UIKs’ protocols by UIKs or even more often by TIKs (territories). Buzin and Lubarev are astonished by how widespread direct falsifications are and about the “courage” of falsificators, who appear confident that they are supported by administration and courts (Buzin and Lubarev 2008, 177). As Buzin and Lubarev point out “the insolence with which the protocols are changed in TIKs, knowing that the copy is already given out to observers, is explained by their confidence in impunity, being assured that falsificators and law machinery, including courts, are acting together” (Buzin and Lubarev 2008, 177). They argue that the federal elections of 2007 and 2008 showed widespread discrepancies between data derived from UIKs and official data produced by TIKs and the Gas VIBORI system (the internet-accessible election reporting system).

In this paper we use UIK-level data from the 2003 and 2007 Duma elections and the 2004 and 2008 presidential elections to show that it is useful to augment analysis of Russian elections that focuses on voter turnout statistics with information about the distribution of the second significant digits in UIK-level vote counts.

Tests of vote counts based on the so-called second-digit Benford’s Law (2BL) distribution have figured prominently in work on election forensics (Mebane 2006a,b, 2007b,a, 2008b). The analysis in Mebane (2007a) ultimately focuses on the conditional means of the second digits in collections of vote counts, measuring how these means differ from the means expected according to the 2BL distribution. The conditioning factors in that analysis, which examined data from the 2006 election in Mexico, were the partisan affiliations of mayors in Mexican municipalities. Mebane (2008a) and Kalinin (2008) combined an examination of UIK vote counts second-digit conditional means with outlier detection methods (Mebane and Sekhon 2004) to try to diagnose which of several hypothesized methods for fraud may have affected the votes reported for Russian presidential candidates in 2004 and 2008.
Nonparametric Regression 2BL Test

The 2BL test used in this paper involves comparing the arithmetic mean of the vote counts’ second digits to the mean value expected if the digits are 2BL-distributed. This test adapts an idea used in Grendar, Judge, and Schechter (2007)’s analysis that focuses on the first significant digit and is intended to identify what they describe as generalized Benford distributions. Grendar et al. suggest that data that do not conform to Benford’s law may have first digits that match a member of a specified class of exponential families. Mebane (2006b) argues that vote counts in general do not have digits that match Benford’s law at all. In particular, the distribution of the first digits of vote counts is undetermined. Mebane (2006b) demonstrates a pair of naturalistic models that produce simulated vote counts with second digits but not first digits that are distributed roughly as specified by Benford’s law. Nonetheless we can use the mean of the second digits to test how closely the digits match the 2BL distribution. According to Benford’s law, the expected relative frequency $q_j$ with which the second significant digit is $j$ is (rounded) $(q_0, \ldots, q_9) = (0.120, 0.114, 0.109, 0.104, 0.100, 0.097, 0.093, 0.090, 0.088, 0.085)$. Given 2BL-distributed counts, the value expected for the second-digit mean $\bar{j}$ is approximately $\bar{j}_B = \sum_{j=0}^{9} j q_j = 4.187$.

Mebane (2006b) and Mebane (2007a) suggest that vote counts whose second digits follow the 2BL distribution are unproblematic, while departures from the 2BL distribution indicate that some kind of manipulation has occurred. Whether the manipulation the second-digit test may detect constitutes any kind of fraud is something that needs to be established by additional evidence.

The test is, first, whether $\bar{j}$ differs from $\bar{j}_B$ and, second, whether it differs in a way that depends on observed conditioning factors. The conditioning factor in the current analysis is reported voter turnout, measured as the proportion of registered voters who voted at each UIK.\(^1\) For vote counts $y_i$ observed for UIKs indexed by $i$, we nonparametrically regress the second digits on the turnout proportion $x_i$. To estimate the nonparametric regressions we use the package \texttt{sm} (Bowman and Azzalini 1997) for the statistical programming environment \texttt{R} (R Development

\(^1\)Specifically the value we use to measure turnout is the sum of the number of ballots given out to voters before election day, the number given to voters in polling places and the number given to voters outside of polling places divided by the number of registered voters.
Turnout, Votes and Manipulations in Russia 2003–2008

Start by considering some of the facts about the distribution of turnout in recent Russian elections that support suspicions that the elections were, increasingly, affected by fraud. Figures 1 and 2 display kernel density estimates\(^2\) for UIK-level turnout in the Duma elections of 2003 and 2007 and the presidential elections of 2004 and 2008.\(^3\) Following Myagkov et al. (2009), we consider separately data from republics and data from other regions (“oblasts”). The figures mirror results presented by Buzin and Lubarev (2008, Appendix, Illustration 38), which they attribute to S. A. Shpilkin.

