Coordination among American Voters with Heterogeneous Expectations

by

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Abstract

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I use National Election Studies data and the coordinating voting model parameter estimates from Mebane (2000) to simulate what would happen in a population of voters who start the final campaign period with extremely heterogeneous beliefs about the election outcome but hear a succession of poll reports up to election day. In the simulation, poll reports summarize the current voting intentions of a random sample of the voters. Only a randomly selected subset of the voters hears the result from each poll. Each voter who hears a new poll result updates his beliefs using Bayes rule (in the form of a Kalman filter). The simulated election results converge to a neighborhood of the common knowledge equilibrium values, but having approached those values the results wander (due to effective ergodicity). The simulation reveals that coordination of voters' beliefs has surprisingly large effects on election outcomes.

Mebane (2000) introduces a model of coordination among voters in American presidential elections that extends the theory developed by Alesina and Rosenthal (1989; 1995; 1996; Alesina, Londregan and Rosenthal 1993) by allowing different voters to have different beliefs about the upcoming election outcome. Mebane (2000) severely limits the heterogeneity in beliefs, however, by assuming that voters have common knowledge about everything except the realized values of random variables that determine each voter's preferences. The result is that voters in effect have common knowledge about everything except each voter's particular choice of candidates. In a large electorate, the variation among voters' expectations about the election outcome is practically negligible.

The idea that all voters have common knowledge about virtually everything that bears on voters' choices is not believable. Mebane (2000) describes that assumption as an approximation that may be practically correct about the mean of voters' strategies and actions but that almost certainly underestimates the variability. Citing the ideas of McKelvey and Ordeshook (1984; 1985),[1] Mebane (2000) observes that among other institutions preelection polls may supply enough information to support coordination by voters each of whom otherwise has quite limited information. Citing Bowden (1987), Mebane (2000) further suggests that the frequent announcement of preelection poll results would mean that near election day the aggregate of voters' beliefs about the results of the upcoming election would behave like the stationary distribution of a Markov process. Mebane (2000) suggested that the mean of that process would approximately equal the fixed point equilibrium values of the fully coordinating model—a model that assumed a basis of common knowledge among all voters.

In this paper I take a few small steps toward a more plausible treatment of the information

[1]Also relevant are the models and experiments of Myerson and Weber (1993) and Forsythe, Myerson, Rietz and Weber (1993).

individual voters have. I use National Election Studies (NES) data and the coordinating voting model parameter estimates from Mebane (2000) to simulate what would happen in a population of voters who start the final campaign period with extremely heterogeneous beliefs about the election outcome but hear a succession of poll reports up to election day. In the simulation exercise, poll reports summarize the current voting intentions of a random sample of the voters. Only a randomly selected subset of the voters hears the result from each poll. Each voter who hears a new poll result updates his beliefs optimally, using Bayes rule. I illustrate how the election results converge to a neighborhood of the common knowledge equilibrium values, but having approached those values the update-driven results wander. The process wanders because it has become effectively ergodic (on the time scale of the campaign period). One virtue of the simulation exercise is that it helps us see how much the coordination of voters' beliefs may affect election outcomes. In some years the effects are surprisingly large.

Motivation and Design

Grunberg and Modigliani (1954) and Simon (1954) use fixed-point arguments to analyze the accuracy of announced predictions when people respond to the announcements. Bowden (1987) embeds the fixed-point analysis in a Markovian statistical context by means of a model of repeated sampling. Bowden (1987) shows that if the mapping from an announced summary statistic to a new population parameter has a Lyapunov function, then the population parameter is a Markovian random variable that converges to a stationary distribution that he characterizes. In particular he shows that the fixed point (assumed to be unique) of the basic mapping from announced statistic to parameter is not in general the same as either the mean of the stationary distribution or the fixed point of the conditional expectation of the mapping.[2]

[2] Bowden (1987) uses notation θ^* to denote the referent fixed point.

In the current analysis the population parameter is a pair of values $(\tilde{H}_t, \tilde{P}_t)$ where $\tilde{H}_t$ denotes the proportion of the national two-party vote Republican candidates would receive at time t in races for House seats and $\tilde{P}_t$ denotes the probability that the Republican would at time t defeat the Democrat for President. I extend the Mebane (2000) analysis to give each voter i a time-varying personal pair of beliefs, $(\bar{H}_{it}, \bar{P}_{it})$, about $(\tilde{H}_t, \tilde{P}_t)$. The pair $(\bar{H}, \bar{P})$ is the common-knowledge fixed point of Mebane (2000). I do not index $(\bar{H}, \bar{P})$ by time because I will be assuming that all of the variables in the Mebane (2000) model are constant over time except the beliefs $(\bar{H}_{it}, \bar{P}_{it})$. Hence the common knowledge equilibrium is the same as in Mebane (2000). Voters in the model of this paper do not have the common knowledge that is assumed in Mebane (2000), but I use $(\bar{H}, \bar{P})$ as a reference point. One implication of Bowden's (1987) analysis is that, given a Markovian repeated sampling specification, it may be that neither the mean of the stationary distribution of $(\tilde{H}_t, \tilde{P}_t)$ nor the fixed point of the expectation $E[(\tilde{H}_t, \tilde{P}_t) \mid (\tilde{H}_{t-1}, \tilde{P}_{t-1})]$ equal $(\bar{H}, \bar{P})$.

