Accounting Discretion, Voluntary Disclosure Informativeness, and Investment Efficiency

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Abstract

Discretion pervades the accounting rules. Proponents argue that allowing discretion enables managers to incorporate more information in their disclosures, while opponents believe that managers can abuse discretion and engage in earnings management at the expense of shareholders. We explicitly model accounting discretion and earnings management in an investment setting and use this setting to study the interaction between management’s voluntary disclosure and the subsequent mandatory disclosure of value-relevant information, and its implications for investment decisions. We show that, in equilibrium, allowing the manager to have some discretion over the mandatory financial reports may enhance the informativeness of the more-timely voluntary disclosure, leading to more efficient investment decisions. However, allowing too much discretion for earnings management may not increase the informativeness of voluntary disclosures and investment efficiency. Thus there may be a hidden benefit of granting some (but not too much) discretion in firms’ mandatory financial statements.

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1. Introduction

We investigate analytically the interaction of a firm’s voluntary disclosure, investment decision, and the subsequent mandatory disclosure of value-relevant information and explore the effect of accounting discretion allowed in the mandatory disclosure on the informativeness of the voluntary disclosure and investment efficiency. We utilize a setting where a firm’s manager has private information about the firm’s value that cannot be directly and verifiably communicated to outsiders and study how introducing accounting discretion affect his voluntary and mandatory disclosures as well as the investment decision. We show that, in equilibrium, allowing the manager some discretion over the mandatory financial reports may enhance the informativeness of the more timely voluntary disclosure, a counterintuitive result. Such increased informativeness of the more timely voluntary disclosure will also have beneficial real effects and increase investment efficiency. We also show that too much discretion can render the voluntary disclosure uninformative and thus will not have such beneficial real effects. Thus from the perspective of maximizing investment efficiency, allowing the manager to manage the reported earnings can be beneficial, but there is a maximal amount of discretion that should be tolerated, consistent with the discretion embedded in the generally accepted accounting principles (GAAP).

Discretion pervades the accounting rules, as reflected by the wide use of estimates in the measurement of various accounting items (e.g., bad debt expense, contingent liabilities), choice of accounting methods for the same economic transactions and assets and liabilities (e.g., straight-line vs. accelerated depreciation, cost model vs. revaluation model for long-lived assets), etc. Proponents argue that such discretion has various benefits. For example, Dutta and Gigler (2002), from a contracting perspective, demonstrate that earnings management reduces the cost of eliciting truthful voluntary disclosures from managers. Dye (1988) shows that agency considerations result in an equilibrium demand for earnings management because compensation contracts designed to motivate managerial effort become more efficient, while Trueman and Titman (1988) show that a firm has an incentive to smooth reported earnings when debtholders use these reported numbers to estimate the volatility of its earnings. In a related paper, Fishman and Hagerty (1990) show in a persuasion game setting that allowing
some reporting discretion may or may not result in improved informativeness of disclosure, depending on which of the equilibria is chosen. In the papers mentioned above, discretion gives managers the option of misreporting what they observe, with arguably desirable results.

In a more recent paper, Drymiotes and Hemmer (2013) investigate the role of accruals in earnings quality from both valuation and stewardship perspectives. They find that neither an “aggressive” nor a “conservative” accrual strategy is optimal. The accrual strategy in their setting generates biased earnings numbers ex ante, which differs from the ex post earnings management that we focus on.

Critics complain that discretion enables managers to engage in “earnings management,” that is, to use the discretion provided by GAAP to manipulate accounting data and report income numbers that reflect managers’ objectives rather than the true economic income numbers of firms (Schipper 1989). The Securities and Exchange Commission (SEC) has publicly stated that earnings management is one of the greatest evils plaguing the accounting profession (Levitt 1998). This concern has been reinforced by such high-profile accounting scandals as those at Enron and WorldCom. Empirical studies provide strong evidence that firms manage earnings to meet or beat earnings targets, such as last years’ earnings or consensus forecasts of earnings provided by financial analysts and firms themselves (see, for example, Burgstahler and Dichev 1997; Degeorge et al. 1999; Bartov et. al. 2002). This line of research argues that earnings management decreases investor confidence and undermines the credibility of accounting reports. To date, there has been a lack of systematic studies on the benefit-cost tradeoff of allowing some degree of discretion in accounting rules in the presence of both voluntary and mandatory disclosures.2