*** Figures 1 and 2 about here ***

The progression of figures shows worse distributions in 2007 and 2008 than in the earlier two years. The distributions are also worse in the presidential election years than in the Duma election years. The top row of Figure 1 shows the distribution for 2003. For both oblasts and republics there is a spike of UIKs with turnout at or very near 100 percent. A higher proportion of UIKs in the republics than in the oblasts have this feature. But in oblasts most of the UIKs have turnout following a relatively smooth unimodal distribution, and in republics many of the UIKs have turnout following such a distribution. In 2004 (the second row of 1), the proportion of UIKs with turnout near 100 percent increases noticeably in oblasts and very substantially in republics. In oblasts the distribution also exhibits spikes at locations corresponding to the excess of turnout values at values of 70%, 80% and 90% noticed by Shpilkin and Shulgin (Buzin and Lubarev 2008, 201). The distributions for 2007 (top row of Figure 2) shows spikes of UIKs at or near 100 percent turnout similar to those observed in 2004. In the distribution for oblasts, spikes are apparent at round number percentages of turnout above 60%. The distributions for 2008 (bottom row of Figure 2) have proportions of UIKs with turnout at or near 100 percent comparable to

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\(^2\)These densities are computed using \texttt{R’s density()} function.

\(^3\)All vote and turnout data were downloaded from the website of the Central Election Commission of the Russian Federation, \url{http://www.vybory.izbirkom.ru/region/izbirkom}. 
2004. The distribution for oblasts shows very pronounced spikes at round number percentages of turnout, and in the distribution for republics a spike is evident near 75% turnout.

Buzin and Lubarev (2008, 201) argue that the only acceptable explanation for the spiked distributions is a wide-spread adjustment of turnout to specific “rounded” figures. Inspecting the last digits of the original UIK-level turnout counts adds to the impression that many of them are faked. If the turnout counts reflected the natural complex of processes that cause people to vote or not to vote, we would expect the counts’ last digits to be uniformly distributed (i.e., each digit zero through nine would occur equally often) (Beber and Scacco 2008). Table 1 shows that the distribution of the last digits in the actual turnout counts from 2003–2008 is very often far from uniform. The table shows for each digit the signed square root of the discrepancy between the observed frequency of the digit and the frequency of 0.1 expected if the distribution is uniform. A value of 2.0 or greater in magnitude represents a significant discrepancy. The table shows that there are always too many zeros, with one exception too few nines, and usually too many fives. Year 2003 for UIKs in oblasts is the only situation where neither the number of fives nor the number of nines is significantly discrepant from the expected uniform distribution, and that subset of UIKs is the only one for which the overall Pearson chi-square statistic is not statistically significant at the .05 test level. As measured by the overall chi-squared statistics, the extent of the discrepancy from the uniform expectation increases monotonically as one moves from 2003 to 2008. Turnout fakery seems to be much worse at the end of the time period than at the beginning.

*** Table 1 about here ***

Myagkov et al. (2009) emphasize the way turnout is associated with votes for the party of power at the rayon level, and Buzin and Lubarev (2008, 204) discuss similar kinds of relationships using UIK-level data. Both discussions make the point that where turnout is very high, support for UR tends also to be very high, and support for other parties—notably the KPRF—tends to be relatively low. Figures 3–6 illustrate these relationships for these two parties. These figures show the scatterplot of the proportion of the vote for candidates of the respective

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If $p_j$ is the observed frequency of digit $j$ and $N$ is the number of UIKs, then the signed square root statistic is $\text{sign}(p_j - 0.1)[N(p_j - 0.1)^2/0.1]^{1/2}$. 