I use $(\hat{H}_t, \hat{P}_t)$ to denote unbiased estimates of $(\tilde{H}_t, \tilde{P}_t)$ that are computed from a sample of the population and announced at each t. Only a fraction u of the population hears the announcement at t and uses the information to update their beliefs $(\bar{H}_{it}, \bar{P}_{it})$. Voters are selected at random to receive each announcement. In general, different voters have different prior beliefs so that two voters who hear exactly the same information will in general have different beliefs after they both update. In the current analysis the updates are Bayesian, and the prior beliefs of all voters are drawn from the same distribution. So the beliefs of two voters who hear the same information become more similar after they update.

Each voter's update is optimal in the sense of being a best adaptive rule given particular distributional stipulations, but the rule is not necessarily part of a noncooperative equilibrium solution of the repeated game that might be defined to comprise the sequence of repeated samples (compare Crawford 1995, 105). I assume that when updating, each voter treats $\tilde{H}_t$ and $\tilde{P}_t$ as i.i.d.

normal variables each having constant mean and variance and being uncorrelated with one another. Such an assumption might be plausible for a large electorate if there is a unique fixed point $(\bar{H}, \bar{P})$ and voters think that $(\tilde{H}_t, \tilde{P}_t)$ is essentially $(\bar{H}, \bar{P})$ subject to independent random wobbles that have mean zero and bounded variance. The variances of the innovations in $\tilde{H}_t$ and $\tilde{P}_t$ are σ_H^2 and σ_P^2.

The announcement of the poll results at time t includes $(\hat{H}_t, \hat{P}_t)$ and $(\hat{\tau}_{Ht}^2, \hat{\tau}_{Pt}^2)$, the latter being the estimated sampling error variances of $\hat{H}_t$ and $\hat{P}_t$ respectively as estimators of $\tilde{H}_t$ and $\tilde{P}_t$. I assume that $\hat{\tau}_{Ht}^2$ is computed using the simple random sampling formula for a sample of size n_t,

$$\hat{\tau}_{Ht}^2 = \hat{H}_t(1 - \hat{H}_t)/n_t, \tag{1}$$

and likewise for $\hat{\tau}_{Pt}^2$. The update of $\bar{H}_{it}$ and $\hat{\sigma}_{Hit}^2$, which is i's estimate at t of σ_H^2, is the Kalman filter update:

$$\hat{\sigma}_{Hit}^2 = \left(\frac{1}{\hat{\tau}_{Ht}^2 w_{Hi,t-1} + \hat{\sigma}_{Hi,t-1}^2} + \frac{1}{\hat{\tau}_{Ht}^2} \right)^{-1} \tag{2a}$$

$$w_{Hit} = \frac{\hat{\tau}_{Ht}^2 w_{Hi,t-1} + \hat{\sigma}_{Hi,t-1}^2}{\hat{\tau}_{Ht}^2 w_{Hi,t-1} + \hat{\sigma}_{Hi,t-1}^2 + \hat{\tau}_{Ht}^2} \tag{2b}$$

$$\bar{H}_{it} = \bar{H}_{i,t-1} + w_{Hit}(\hat{H}_t - \bar{H}_{i,t-1}) \tag{2c}$$

(Harvey 1989, 104–106; Berger 1985, 128, Example 2). The update of $\bar{P}_{it}$ and $\hat{\sigma}_{Pit}^2$ is analogous.

To get a population of voters for the simulation exercises I use the NES data analyzed in Mebane (2000). I replicate the study for each of the six election years included in the data: 1976, 1980, 1984, 1988, 1992 and 1996. The model for individuals' vote choices is the empirical coordinating model for which parameter estimates are reported in Mebane (2000, Table 2). The data are exactly the same as in the Mebane (2000) analysis and all parameters of the model have the values estimated there. For the simulations I vary only the values of the beliefs about the election outcome. In the notation of the empirical model of Mebane (2000), those beliefs are $\hat{\bar{H}}$ and $\hat{\bar{P}}$.

Simulation Results

Bowden (1987) shows several ways in which the fixed points of the process with repeated sampling and announced poll results depend on the form of the basic mapping from announced summary statistic to parameter. That mapping does *not* involve the sampling and announcement sequence. Rather the question is what value of the parameter is produced when *everyone* responds to the same value of the summary statistic. The full mapping is defined by generating such parameter values for every possible value of the statistic. Regarding the current model the question is what values of $(\tilde{H}_t, \tilde{P}_t)$ are produced when the beliefs $(\bar{H}_{it}, \bar{P}_{it})$ are the same for all voters, for all possible pairs of initial beliefs. $\tilde{H}_t$ and $\tilde{P}_t$ each varies over the open unit interval, $(0, 1)$, so the domain of values to use for the initial beliefs is the open unit square, $(0, 1)^2$.