The empirical literature indicates that earnings management is strongly related to earnings forecasts (e.g., Degeorge et al. 1999) and that voluntary disclosure and mandatory financial reports complement each other because the primarily backward-looking and less

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1 Other examples include Carter et al. (2007) and Roychowdhury (2006). Carter et al. (2007) document that firms facing financial reporting concerns, such as meeting or beating earnings benchmarks and avoiding violation of debt covenants, overused stock options to compensate their employees before the enactment of SFAS 123R, which required the expensing of stock options. Roychowdhury (2006) provides evidence consistent with managers manipulating real activities (e.g., offering price discounts to temporarily increase sales or overproducing to report lower cost of goods sold) to avoid reporting annual losses or to meet annual analyst forecasts.

2 Dye and Verrecchia (1995), in a principal-agent setting, study the effect of allowing managerial discretion in reporting current period expenses. They show this can be desirable sometimes and undesirable other times. Fischer and Verrecchia (2000), Sankar and Subramanyam (2001), and Ewert and Wagenhofer (2005, 2013) study the effect of accounting discretion on the properties of mandatory accounting reports in market settings.
timely mandatory reports encourage managers to disclose voluntarily (and credibly) their forward-looking and timely private information (Ball et al. 2012). However, there is little theoretical research so far that simultaneously examines managers’ voluntary earnings forecasts and the subsequent management of the mandatorily announced earnings. The voluntary and mandatory disclosures made by managers, as well as the equilibrium price response to them, are endogenously determined. Thus if there are relationships among them, they must be shown to hold in equilibrium. In fact, Ball et al. (2012) strongly assert that the economic roles of (audited) mandatory and (unaudited) voluntary disclosures cannot be evaluated separately. Beyer et al. (2010) also emphasize the need for models to examine voluntary and mandatory disclosures simultaneously. We respond to such calls and propose an analytical model that jointly determines the equilibrium voluntary and mandatory disclosures.

Our model builds on early voluntary disclosure literature which generally assumes that the manager is either uninformed or perfectly informed (e.g., Grossman 1981; Dye 1985; Jung and Kwon 1988). This stream of work demonstrates that, in equilibrium, the manager either discloses the entire truth or keeps silent. Shin (1994) studies a setting in which the manager simultaneously learns both good and bad news and shows that the optimal disclosure strategy is the “sanitization strategy,” that is, disclosing only good news (the lower bound of the firm’s value that can be justified by his signals) and withholding all bad news (the upper bound of the firm’s value that can be justified by his signals). Shin (1994) introduces imprecision to the manager’s information set; that is, the manager learns a possible set (or a range) of values that the underlying “true” earnings will fall in, and as a result, his voluntary disclosure can be interpreted as either a range or a point disclosure and does not have to be the entire truth. This feature is empirically appealing, and our setting allows us to preserve this interpretation of the manager’s voluntary disclosure. We build our model using the private information structure motivated by Shin (1994), but we relax the assumption that the true earnings is verifiable, thus allowing the manager to manage the reported earnings when necessary.

Specifically, the manager of a firm observes privately some information regarding the underlying state which determines both the value of the firms’ asset in place as well as the

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3 “[W]e encourage researchers to investigate the interplay between management forecasts and mandatory disclosures in general … [It] would be useful to combine analyses of voluntary disclosures with mandatory disclosure to investigate … the extent to which mandatory disclosure requirements affect … the amount of voluntary disclosures” (Beyer et al. 2010, 314).
value of the firm’s growth options. If the state is good, it is optimal to exercise the growth option immediately because delayed exercise decreases the value of the option, for example, due to competition. If the state is bad, it is optimal to not exercise the growth option. The manager can make a voluntary disclosure of the underlying state based on his private information and choose whether to exercise the growth option early. The manager then learns the realization of the true state and has to issue a mandatory earnings report. The manager chooses the disclosure strategies and the investment strategies to maximize a weighted average of the short-term price (i.e., price after voluntary disclosure) and long-term price (i.e., price after mandatory disclosure). We assume that the true state which determines the true earnings can be observed only by the manager and is never observed by the outside investor, at least during the manager’s horizon. What the outside investor observes are the voluntary disclosure and the reported earnings, which can be managed by the manager. We assume that earnings management is personally costly to the manager and interpret (the inverse of) the cost of successful earnings management as the discretion granted by GAAP to the manager. In other words, for a given amount of earnings management, higher personal cost represents lower accounting discretion.