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party against turnout proportions (the data points appear as yellow dots). Superimposed on the plots are a line showing the nonparametric regression of the vote proportions on the turnout proportions and a shaded (cyan-colored) region showing the 95 percent confidence region for a model that hypothesizes there is no relationship between the vote proportions and turnout. Figure 3 shows the results for UR in republics, with one plot for the UIKs in each year. Clearly mean support for UR is much greater in UIKs where turnout is very high. The increase in mean UR vote share from its approximate floor (for turnout roughly 50 percent) to its peak at turnout equalling 100 percent is greater in 2003 than in 2007 but also greater in 2008 than in 2004. Figure 4 shows the results for UR in oblasts. In the Duma election years, then mean support for UR no longer peaks at turnout equal to 100 percent but instead reaches a maximum for turnout at around 90 percent. In the presidential election years, mean support for UR does have a maximum at the highest level of turnout. The gain in mean support from floor to maximum is now greater in 2007 than in 2003 and in 2008 than in 2004.

*** Figures 3 and 4 about here ***

Figures 5 and 6, which show the same kinds of scatterplots and nonparametric regression lines, in contrast show mean support for the KPRF decreasing once turnout increases beyond a certain level. The relationships for UIKs in republics, in Figure 5, show mean KPRF support declining throughout the distribution of turnout in 2003, but 2004, 2007 and 2008 a decline in mean support sets in only for turnout greater than about 60 percent. The decline from ceiling to minimum is greater in 2003 than in 2007 but greater in 2008 than in 2004. The relationships for UIKs in oblasts, in Figure 6, show mean KPRF support declining only turnout greater than a certain level in all four years. In 2003 and 2004 the decline begins once turnout reaches about 80 percent, but in 2007 and 2008 the decline starts when turnout reaches about 50 percent. The ceiling to minimum declines in mean KPRF support are also larger in 2007 than in 2003 and in 2008 than in 2004.

*** Figures 5 and 6 about here ***

5For 2003 we use the proportional representation votes, to match the electoral system in place in 2007.
Reported turnout certainly looks suspicious when its distribution and the distribution of turnout counts’ last digits are viewed on their own, and turnout is clearly related to the mean support for UR and the KPRF. Plots computed for other parties resemble the ones shown here for the KPRF. Such a pattern of UR tending to gain support in places where the KPRF and other parties are tending to lose support strongly suggests that vote switching is possibly occurring.

The 2BL test may provide further evidence on this point. Simulations reported in Mebane (2006b) and Mebane (2008a) suggest that variations from the 2BL distribution can occur both when vote counts are artificially increased and when they are artificially reduced. As Mebane (2008a) observes, it is unclear whether an artificial increase in vote counts will mean that the mean second digit, $\bar{j}$, also increases, or whether an artificial decrease implies that $\bar{j}$ decreases. But we might expect that if substantial vote switching is occurring, we should see significant departures from the 2BL expected mean, $\bar{j}_B$, for both the receiver party and the donor party in places where the vote switching is happening. In the current case, we might expect nonparametric regression lines to show that the expected second digit differs from the 2Bl expected value for both UR and the KPRF for the same values of turnout, if vote switching is taking place.

Figure 7 shows the first of a series of graphs intended to allow such parallel assessments. Each plot in the figure shows the scatterplot of the second digit of the UIK-level vote count for UR against each UIK’s turnout proposition (yellow dots indicated data points). A solid line shows the nonparametric regression of the second digits on the turnout proportions, with a pair of dotted lines indicating the boundaries of a 95% confidence interval. A horizontal dashed line locates $\bar{j}_B = 4.187$. The first question is whether there is any range of turnout values for which the nonparametric regression curve’s confidence interval does not contain $\bar{j}_B$. If so, we will then ask whether the same region of discrepancy is found for both UR and KPRF.

*** Figure 7 about here ***

In all four years in republics, shown in Figure 7, the nonparametric regression curves for the UR vote counts’ second digits have roughly the same shape, but across the years the regions where $\bar{j}$ significantly differs from $\bar{j}_B$ varies somewhat. In 2003, $\bar{j}$ differs significantly from $\bar{j}_B$
only for turnout in the interval roughly (0.45–0.6).\textsuperscript{6} In this region, \( \bar{j} > \bar{j}_B \). In 2007, \( \bar{j} \) differs from \( \bar{j}_B \) for roughly the same values of turnout in the same way, but \( \bar{j} \) also is significantly less than \( \bar{j}_B \) for turnout in the interval roughly (0.7–0.9). In 2004, \( \bar{j} \) differs from \( \bar{j}_B \) for turnout in the interval roughly (0.6–0.8), and in that interval \( \bar{j} < \bar{j}_B \). For 2008, \( \bar{j} < \bar{j}_B \) significantly for turnout in roughly (0.65–0.9), and \( \bar{j} > \bar{j}_B \) significantly for turnout greater than roughly 0.975. The graphs for UIKs in oblasts, shown in Figure 8, show roughly the same pattern as in the republics for 2003 and 2007. For 2004, the oblasts graph shows \( \bar{j} > \bar{j}_B \) significantly for turnout in the interval roughly (0.3–0.5) and \( \bar{j} < \bar{j}_B \) significantly for turnout in roughly (0.55–0.8). The graph for 2008 is similar. None of the graphs for oblasts shows a significant discrepancy between \( \bar{j} \) and \( \bar{j}_B \) at the very highest levels of turnout.

