Big Topic: Hyperbolic Geometry (Benedetti and Petronio, Chapters A–E)

- *Hyperbolic n–space.* Metric, geodesics, classification of isometries, properties of hyperbolic space (e.g. thin triangles).
- *Examples of hyperbolic 3–manifolds.* (Not in [BP].) Contrasting the mapping torus of a surface under reducible map versus pseudo-Anasov map, statement and discussion of: Hyperbolicity Theorem, Poincaré 's gluing theorem, Borel's theorem, and discussion of several specific examples of hyperbolic manifolds, both of finite volume and infinite volume.
- *Margulis Lemma and Thick–Thin Decomposition.* Bieberbach's Theorems (Buser's proof) as an "easy" version of Margulis's Lemma, Margulis' Lemma, classification of torsion–free nilpotent subgroups, Thick-Thin, universal horoball theorem.
- *Teichmüller Theory.* Definition of Teichmüller space and proof of Fenchel Nielsen coordinates parametrization.
- *Mostow Rigidity.* The proof presented in [BP] uses the following sequence of intermediate steps.

  1. If $f : M_1 \longrightarrow M_2$ is a homotopy equivalence of closed n–dimensional hyperbolic manifolds, it induces an isomorphism $f_* : \pi_1(M_1) \longrightarrow \pi_2(M_2)$. We fix coverings $\mathbb{H}^n \longrightarrow M_i$, giving us identifications $\pi_1(M_i) \cong \Gamma_i \subset \mathrm{Isom}^+ \mathbb{H}^n$.

     Lemma. *We can lift $f : M \longrightarrow N$ to a map $\tilde{f} : \mathbb{H}^n \longrightarrow \mathbb{H}^n$ such that the following hold:*

     (a) $\tilde{f}$ is a quasi–isometry.

     (b) $\tilde{f}$ extends to a continuous mapping $\overline{\mathbb{H}^n} \longrightarrow \overline{\mathbb{H}^n}$ which is one–to–one on the boundary. (See also Gromov Hyperbolic Spaces.*)*

     (c) The relation $\tilde{f} \circ \gamma = f_*(\gamma) \circ \tilde{f}$, $\gamma \in \Gamma_1$, holds on all of $\overline{\mathbb{H}^n}$.

  2. We use the definition of the Gromov Norm of a closed hyperbolic manifold M and the fact that

     $$||M|| = \mathrm{vol}\, M / v_n. \tag{1}$$

     to prove the following lemma.

Lemma. *The map $\tilde{f}$ has the property that it maps the vertices of ideal regular simplices in $\mathbb{H}^n$ to ideal regular simplices.*

3. Next, we use the previous lemma and barycentric subdivision to construct an isometry, $q : \mathbb{H}^n \longrightarrow \mathbb{H}^n$.
4. Lastly, we show that q induces and isometry between M_1 and M_2 which is homotopic to f.

- *Volume of Hyperbolic Manifolds.* Volume of triangles in $\mathbb{H}^2$, volume of closed hyperbolic surfaces, Wang's Finiteness Theorem, the statement of Thurston's Dehn filling theorem and how it applies to the proof that convergent sequences of hyperbolic 3–manifolds exist .

Little Topics

- *Spectrum of the Laplace Operator.* Cheeger constant, Buser's theorems.
- *Gromov Hyperbolic Spaces.* Stability of quasi–geodesics, local quasi–geodesics are quasi–geodesics, Gromov boundary.
- *Minimal surface theory.* Properties of harmonic and holomorphic functions, properties of minimal surfaces: monotonicity, maximum principle and applications, Dehn's lemma in the case of manifolds with convex boundary, existence of the solution to the Plateau problem in $\mathbb{R}^3$.
- *Foliation Theory.* Examples of foliations, every 3–manifold admits a foliation (using that every 3–manifold can be obtained from any other by Dehn surgery), Novikov's (little) theorem: "A 3–manifold which admits a foliation with no compact leaf has trivial π_2," (using Sullivan's theorem about minimal spheres without proof).