We show that, relaxing the maintained assumption in the prior disclosure literature that the manager cannot manage the reported earnings in a setting of a joint determination of voluntary and mandatory disclosures can generate interesting insights. Specifically, in a pure exchange economy, allowing a limited degree of discretion to the manager to manage the mandatorily reported earnings does not lead to enhanced voluntary disclosure. However, when real investment decisions are explicitly considered, we demonstrate that such discretion over mandatory earnings report enables the manager to incorporate more of his private information in his voluntary disclosure and make more efficient investment.

The intuition underlying the result is as follows. In a pure exchange economy, the manager’s voluntary disclosure does not affect the realization of the true earnings and is thus costless. Thus, the manager will always make the voluntary disclosure that generates the highest price response to such a disclosure, regardless of whether he will manage the subsequent mandatory disclosure. This results in completely uninformative voluntary disclosure both in the presence of and absent discretion over mandatory disclosure.
In an economy where voluntary disclosure has to go in hand with real investment choice, however, voluntary disclosure is no longer costless because it affects the realization of true earnings through the investment decision. As a result, allowing some discretion to manage reported earnings gives an opportunity for a manager with favorable private information (or, a High type manager) to separate himself from a manager with bad private information (or, a Low type manager). More specifically, when no discretion is granted, and if the manager cares sufficiently more about short-term price response to voluntary disclosures, both the High and Low types will choose again the (same) voluntary disclosure that results in the highest price response. If discretion over mandatory disclosure is allowed, a High type manager is able to communicate a more optimistic voluntary disclosure and choose to invest immediately (i.e., to exercise the growth option) to inflate the price of the firm. He would do this knowing that the chance of the true state being low would be small and therefore the chance of managing the reported earnings would also be small. A Low type manager, however, knowing that the chance of the true state being low is high, would in expectation find it too costly to mimic the voluntary disclosure and investment strategy of a High type. This is because if he choose to mimic a High type and issue a more optimistic voluntary disclosure and exercise the growth option immediately, there is high probability that underlying outcome is actually low. If this turns out to be the case, he has two options. He can either choose to incur a cost to engage in earnings management to cover the fact that he has made an inefficient investment, or choose not to manage reported earnings so that investors know the state is actually low. In either case, there is an endogenous cost of the inconsistency between the state inferred from his private information and that revealed in his voluntary disclosure. As a result, a Low type chooses a more pessimistic forecast and refrains from exercising the growth option. Therefore, allowing the manager some discretion to manage the reported earnings can lead to a (counterintuitive) result: the earlier voluntary disclosure reveals more of the manager’s private information—the probability distribution of the earnings, resulting in more efficient investment decision.

This intuition, however, does not hold at the other extreme. When the manager is granted too much discretion, he can almost always manage the reported earnings to a target level with no reference to the true earnings. As a result, he will always issue an optimistic

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4 If the voluntary disclosure is inconsistent with the investment decision, i.e., the manager discloses that the state is high but does not invest, investors will rationally ignore such voluntary disclosure. Equivalently, we can interpret the investment decision as voluntarily disclosing value-relevant information.
forecast regardless of his private information and invest immediately, knowing that he can always manage the reported earnings later if the true state turns out to be low. The voluntary disclosure and the investment decision will then have no information content, leading to inefficient investment behavior. Thus an intermediate degree of discretion allows for the improved informativeness of voluntary disclosure and enhanced investment efficiency.\(^5\)

Our paper makes several contributions to the literature. First, we uncover a hidden benefit to earnings management, thus providing some rationale for the allowance of discretion imbedded in GAAP. In this sense, our paper adds to the literature on the desirability of some tolerance for earnings management, although we derive this intuition in a market setting with real effects as opposed to an agency setting, as in Arya et al. (1998), Dutta and Gigler (2002) and Arya et al. (2003). Sankar and Subramanyam (2001) consider a two-period model and focus on mandatory disclosure with the possibility that part of the earnings management has to be reversed in the second period, whereas we focus on a single period and show that discretion results in the manager incorporating his private information in his voluntary disclosure without such reversal. Stocken and Verrecchia (2004), and Ewert and Wagenhafer (2013) introduce nonfinancial information privately owned by a manager that can at most be partially captured by the firm’s accounting system. The former show that a privately informed manager may endogenously choose an imprecise accounting system and the efficiency of this choice depends on the usefulness of accounting information. The latter show that, under certain conditions, less discretion allowed in reporting earnings reduces earnings quality. Both papers focus on mandatory disclosure, while we consider both mandatory and voluntary disclosures and the effect on investment of the interaction of the two disclosures. Finally, Fishman and Hagerty (1990) study the effect of discretion on the property of disclosure. However, in their model, there is only one managerial disclosure decision, and the discretion allowed is directly over that single disclosure. The manager is also required to disclose a signal from a pre-specified subset of available signals (which makes the disclosure more of a mandatory one in nature). Thus the disclosure itself must be truthful. The authors are therefore silent about the interaction between the two distinct forms of disclosure that are central to accounting. Our contribution is to