The (a) and (b) plots in Figures 1 through 6 display the basic mappings in the NES data. The (a) and (b) plots show contour lines for the values of $(\tilde{H}_t, \tilde{P}_t)$ that were produced as the initial values varied over $(0, 1)^2$. The (a) plot shows the values of $\tilde{P}_t$ and the (b) plot shows the values of $\tilde{H}_t$. The axes of the plots indicate the relevant initial values.[3] The contours show that in most years the basic mapping is not linear.[4] The closest approximations to linearity occur in 1984 and 1988. The analysis of Bowden (1987) implies that if the mapping is not linear then in general the repeated sampling and announcing process will not end up at the basic mapping's fixed point. That fixed point corresponds to the common knowledge fixed point $(\bar{H}, \bar{P})$. If the mapping is not linear, then in general the mean of the stationary distribution of the sampling-and-announcing process is

[3] The plot axis labeled "H0" shows the value every voter in the NES data was assigned for $\bar{H}_{it}$ and the axis labeled "P0" shows the value assigned for $\bar{P}_{it}$. Given those values, given the regressors data for each voter and given the values estimated by Mebane (2000) for the model parameters, I compute vote choice probabilities for each voter and hence expected vote results among all the voters in each year of NES data.

[4] Jagged or wavy features in the lines in the plots do not correspond to real variations in the $(\tilde{H}_t, \tilde{P}_t)$ values but are artifacts of numerical interpolations used to complete a fine grid of $\tilde{H}_t$ and $\tilde{P}_t$ values.

not $(\bar{H}, \bar{P})$.

In every year the range of the basic mapping is much smaller than the unit square. The range of possible election results is narrowly constrained when there is perfect coordination of the kind that corresponds to all voters having identical expectations about the election outcome.

The dotted lines that appear in the (a) and (b) plots of Figures 1–6 correspond to conditional fixed points of the basic mapping. In the (a) plot, the line shows the values of $\bar{P}_{it}$ that are fixed points if the value of $\bar{H}_{it}$ is pinned to the corresponding value on the "H0" axis, in the case where $\bar{P}_{it}$ and $\bar{H}_{it}$ are each constant over all voters. And in the (b) plot the dotted line shows the values of $\bar{H}_{it}$ that are fixed points if the value of $\bar{P}_{it}$ is held constant. The intersection of those lines is $(\bar{H}, \bar{P})$. The (c) plot shows the lines and their intersection. The rectangle that appears in each (c) plot shows the boundary of the range of the basic mapping—i.e., the boundary of the values of $\tilde{P}_t$ and $\tilde{H}_t$ depicted in the (a) and (b) plots. In the data for 1976 the bounded region is remarkably small.

Each of the (d) plots in Figures 1–6 shows the results of one simulation of repeated sampling and announcing. For these simulations each preelection poll was produced by drawing a bootstrap resample (with replacement) of size $n_t = 500$ of the voters in the NES data.[5] The rate at which voters updated their beliefs from the result of each poll was $u = 0.2$: one-fifth of the voters updated from each poll. To try to approximate the duration of the fall campaigns from Labor Day to election day—a period of roughly 60 days—each simulation performs 60 iterations of the sampling-and-announcing process. To illustrate the maximum possible amount of heterogeneity in voters' beliefs, the initial values of $(\bar{H}_{it}, \bar{P}_{it})$ were drawn uniformly from the unit square and assigned to voters at random. In each (d) plot a small circle is plotted to show the values of $(\tilde{H}_t, \tilde{P}_t)$ at which

[5] Implicit here are the contrary-to-fact assumptions that all who are sampled do provide data and that all who provide data are telling the truth. Cf., e.g., Chilton 1998.

the process commenced. The line that proceeds irregularly out of that circle shows the sequence of $(\tilde{H}_t, \tilde{P}_t)$ values that occurred. The dotted lines in each plot show the parts of the conditional fixed point curves (from the (c) plots) that occur in the range of values included in the plot. The solid, rectangularly connected lines show the boundary of the range of the basic mapping (also from the (c) plots).

The most interesting feature of the simulations is the often substantial changes that occur in the election results as the heterogeneity among voters' beliefs decreases. The values of the results move to the vicinity of the common-knowledge fixed point. In Figure 2, $\tilde{P}_t$ starts at 0.51 and ends up at 0.62. In Figure 3, $\tilde{P}_t$ goes from 0.50 and to 0.63. In Figure 5, $\tilde{P}_t$ goes from 0.50 and to 0.42 while $\tilde{H}_t$ goes from 0.49 to 0.43. In other years there are other large changes. Replications of the simulations using different pseudorandom sequences produce comparable effects. Changes of more than ten percent in the support for candidates are remarkably large when one recalls that *the only things that are changing across the iterations of the simulation are voters' beliefs about the election outcome*. Policy preferences, party identification, evaluations of the economy, House candidate incumbent status: all are constant throughout. The effects seen here, which are due purely to the increasing similarity of the beliefs voters have, are surprisingly large.

