Wolverine Pathways: Math Circle
October 8, 2016

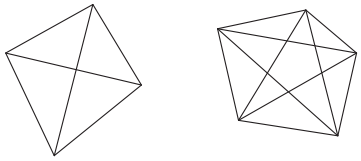
Problem 1 (The Handshake Problem)**.** (a) There are 100 people in a room. Each person shakes the hand of every other person in the room. How many handshakes are made?

(b) What if there are 500 people in the room?

(c) What if there are n people in the room?

(d) What are some things your group did that were helpful in solving the problem?

Problem 2 (Counting Diagonals)**.** A *diagonal* of a polygon is a straight line connecting to non-adjacent vertices. For example, a square (or any quadrilateral) has two diagonals. A pentagon has 5 diagonals.



(a) Complete the following table.

Keep track of shortcuts you may be using for calculating the number of diagonals, as you will need them in the next two parts of the problem.

Number of sides	Number of diagonals
4	2
5	5
6	9
7	
8	
9	

(b) A polygon with n sides is called an n-gon. How many diagonals does a 20-gon have? (Don't try to draw it!)

(c) How many diagonals does an n-gon have? Express your answer in terms of n. Explain your work.

Problem 3 (Tiling with Dominoes)**.** (a) How many ways can you fill up (or *tile*) the grid in Figure 1(A) using dominoes (a domino is shown in Figure 1(D))? How about the grid in Figure 1(B)? How about the grid in Figure 1(C)? Note: You must place the dominoes either vertically or horizontally and they have to stay inside the rectangles.

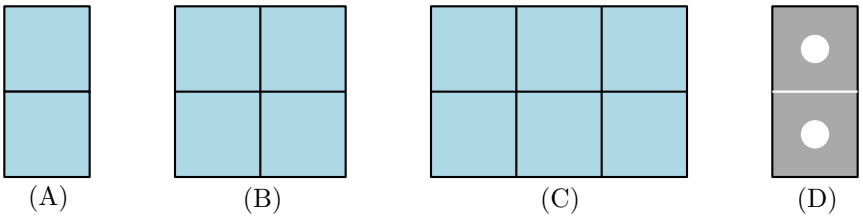


FIGURE 1. (A) 2×1 grid, (B) 2×2 grid, (C) 2×3 grid, (D) a domino.

(b) Did you see a pattern from part (a)? How many ways do you think you can tile the 2×4 grid? Do it out; did you get what you expected?

(c) How many ways can you tile the $2 \times n$ grid?

Problem 4 (The Locker Problem)**.** A school hallway has a row of lockers numbered 1 to 100. While cleaning the building over the summer, a janitor opens all of the lockers. The janitor then closes every second locker (that is, those numbered $2, 4, 6$, etc.), walks back to the start of the row, and changes every third locker: closing it if it is open and opening it if it is closed. In the next stage the janitor changes every fourth locker, and so on. At the last stage, the janitor changes every 100^{th} locker (that is, only the last locker). Which lockers are open at the end?

Logic Puzzles

Problem 5. (a) A farmer has to transport a wolf, a sheep, and a head of cabbage across a river. He has one boat, but he can only fit one of these objects with him in the boat. The problem is: if he leaves the cabbage alone with the sheep, the sheep will eat the cabbage, and if he leaves the sheep alone with the wolf, the wolf will eat the sheep. How can the farmer get the wolf, sheep, and cabbage safely across the river?

(b) The farmer wises up and trades in for a bigger boat. He can now fit two objects in the boat with him. Unfortunately for him, now the wolf is very hungry, and if he is left alone with the cabbage, he'll eat it. The farmer still can't leave the cabbage alone with the sheep, or the sheep alone with the wolf. Now how can the farmer get the wolf, sheep, and cabbage safely across the river?

(c) Now, a man and woman and their two children want to cross a river. They have a boat that can hold no more weight than the weight of one adult. Assuming that the two adults weigh the same amount, that each of the children way half of that amount, and that the chilren are old enough to paddle the boat themselves. What is the smallest number of times the boat has to go across the river (in either direction) to get them all across?

Problem 6. (a) In ye olde town of Avershire, there are only knights and jokers. Knights always tell the truth, and jokers always lie. We meet Larry and Moe. Larry says "We are both jokers." What is Larry? What is Moe?

(b) (a) We meet a man named Curly. Curly says, "I am a knight." Then
(i) Curly is a knight
(ii) Curly is a joker
(iii) We cannot tell what Curly is

Then Curly says, "Larry is a knight." Then
(i) Curly is a knight
(ii) Curly is a joker
(iii) We cannot tell what Curly is

(b) Next, we meet Arthur and Gwen.
Arthur says: Either we are both knights, or we are both jokers; we are of the same kind.
Then Gwen says: We are of different kinds.

What is Gwen?

What is Arthur?