a) Part a asked us to write a formula for the power produced by the solar panel after t hours on a summer day, meaning they want us to express the power output as a function of time. They tell us that the power output varies depending on the angle of the sun and give us two formulas: one for the output of the solar panel and one for the angle of the sun. If we plug the equation of the angle of the sun into the output formula we get f(t) and function that represents the output formula as a function of time. We demonstrate this below:

\[ P = 10\sin\theta, \quad \theta(t) = \frac{\pi}{14} t \quad f(t) = 10\sin\left(\frac{\pi}{14} t\right) \quad f(t) = P(\theta(t)) \]

Below is the graph this formula starting at sunrise (6am) and ending at sunset (8pm).

b) According to the graph the power output is the greatest when t=7 hours or 1pm. At that point the power output is 10 Watts.

c) The function seems to be increasing the fastest from 0 to 1 hours because there the slope is \( \frac{2.2252-0}{1} = 2.2252 \). The slope seems to decrease the fastest between 13 and 14 hours because there the slope is \( \frac{0-(-2.2252)}{1} = -2.2252 \).

d) Lastly, they ask us to give the power out formula on a winter day when the sunrise and sunset span from 8 am to 5pm (9 hours instead of 14). The range become smaller, instead
of being $0 < t < 14$ its now $0 < t < 9$. Which simply changes the formula because it changes the period in this way:

Original Formula: $f(t) = 10\sin\left(\frac{\pi}{14}t\right)$,  
New Formula: $g(t) = 10\sin\left(\frac{\pi}{9}t\right)$
The air in a factory is being filtered so that the quantity of a pollutant, \( P \) (in mg/L), is decreasing according to the function \( P = P_0 \cdot e^{-kt} \), where \( t \) is time in hours. If 100% of the pollution is removed in the first five hours:

1) What percentage of the pollution is left after 10 hours?

\[
A = P = P_0 \cdot e^{-kt} \quad 9 = 1 \cdot e^{k(5)} \Rightarrow 1.9 = e^{5k} \ln(1.9) = 5k \Rightarrow k = \frac{\ln(1.9)}{5} = 0.2107
\]

\[
P = 1e^{-0.2107 \cdot 10} = 0.813 \text{ or } 81.3\%	ext{ when } t = 10
\]

We used the given exponential equation \( P = P_0 \cdot e^{-kt} \) and the given information to solve for the amount of decay. This was calculated to be 0.2107. Once found we plugged in 10 for \( t \) to solve for the amount of pollution left which was 81.3%. 9 was used as \( P \) for that was the total pollution left after 5 hours for total was 1. 5 was used as \( t \) because it was time for 100% of the pollution to be removed. \( P_0 \) was 1 as it was original total.

Why are you allowed to do this?

2) How long is it before the pollution is reduced by 50%?

\[
A = s = 1e^{-0.2107t} \Rightarrow \ln(s) = \ln(0.5) = -0.2107t \Rightarrow t = \frac{-\ln(0.5)}{0.2107} = 33 \text{ hours}
\]

Using the obtained equation from part a we solved for time using 1 as \( P_0 \) and 0.5 as \( P \). \( k \) was precalculated from part a. The answer is 33 hours.
0) Plot a graph of pollution against time. Show the results of your calculations on the graph.

\[ A = \frac{1}{2} \sqrt{2} \]

The \( y \)-intercept is 1 because that is the initial amount of pollution after no time has passed. \( I = 1 \times 1000 \text{W} \). ISO was chosen because at this point very little of the pollution is left after that many hours. Time is input pollution left as percentage output.

d) Explain why the quantity of pollutant might decrease this way.

\[ A = \text{The excaustion is exponentially decreasing the value of pollution. The amount present directly affects the amount filtered. As less pollutant is present less can be filtered.} \]
1.5 #46.

In this problem, the variations of a desert temperature are described. The temperature of this desert oscillates between a low of 40 degrees Fahrenheit and a high of 80 degrees Fahrenheit every day. The low temperature occurs at 5am, and the high temperature occurs 12 hours later, at 5pm.

We are asked to construct a formula for \( T \), the desert temperature, in terms of \( t \), which is measured in hours starting at 5am.

First, we drew a graph portraying the temperature of the desert over 24 hours, starting at 5am.

![Graph of desert temperature](image)

Then, in order to figure out the formula for this graph, we had to figure out what kind of graph it is, whether the graph is positive or negative, the period, the amplitude, and any vertical or horizontal shifts that occur. We know the graph is going to be a sine or cosine because of the oscillating shape it would make over a certain amount of time (the period).

After drawing the graph, we concluded that the graph is a negative cosine graph. We know this because a positive cosine graph looks like this:

![Positive cosine graph](image)

Since our graph begins going upward (flipped vertically) we must put a negative sign in front.