That the heterogeneity of voters' beliefs decreases sharply over the course of each simulation is documented in Figure 7, which plots the cross-voter standard deviation of the beliefs $\bar{H}_{it}$ and $\bar{P}_{it}$. The story is essentially the same for all of the series. In iteration 1 the standard deviations are all about 0.27 but by the end they are all about 0.04.

The other noteworthy feature of the simulations is that the election results all wander once they reach the vicinity of the common-knowledge fixed point. Once the major transient toward the fixed point has occurred, the behavior of the process becomes approximately ergodic: the variability of the process over time is roughly proportional to the standard deviation of beliefs across voters.

Discussion

The results reported here are not sufficient to support any conclusion about whether the model of Mebane (2000) at least gets the mean right. The principal trajectory of movement in the simulations is toward the outcomes that are fixed point equilibria under the extremely strong common knowledge assumptions that Mebane (2000) uses. But the setup of the simulation still has several features that seem unrealistic. Principal among such features is the specification that voters use Bayesian updating. The success of the analysis in Crawford (1995) suggests that a weaker learning model need not prevent convergence to behavior that noncooperative equilibrium concepts imply. NES survey data can be used as a test bed for simulations to investigate many ideas.

The biggest surprise from the simulations is the apparently large magnitude of the effects different configurations of voters' beliefs may have on election results. Changes of more than ten percent in support for a presidential candidate, due solely to convergence in voters' expectations, are potentially very important. It remains to be seen whether the effects describe the actual world or are artifacts of some aspect of the simulation methods I have introduced here.

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Figure 1: Statistics of American Voters' Strategies and Beliefs in 1976