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\(^5\) Beyer et al. (2010) document that a great majority of the firm-provided financial information is conveyed through managers’ voluntary disclosure. Thus improving the informativeness of voluntary disclosure is potentially important for resource allocation in capital markets. We show here that the increased informativeness of voluntary disclosure has direct real effects of improving firms’ investment decisions.
demonstrate the desirability of granting the manager discretion over his mandatory disclosure as it enhances the informativeness of the more timely voluntary disclosure, resulting in higher investment efficiency.

Second, we jointly study voluntary and mandatory disclosures, while focusing on the interaction between those two types of disclosures. Our result shows that investigating the desirability of accounting discretion in the sole framework of earnings management and whether it impairs the integrity of mandatory disclosures can be misleading, because prohibiting the management of mandatory reports can have some unintended consequences for the voluntary disclosures that have implications for investment efficiency. We propose that studies of the benefits or costs of earnings management should jointly consider the tradeoff between those two types of disclosures.

Einhorn and Ziv (2012) examine a double-tier disclosure decision and explore the interactions between the decision to disclose and the bias in the disclosure if it is made. The two dimensions of disclosures are assumed to be made simultaneously in their model, which ignores the potential disciplinary effect of subsequent mandatory disclosure has on the voluntary disclosure that we focus on. Kwon et al. (2009) explicitly model such an effect. However, in their model, mandatory disclosure cannot be managed. Several other papers maintain the assumption that mandatory disclosure cannot be managed or management (if allowed) of mandatory disclosure may be detected and penalized and demonstrate that the credibility of voluntary disclosure can be sustained (Sansing 1992; Stocken 2000; Lundholm 2003; Korn 2004). Beyer (2009) also jointly studies managers’ optimal voluntary and mandatory disclosure strategies in a setting where the capital market is ignorant of both the mean and the variance of firms’ underlying cash flows. However, her focus is on the properties of capital market responses, while ours is on the properties of managers’ disclosures and, in particular, the relationship between mandatory and voluntary disclosures. Other related studies include Einhorn (2005), Bagnoli and Watts (2007), and Beyer and Guttman (2012), where voluntary disclosures are required to be truthful. Stocken (2013) provides a synthesized review of the theoretical literature on managers’ strategic voluntary disclosure as the scope of discretion retained by managers varies from zero to infinity. A closely related paper is Boisits (2013), who studies the interplay between the two forms of disclosures by restricting to a constant bias.

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6 We thank Phil Stocken for pointing this out.
mandatory reporting strategy and imposing two exogenous costs related to the disclosures to the model: 1) cost as a function of the difference between voluntary and mandatory disclosures and 2) cost as a function of the bias in mandatory disclosure. We endogenously derive the cost of discrepancy between the voluntary and mandatory disclosures,\(^7\) and we impose no constraint on the form of mandatory disclosure.

By investigating the interaction between mandatory and voluntary disclosures, we also add to the literature on the economic role of accounting disclosures. In this regard, we take the same stand as Gigler and Hemmer (1998) and Gigler and Jiang (2015). Their studies emphasize the “confirmatory role” of mandatory accounting reports: by providing backward-looking information, they confirm the credibility of firms’ more timely voluntary disclosures of forward-looking information. This view is empirically supported by Ball et al. (2012) and is strongly advocated by Ball et al. (2012) and Beyer et al. (2010). Our paper further shows that, even though limiting managers’ discretion over financial reporting strengthens the “confirmatory role” of the mandatory reports, overly restricting managers’ reporting behavior can lead to decreased informativeness of the voluntary disclosures. This is new in the literature.