Next, we had to determine the amplitude of the graph. Because the graph ranges from 40 degrees to 80 degrees, we know that the amplitude of the graph. The formula to find amplitude is the largest y value minus the smallest y value divided by 2. This would be \( (80-40)/2 = 20 \). So the amplitude of the graph is 20.

We know that the period of this graph is 24 hours, because it takes 12 hours for the temperature to rise from 40 degrees to 80 degrees, and then another 12
hours for it to drop from 80 degrees back to 40 degrees, where it started. So, in order to figure out “B” or the period of this graph we must use the equation \( P=\frac{2\pi}{B} \).

\[
P=\frac{2\pi}{B} \quad \text{multiply both sides by } B \text{ to get } \]
\[
B=\frac{2\pi}{P}
\]
\[
B=\frac{2\pi}{24} \quad \text{24 hours is the period as previously stated}
\]
\[
B=\frac{12\pi}{24} = \frac{\pi}{2}
\]

Finally, we had to figure out if there were any horizontal or vertical shifts. In our graph, we consider 5am to be “Zero” on the x-axis, so there is no horizontal shift. However, there is a vertical shift. The graph is shifted up 60 units. We found this because we drew an imaginary x-axis where the middle of the graph would be, and so this would be at 60 degrees.

The equation for the trigonometric function of cosine is \( f(t)=A\cos(B(t-h))+k \), where...

- \( A \) = amplitude
- \( 2\pi/B \) = period
- \( h \) = horizontal shift
- \( k \) = vertical shift

We can now plug in our information from previous equations to get the full equation:

\[
H(t) = -20\cos\left(\frac{\pi}{12}t\right)+60
\]

Conclusion: After calculating this equation, we checked to make sure that this was the right equation by substituting 0 hours (which is technically 5am) in for \( t \) to get 40 degrees, and 12 hours (which is technically 5pm) in for \( t \) to get 80 degrees. We can conclude that the temperature oscillates daily between 5am and 5pm, ranging form 40 degrees to 80 degrees Fahrenheit. We can also conclude that the graph is a negative cosine graph, that the amplitude is 20, that the period is 24 hours, that \( B \) is equal to \( \pi/12 \), and that the graph is shifted vertically upward 60 units.

\[
H(0) = -20\cos\left(\frac{\pi}{12}\right)+60 = 40^\circ F
\]
\[
H(12) = -20\cos\left(\frac{\pi}{12}12\right)+60 = 80^\circ F
\]
\[
H(24) = -20\cos\left(\frac{\pi}{12}24\right)+60 = 40^\circ F
\]

Cosine and sine graphs are common with things that fluctuate in the same manner over a given period of time, such as the tide height and temperature in our case.
When the object of mass $m$ moves with a velocity $v$ that is small compared to the velocity of light, $c$, its energy is given approximately by:

$$E = \frac{1}{2}mv^2$$

If $v$ is comparable in size to $c$, then the energy must be computed by the formula:

$$E = mc^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

This question asked us to plot both equations for $E$ against $v$ for the intervals:

$v \in [0, 5 \times 10^8]$ and $E \in [0, 5 \times 10^{17}]$

However, since there were too many variables in the equations ($m, v, c$) they gave us values to substitute $m$ and $c$. For $c$ we put in $3 \times 10^8$ m/sec and for $m$ we put 1 kg:

- $E = \frac{1}{2}mv^2$ and
- $E = (0)(3 \times 10^8)^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{3 \times 10^8}}} - 1 \right)$

The question next asks us to explain how you can predict from the exact formula the position of the vertical asymptote. A vertical asymptote is where the end behavior of the graph goes to either negative infinity or positive infinity. From the graph above you can see that the graph of

$$E = mc^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

begins to take off to positive infinity. To find this x value of the vertical asymptote, we need to find when the equation will be "undefined" or when there is a 0 in the denominator. In the bottom of the square root you can see that $(1 - v^2/c^2)$. To get a zero you need to have $1 - 1$ and to get another "1" you will need $v^2/c^2 = 1$. To do this you have $v^2 = c^2$ and this will give you "1". Because of this we know that the vertical asymptote is at $x = (3 \times 10^8)^2$. Because the graph goes on and on to infinity, this tells
us that it takes an infinite amount of energy to travel at the speed of light – which basically means it is impossible to travel faster than the speed of light.

b: The next question asks us what the graph tells us about the approximation. The approximation formula and the exact formula look very similar when using smaller numbers, but when you use a larger interval you can see the difference. (see graph). For smaller values of $v$ will give you a good approximation to $E$.

**Conclusion**

In context with the real world, when velocity of something is comparable to the speed of light we can use an exact equation to find the energy, unlike the inexact equation when the velocity is much smaller than that of light. This results in a more accurate answer. The energy equation used here is very common in physics equations with masses moving extremely fast and needing to know the amount of energy to allow them to move that quickly.