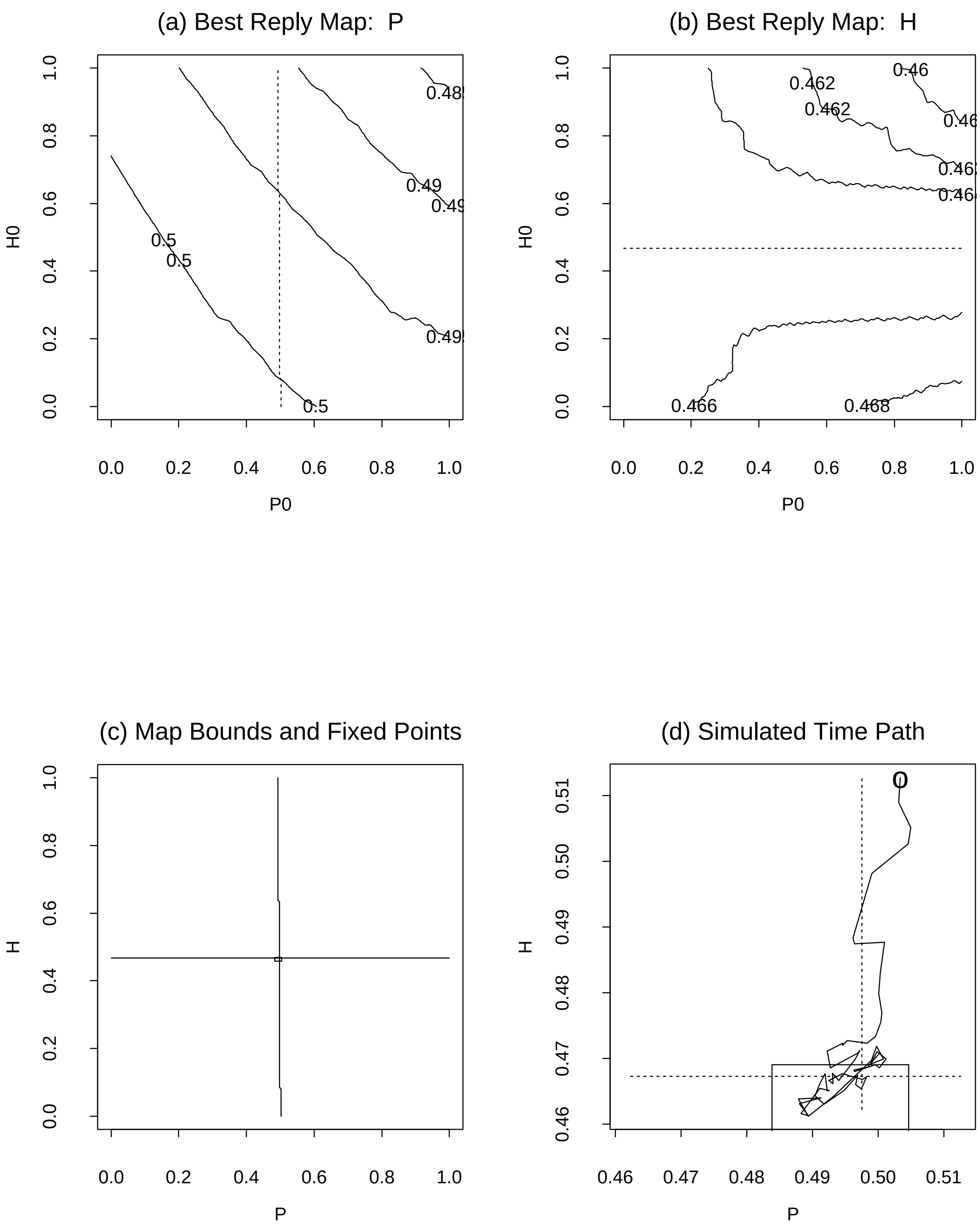

Figure 2: Statistics of American Voters' Strategies and Beliefs in 1980

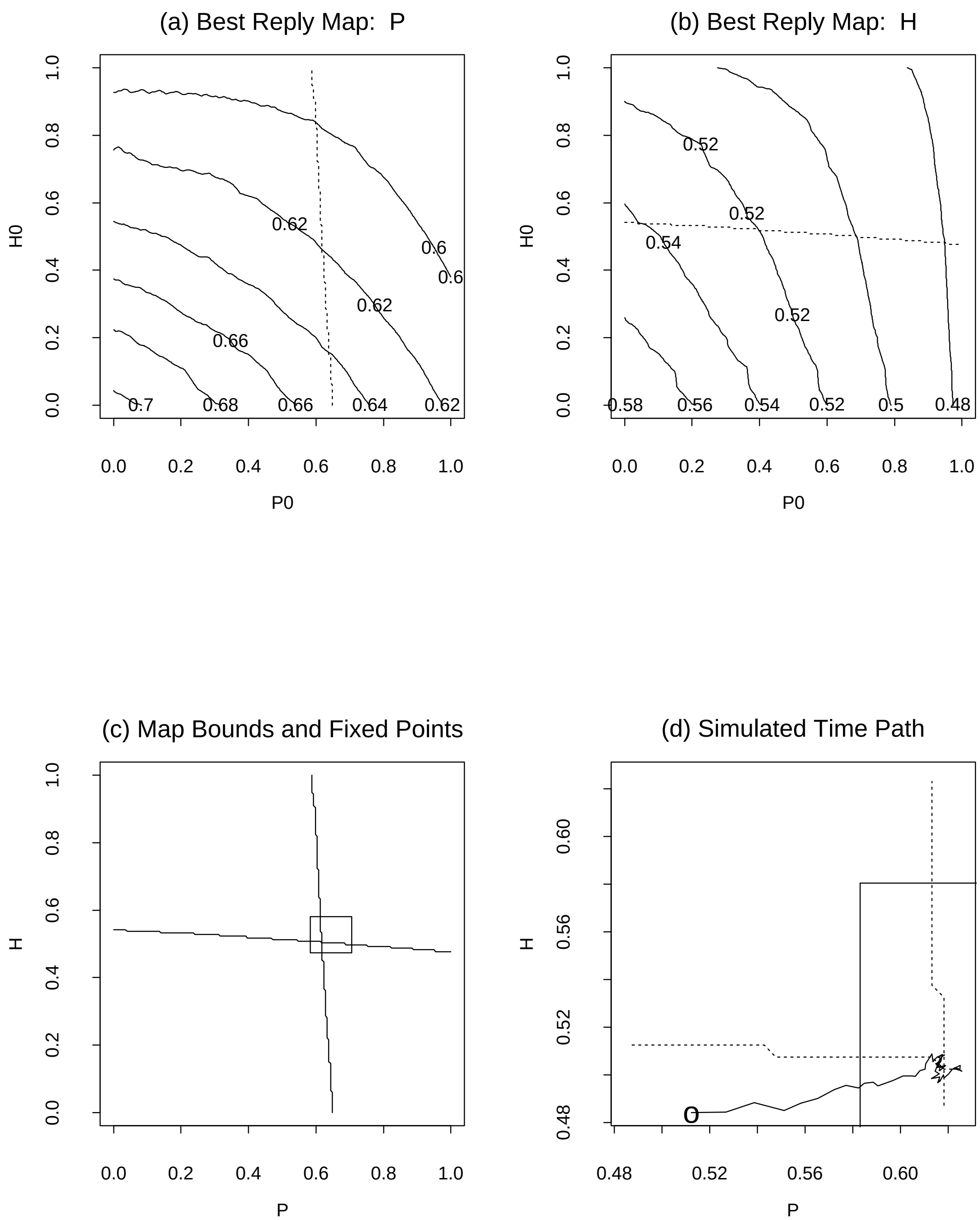

