Online Problem Number 6

(Angle of Solar Panels Revisited) A panel outputs power $P(\theta)$ in watts depending on the angle to the sun $\theta \geq \theta \leq \pi$. On a typical summer day in Ann Arbor, the angle between the panel and the sun $t$ hours after 6 am is $\theta(t)$ for $0 \leq t \leq 14$. Assume sunset is 8 pm and sunrise 6 am.

a) Calculate $\frac{dP}{dt}$ using the chain rule. Give interpretations of each part of the calculation.

$\frac{dP}{dt}$ is the change in voltage over time.

This would mean it is a composite function with the variable as $t$ which must go through the calculation of angle measurement which will go on to the measuring of voltage $P(\theta(t))$ is such equation.
Using the chain rule to calculate \( \frac{df}{dt} \) resulted in
\[
\frac{df}{dt} = \frac{d}{d\theta} (C(\theta(t))) \cdot \theta'(t)
\]
This value is the change in the change in voltage with respect to angle, read over time.

This value is equal to angle read over time.

To differentiate the equation \( \sin(\cos(\theta(t)) - \frac{\pi}{2}) = \frac{1}{2} \) the chain rule and the notion that \( f'(x) \) of sine is equal to cosine must be used. So following these rules the derivative unsimplified is:
\[
\cos(\cos(\theta(t)) - \frac{\pi}{2}) \cdot \theta'(t) = \frac{1}{2}
\]
Simplifying this yields:
\[
\theta'(t) = \cos(\cos(\theta(t)) - \frac{\pi}{2})
\]

3) Suppose \( \frac{df}{d\theta} \left( \frac{2\pi}{3} \right) = 12 \) and \( \theta(t) \) is as in part (b).

Find the approximate change in power outage of the panel between 4:30 PM and 5:30 PM.

Using the given value \( \frac{df}{d\theta} \left( \frac{2\pi}{3} \right) = 12 \) we calculated...
solve for \( t \). We found that \( t \) to be 10.5 or 9:30 PM which is close enough to the actual time we are searching for to use. \( \theta(t) \) must also be calculated so using 11 as this represents 5 o'clock we plugged 11 into the original \( \theta(t) \) equation to calculate the angle present. Then we plugged this value into the \( \theta(t) \) equation to calculate \( \theta(t) \) at 1177. Now we have \( \frac{d\theta}{dt} \) as 12 and \( \theta \) as 1177. Using the original chain rule equation for part (a), 

\[
\frac{d\theta}{dt} = \frac{d\theta}{d\theta} \cdot \frac{d\theta}{dt}
\]

we plugged the values of \( \frac{d\theta}{d\theta} \) and \( \theta \). We got \( \frac{d\theta}{dt} \) at 1.412 m/s per hour, which is the approximate change in voltage between 9:30 PM and 5:30 PM.
This problem tells us that \( f(v) \) is the gas consumption (L/km) of a car at velocity \( v \) (km/hr). \( f(v) \) is the number of liters of gas the car uses to go one kilometer at \( v \).

We are given: \( f(80) = .05 \) and \( f'(80) = .0005 \)

a) Asks us to find \( g(80) \) and \( g'(80) \) given \( g(v) \) is the distance the car goes on 1 L of gas at \( v \). To help us do this we must also find the relationship between \( f(v) \) and \( g(v) \).

First, looking at the relationship between \( f(v) \) and \( g(v) \) we understand that \( f(v) \) 's units are L/km and \( g(v) \) 's units are km/L so one of them is a reciprocal of the other. Therefore, we can set \( f(v) = \frac{1}{g(v)} \) to get them to be the same. Because of this we can divide 1 by what we know, \( f(80) = .05 \) to find \( g(80) \).

\[
\frac{1}{.05} = \frac{20}{L} = \text{km/L}
\]

Now that we know \( f(80) \), \( g(80) \), and \( f'(80) \) we can use the product rule to solve for \( g'(80) \). We use this because we have a derivative and two values which we can solve to find the other value being the other derivative.

\[
f'(v) \cdot g(v) + f(v) \cdot g'(v) = 0
\]

\[
.0005(20) + .05 \cdot g'(80) = 0
\]

\[
.01 + .05 \cdot g'(80) = 0
\]

\[
.05 \cdot g'(80) = - .01
\]

\[
g'(80) = - .2 \text{ hr/L (units are hr/L because the units are km/L per km/hr, the kms cancel leaving hr/L)}
\]

b) Asks us to do the same thing as part a but now with \( h(v) \) as the gas consumption in L/hr. We must again find the relationship between \( h(v) \) and \( f(v) \) and find \( h(80) \) and \( h'(80) \).

