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Existence of Parity Experiments in Multiparticle Reactions*

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December, 1966†

Abstract

We discuss a theorem of Csonka, Moravcsik, and Scadron on the nonexistence of purely kinematic parity experiments involving scattering processes with more than four particles. Such experiments can be carried out, even in the case of noncoplanar momenta in the center of mass frame, if the experimenter can construct coherent superpositions of some asymptotic states and their parity inverses, as one ideally supposes. If such information is not available, the result of Csonka, Moravcsik, and Scadron appears to stand.

*Work supported by the National Science Foundation.

†March 4, 2008: This LaTeX version has only cosmetic changes from the original preprint.

Csonka, Moravcsik, and Scadron [1], hereafter referred to as CMS, have proposed a theorem to the effect that the sign of the "process intrinsic parity"[1] in a scattering reaction involving more than four particles, incoming plus outgoing, cannot be measured by kinematical means alone, unless no more than three of the particle four-momenta are linearly independent. Their theorem seems to contradict a result give by Stapp [2] in a rather thorough analysis of the concept of intrinsic parity for parity-invariant, S-matrix theories.

The reason for the conflict is not far to seek; CMS have implicitly assumed that information coming from coherence among asymptotic states of different energy-momentum is unavailable, while Stapp makes the more conventional assumption that it is available.

To make this clear, we show how the process intrinsic parities can be measured on the latter assumption, without any knowledge of dynamics.

We assume that the transition probabilities

$$\left|\left\langle f_m\right| S \left|g_n\right\rangle\right|^2$$

are observable, where f_m and g_n lie in m– and n–particle, coherent subspaces of the Fock space of (normalizable) asymptotic states. We suppose that there is a parity operator P defined in the usual way on each subspace having a definite number of particles of definite types, such that

$$\left|\left\langle Pf_m\right| S \left|Pg_n\right\rangle\right| = \left|\left\langle f_m\right| S \left|g_n\right\rangle\right| . \tag{1}$$

The conventions by which the phases of P are defined on each definite particle subspace need not be specified, as long as they are fixed for the discussion.

By standard arguments [3], it follows from Eq. (1) that

$$\left\langle f_m\right| P^\dagger S P \left|g_n\right\rangle = \omega_{mn} \left\langle f_m\right| S \left|g_n\right\rangle , \tag{2}$$

where $|\omega_{mn}| = 1$, and ω_{mn} is independent of the particular vectors f_m and g_n. In nonpathological theories, where we can choose $P^2 = 1$, we can correspondingly choose ${\omega_{mn}}^2 = 1$. The process intrincic parity

$$\begin{aligned} \omega_{mn} &= \frac{\left\langle Pf_m\right| S \left|Pg_n\right\rangle}{\left\langle f_m\right| S \left|g_n\right\rangle} \\ &= \frac{\mathrm{Tr}\left\{ S \left|Pg_n\right\rangle \left\langle g_n\right| S^\dagger \left|f_m\right\rangle \left\langle Pf_m\right| \right\}}{\left|\left\langle f_m\right| S \left|g_n\right\rangle\right|^2} \end{aligned} \tag{3}$$

[1] We use the terminology of Stapp [2].

is manifestly observable, relative to any fixed convention for P, because by "polarization" we can write, for example,

$$\begin{aligned} |Pg\rangle\langle g| = \frac{1}{2}|Pg+g\rangle\langle Pg+g| + \frac{i}{2}|Pg+ig\rangle\langle Pg+ig| \\ - \frac{1+i}{2}\{|Pg\rangle\langle Pg| + |g\rangle\langle g|\}\,. \end{aligned} \tag{4}$$

The formal argument of CMS assumes that the only observable quantities are the absolute squares of S-matrix elements in momentum space. Of course, such S-matrix elements are not functions in genereal, but are rather tempered distributions (in particular, kernels of bounded operators), which cannot in general be squared; but that is a relatively harmless side issue. The basic point is that, even if the momentum-space amplitudes would not have singularities, such as the energy-momentum conservation delta function in each "connected part", so that squaring would be legitimate, to consider only their absolute squares would be to throw away information that could be obtained from interference measurements involving states such as

$$\alpha\,|f\rangle + \beta\,|Pf\rangle\,.$$

The argument just given shows that

$$\langle f_m|\,S\,|g_n\rangle\,\langle g'_n|\,S^\dagger\,|f'_m\rangle$$

are observable quantities which contain the same information as the original quantities. The equivalent quantities in momentum space, taking spinless bosons as an example, are the tempered distributions

$$\langle p_1,\ldots,p_m|\,S\,|q_1,\ldots,q_n\rangle\,\langle q'_1,\ldots,q'_n|\,S^\dagger\,|p'_1,\ldots,p'_m\rangle\,,$$

where the momentum variables in the two factors are *different*. If one begins with these quantities, the discussion of CMS can be recast to show that the process intrinsic parities are observable.

On the other hand, the argument of CMS establishes at least formally the following point: if information coming from coherence among the various momenta (there is always a spread) in the asymptotic states is ignored, or not available, the process intrinsic parities cannot be measured for reactions of more than four particles, when four of the four-momenta are linearly independent.

Such information is ignored in practise. Whether it can actually be retrieved would seem to be largely a matter of experimental technique and imagination, a question of the "state of the art" rather than a question of principle. That is, the observability of coherence might be validated by a clever experimenter; but we do not see any way to invalidate it.

Thus, there appears to be no harm in taking it as a theoretical principle that coherent superpositions of the states in definite particle sectors of the Fock space can be realized. Moreover, there are technical advantages in doing so; we have already seen that it permits a simple and precise description of the observable quantities in momentum space. Finally, it is worth recalling that some coherence is always present, in principle, due to the approximate localization of experiments in spacetime.[2]

Acknowledgements. I wish to thank Dr. Carl Kaysen for the hospitality of the Institute for Advanced Study, where this work was performed.

References

[1] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, "Nonexistence of parity experiments in multiparticle reactions", *Phys. Rev. Letters* **14**, 861–862 (1965).

[2] H. P. Stapp, "Intrinsic Parity from the S-Matrix Viewpoint", *Phys. Rev.* **128**, 1963–1969 (1962).

[3] A. Messiah, *Quantum Mechanics*, North-Holland Publishing Company (Amsterdam, 1962), Chap. XV.

[2] I am indebted to G. F. Dell'Antonio for reminding me of this fact.