Figure 3: Statistics of American Voters' Strategies and Beliefs in 1984

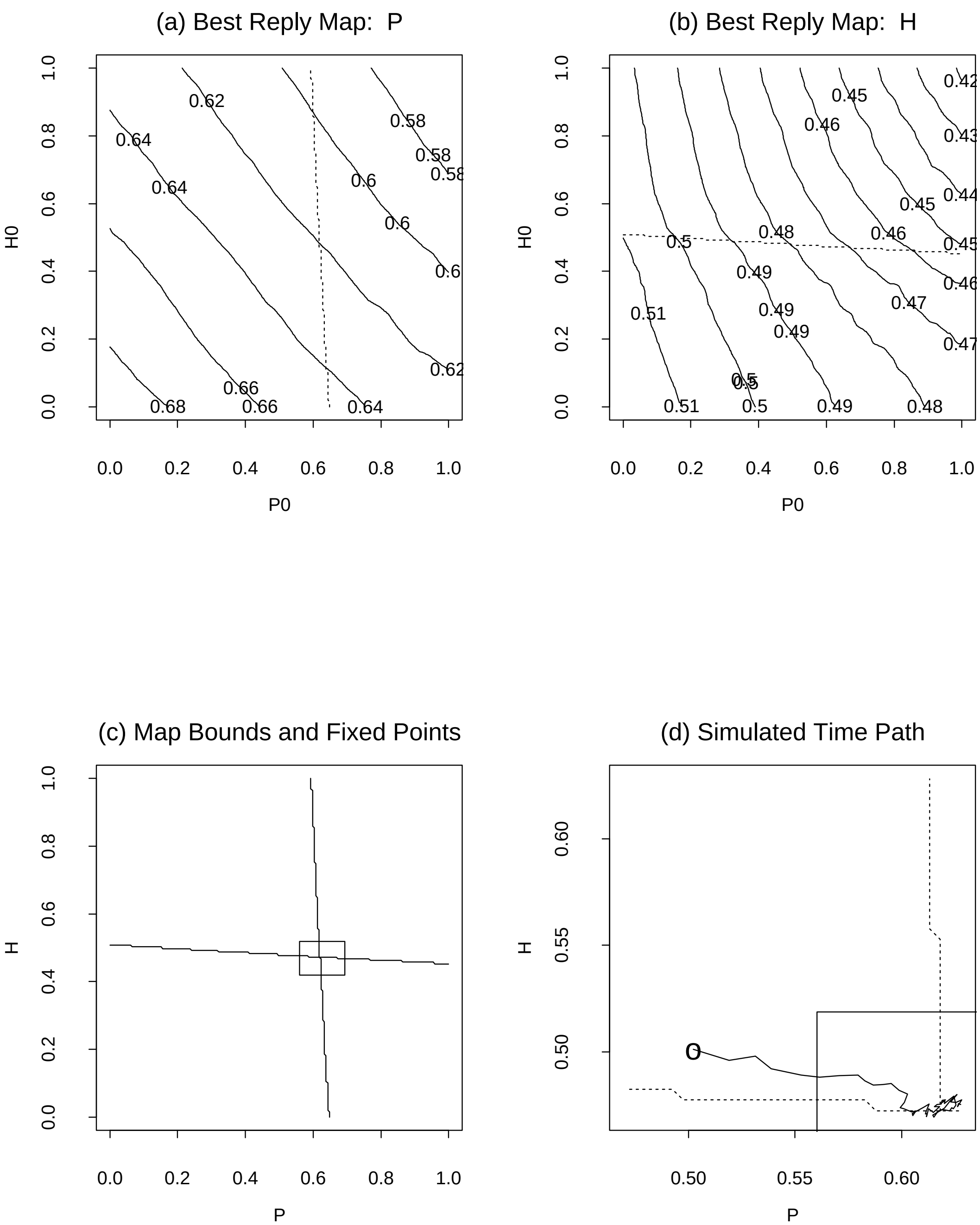

Figure 4: Statistics of American Voters' Strategies and Beliefs in 1988