From the previous problem we know the units of \( g'(v) \) are hr/L. We can use this to create a relationship with \( h(v) \) being \( g'(v) = \frac{1}{h(v)} \) like the former question. We can do the same thing as before, plugging in \(-.2\) for \( g'(8) \) to get \( h(80) \). \(-.2 = \frac{1}{h(80)} \) which equals \(-5 \text{ L/hr}\). Now we can again use the product rule to find \( h'(v) \) or \( h'(80) \).

\[
h(v) \cdot g'(v) = 1
\]

\[
g'(v) \cdot h(v) + g(v) \cdot h'(v) = 0
\]

\[
-.2 \cdot 5 + 20 \cdot h'(80) = 0
\]

\[
1 + 20 \cdot h(80) = 0
\]

\[
h'(80) = -.05 \text{ km/h}
\]
c) Is basically a conclusion to this problem, explaining the practical meaning of these functions and their derivatives. These values show the change in speeds and how they change fuel efficiency, based on the velocity as well. All of these are related to each other and the derivatives of them will show how when one aspect of them is changed slightly, how it affects another aspect respectively.

Specifically, \( f(\alpha), f'(\alpha), g(\alpha), \ldots \).
Section 3.4

60. Find the equation of the line tangent to \( f(x) = 6e^{x}(5x) + e^{-x^2} \) at \( x = 1 \).

We used the first derivative of \( f(x) \) and then plugged 1 in for \( x \) to find the slope of the tangent line of \( f(x) \) at 1. To find the first derivative I used the chain rule and the rules for \( e^x \) \( (e^x = e^x) \).

\[
f'(x) = 30e^{5x} - 2xe^{-x^2}
\]

Then we plugged 1 into \( f(x) \) to get a \( y \) coordinate.

\[
f(1) = 890.847
\]

Then we plugged the \( x \) and \( y \) coordinates \((1, 890.847)\) into the \( y = mx + b \) equation to get the \( y \)-intercept using the slope originally found.

\[
890.847 = (4451.659)(1) + b \quad b = -3560.812
\]

So the equation to the line tangent to \( f(x) \) is

\[
y = 4451.659(x) - 3560.812
\]
3.4 \#16

\[ m(t) = e^{\mu t + \sigma^2 t^2 / 2} \]

Firstly, when using statistics we know that \( \mu \) represents the mean and \( \sigma^2 \) represents the variance (standard deviation). In this problem we will mathematically prove both of these statements. Part (a) asked us to find \( \text{mean} = m'(0) \) so first we must find the derivative of this function.

\[ m(t) = e^{\mu t + \sigma^2 t^2 / 2} \]

\[ m'(t) = \left( \mu + \sigma^2 t \right) e^{\mu t + \sigma^2 t^2 / 2} \]

Now we input zero for \( t \) to find the mean.

\[ m'(0) = \left( \mu + \sigma^2 (0) \right) e^{\mu (0) + \sigma^2 (0)^2} \]

\[ m'(0) = \mu e^{(0)} \]

\[ m'(0) = \mu \]

\( \mu = \text{mean} \)

b) Part asks us to find the variance \( = m''(0) - (m'(0))^2 \). From part (a) we already know the value of \( m'(0) \) so the only new information we must find is \( m''(0) \).
\[ m'(t) = (\mu + \sigma^2 t) e^{\mu t + \sigma^2 t^2 / 2} \]

\[ m''(t) = \text{requires us to use the product rule to find the derivative of } m'(t) \]

Product Rule: \( f(x)g(x) = f(x)g'(x) + f'(x)g(x) \)

\[ m''(t) = (\sigma^2) e^{\mu t + \sigma^2 t^2 / 2} + (\mu + \sigma^2 t)(\mu + \sigma^2 t) e^{\mu t + \sigma^2 t^2 / 2} \]

Now, we input 0 for \( t \)

\[ m''(0) = (\sigma^2) e^{\mu(0) + \sigma^2 0^2 / 2} + (\mu + \sigma^2(0))(\mu + \sigma^2(0)) e^{\mu(0) + \sigma^2(0)} \]

\[ m''(0) = (\sigma^2) e^{\mu(0)} + (\mu(0))(\mu(0)) e^{\mu(0)} \]

\[ m''(0) = \sigma^2 + \mu^2 \]

Variance = \( m''(0) - (m'(0))^2 \)

Variance = \( \sigma^2 + \mu^2 - (\mu)^2 \)

Variance = \( \sigma^2 \)