*** Figure 8 about here ***

Comparing these figures to those for the second digits of the KPRF vote counts, we see in the plot for republics (Figure 9) very different patterns. In 2003, \( \bar{j} < \bar{j}_B \) significantly for turnout in two intervals, roughly (0.3–0.55) and greater than 0.75. The results for 2004 are approximately similar. In 2007 the lower interval shrinks to roughly (0.55–0.6) and the upper interval is also slightly smaller (greater than about 0.85). In 2008, \( \bar{j} > \bar{j}_B \) significantly for turnout in roughly (0.6–0.7), while \( \bar{j} < \bar{j}_B \) significantly for turnout greater than about 0.9.

*** Figure 9 about here ***

The intervals of turnout for which \( \bar{j} \neq \bar{j}_B \) significantly for UR overlap with the intervals for which \( \bar{j} \neq \bar{j}_B \) significantly for the KPRF in all four years. For 2003, 2004 and 2007, the overlaps occur for turnout values in the vicinity of 0.5, and \( \bar{j} > \bar{j}_B \) for UR but \( \bar{j} < \bar{j}_B \) for the KPRF. For 2008 overlaps occur for most of the turnout values greater than about 0.65, and once again \( \bar{j} - \bar{j}_B \) has opposite signs for UR and for the KPRF. Such patterns strongly suggest vote switching. Notably, in every year except 2008, the pattern that suggests vote switching occurs for moderate levels of turnout and not at the highest levels. The pattern of \( \bar{j} \) for the KPRF in republics in the earlier years clearly suggest something irregular was happening in the highest turnout UIKs.

\textsuperscript{6}Magnification is probably needed to see the differences discussed in this section.
Perhaps, as in some of the simulations reported by Mebane (2006b) and Mebane (2008a), vote switching was also occurring in those years but it did not rise to levels sufficient to trigger a 2BL signal in the receiver party’s vote counts.

The results for $\bar{y}$ for the KPRF for UIKs in oblasts, shown in Figure 10, are similar to those for republics. With minor differences the comparison between those conditional means and the conditional means for UR suggest the same kind of conclusion.

Anomalies the methods detect are worse by the end of the period under study than at the beginning. The second-digit test detects anomalies beyond those suggested by a simple idea that turnout in many places was fraudulently inflated.
References


URL http://www.R-project.org
Table 1: Distribution of last Digits for UIK Vote Totals in Russian Elections 2003–2008

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Notes: Entries for each digit show signed square roots of chi-squared statistics implied by the null hypothesis that the total number of votes cast at each UIK (polling station) have uniformly distributed last digits. The $\chi^2_L$ statistics show the overall Pearson chi-squared statistic (9 degrees of freedom). $n$ shows the number of UIKs.
Figure 1: UIK turnout, 2003 and 2004
Figure 2: UIK turnout, 2007 and 2008
Figure 3: UIK United Russia vote proportion by turnout, 2003, 2004, 2007 and 2008, republics
Figure 4: UIK United Russia vote proportion by turnout, 2003, 2004, 2007 and 2008, oblasts
Figure 5: UIK Communist vote proportion by turnout, 2003, 2004, 2007 and 2008, republics
Figure 6: UIK Communist vote proportion by turnout, 2003, 2004, 2007 and 2008, oblasts
Figure 7: UIK United Russia second-digit by turnout, 2003, 2004, 2007 and 2008, republics
Figure 8: UIK United Russia second-digit by turnout, 2003, 2004, 2007 and 2008, oblasts
Figure 9: UIK Communist second-digit by turnout, 2003, 2004, 2007 and 2008, republics
Figure 10: UIK Communist second-digit by turnout, 2003, 2004, 2007 and 2008, oblasts