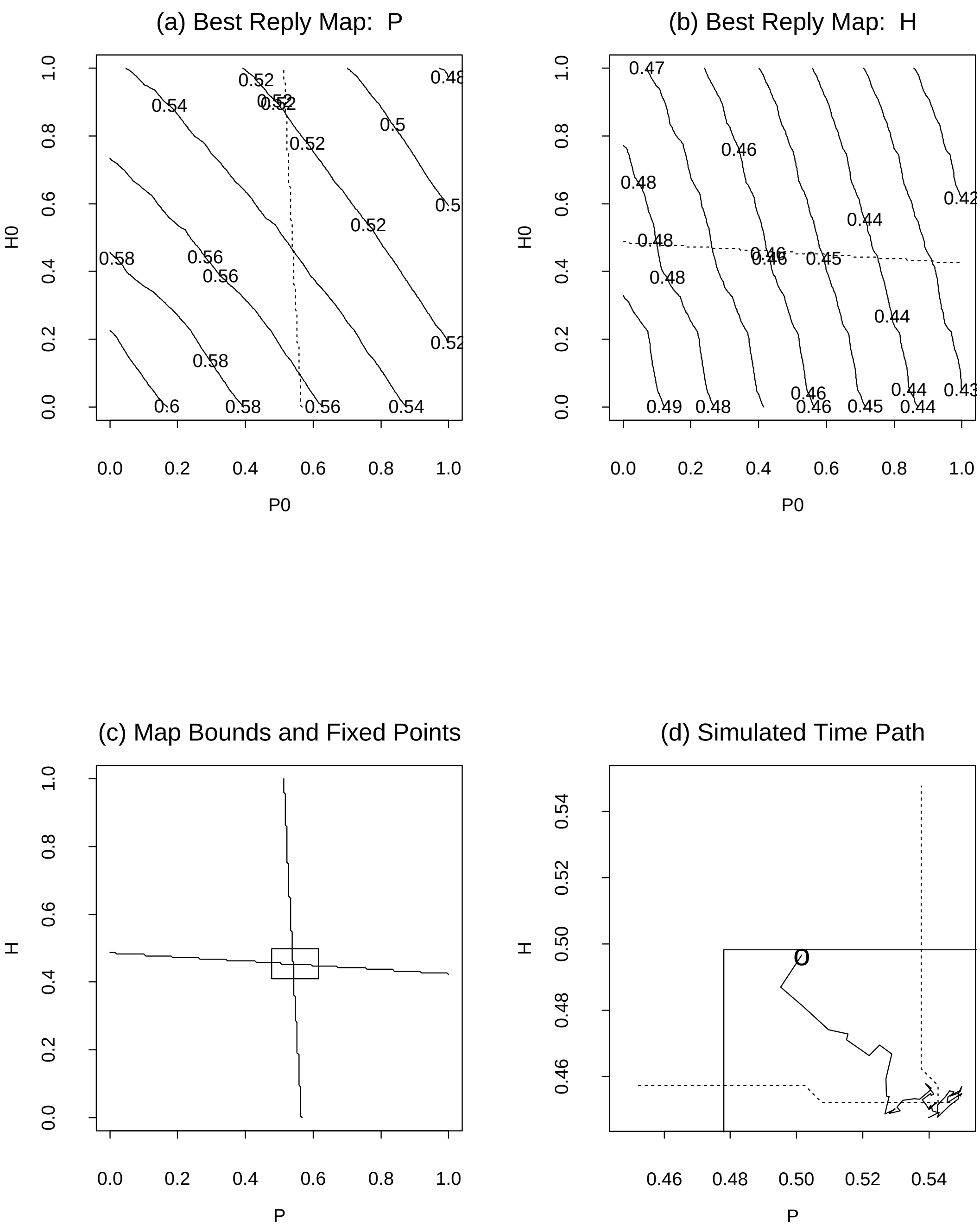

Figure 5: Statistics of American Voters' Strategies and Beliefs in 1992

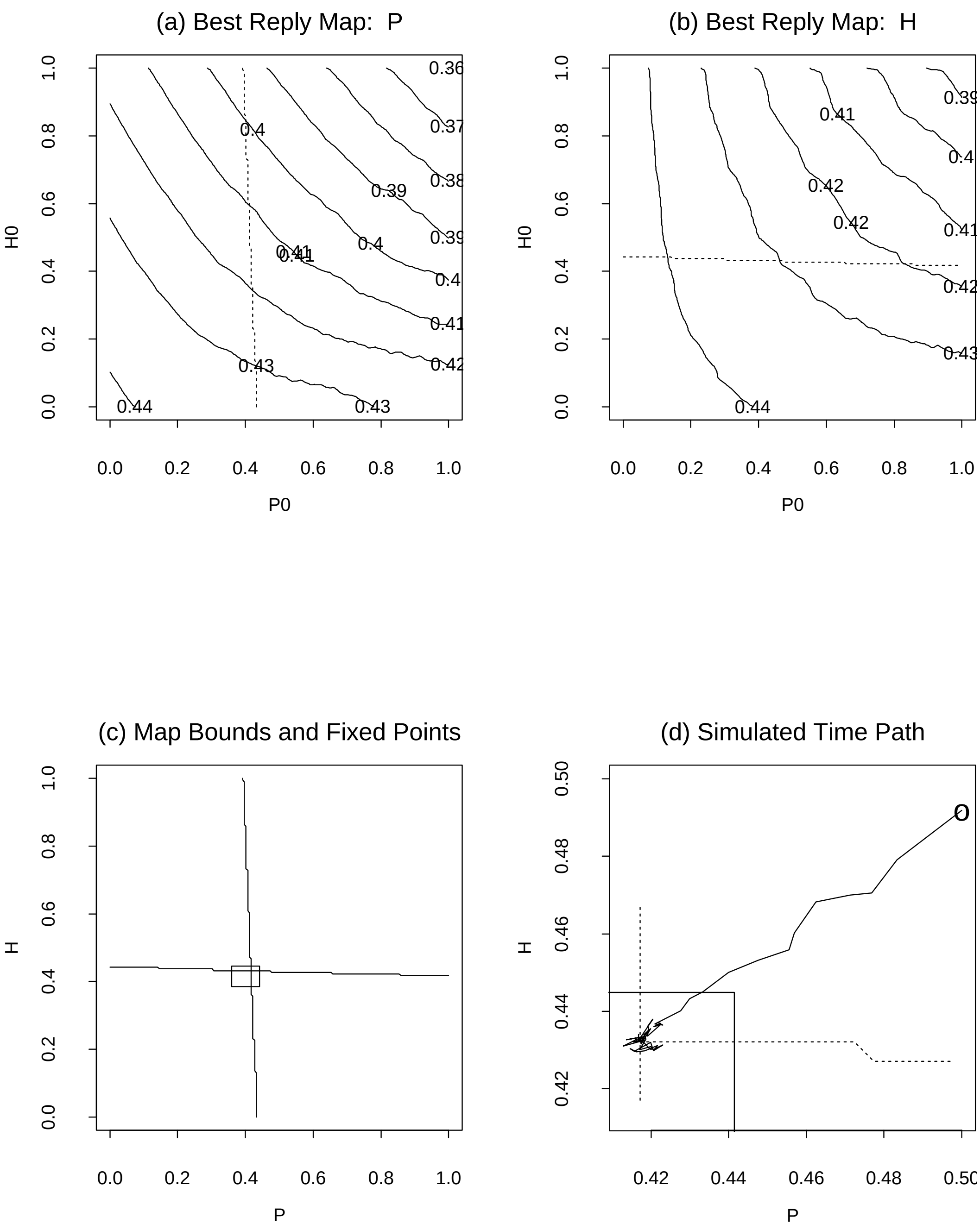

Figure 6: Statistics of American Voters' Strategies and Beliefs in 1996

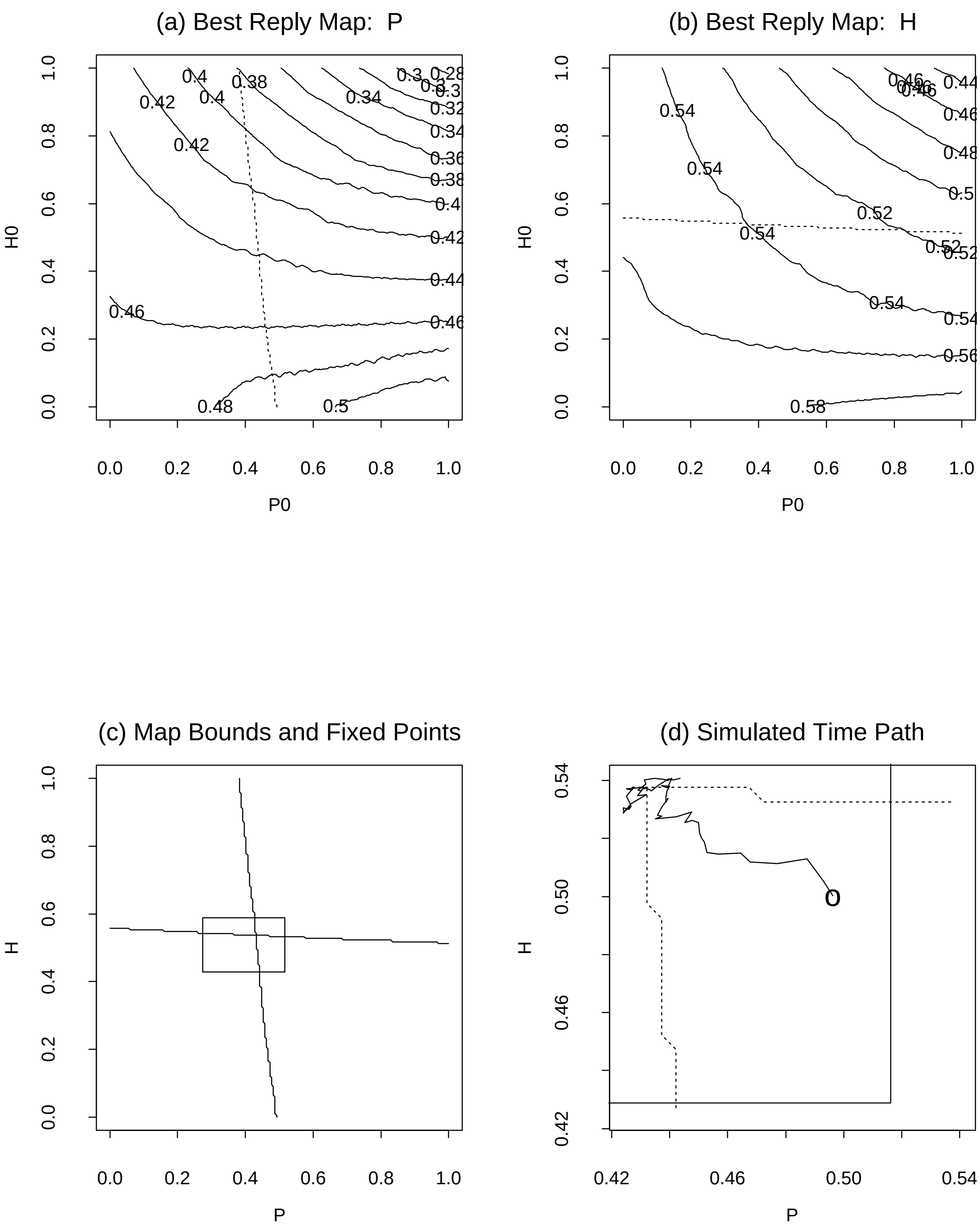

Figure 7: Standard Deviations, Across Voters, of Simulated Beliefs