Prior accounting studies on voluntary disclosure assume that any managerial disclosure can be costlessly and immediately verified by external parties but that a claim of ignorance cannot be verified. They demonstrate that, in equilibrium, the manager either discloses the entire truth or only discloses if the earnings news is sufficiently good (e.g., Grossman 1981; Verrecchia 1983; Dye 1985; Jung and Kwon 1988; Shin 1994). These studies show that voluntary disclosures are themselves managed, but they do not investigate the strategic interaction between voluntary disclosures and the mandatory disclosures that follow, because they assume that voluntary disclosures can be verified by the subsequent mandatory disclosure and that the mandatory disclosure cannot be managed. We relax this assumption and show that introducing such an important institutional feature of accounting generates interesting novel insights.

Third, we derive the conditions under which allowing reporting discretion can have real benefits in the sense of investment efficiency in the presence of myopic managers. There is a huge stream of literature documenting investment inefficiencies generated by managerial

\[^7\] Most of the studies assume an exogenous cost if voluntary disclosure differs from the mandatory disclosure. Notable exceptions are Stocken (2000) and Lundholm (2003), who endogenize the cost in a repeated game setting. Beyer and Guttman (2012) introduce (endogenous) real costs to discipline the voluntary disclosure.
myopia (e.g., Gigler et al. 2014, Edmans et al. 2014, and Kraft et al. 2014). Most of those papers argue that managers’ concern about short-term price response leads them to engage in both accrual-based and real earnings management for their own benefit. In our setting, taking away manager’s reporting discretion results in both High and Low type managers issuing the same optimistic voluntary disclosure and investing immediately. Such inefficient overinvestment behavior can be avoided if some degree of reporting discretion is granted so that High type managers can separate themselves from Low type managers. Thus, our paper shows that restricting managers’ ability to manage earnings as a way of curbing managerial myopia may harm the investment efficiency.

Finally, our paper also provides some novel—and potentially testable—empirical implications regarding the relationship between earnings management and investment efficiency. The implication that the discretion embedded in the accounting rules could be positively associated with investment efficiency has, to the best of our knowledge, not been tested.

The rest of the paper is organized as follows. Section 2 outlines the basic model without real effects. Section 3 defines and derives the equilibrium disclosure strategy and market response of the basic model. Section 4 extends the basic model to incorporate real investment decisions and show that in this setting, allowing reporting discretion can result in more efficient investment decisions. Section 5 discusses some empirical implications and Section 6 concludes. The appendix contains all the proofs.

2. The basic model without investment

There is a risk-neutral manager who runs the firm, and a continuum of risk-neutral shareholders of the firm. For exogenous reasons (for example, liquidity needs), all investors need to sell their shares before consuming the true income of the firm. More specifically, a proportion $\alpha$ of investors need to sell immediately at the anticipated date of the firm’s (possible) voluntary disclosure, and the rest $(1 - \alpha)$ will sell after the firm’s mandatory disclosure. Disclosure thus plays a role in determining the selling price. We can also view the proportion $\alpha$ of investors who sell earlier as short-term investors and the rest long-term investors. There are

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8 The timeline of the voluntary and mandatory disclosures will be discussed later.
two states of nature, denoted by $H$ and $L$, respectively, that occur with equal probability. The firm is endowed with a project that generates underlying unmanaged true earnings of $x_i$ in state $i$, where $i = H$ or $L$. Without loss of generality, we assume that $x_H > x_L > 0$. The manager observes a noisy signal $\tilde{s} \in \{h, l\}$ that is informative of the underlying state in the following manner:

$$p(s = h| i = H) = p(s = l| i = L) = q \in \left(\frac{1}{2}, 1\right).$$

(1)

We further assume that the manager cannot credibly convey $\tilde{s}$ to the public directly due to their non-verifiable nature. Other than the manager’s observed signal and, later on, the realized true earnings, everything else is common knowledge. Our introduction of the noisy signals observed by the manager is a parsimonious way of introducing one of the key features of our model: even if the manager does not know the underlying state precisely, he may still possess more information than outside investors about what the true earnings will be.

After observing the private signal, the manager has the option to issue a voluntary disclosure, $D$, at some anticipated date. Because the state of nature is binary, there are two possible voluntary disclosures: $D_L \equiv \{x \geq x_L\}$, interpreted in equilibrium as earnings is at least $x_i$ as in Shin (1994), and $D_H \equiv \{x \geq x_H\}$, interpreted as earnings is at least $x_H$ (which, in our case, is equivalent to $x = x_H$). $^9$ The true earnings number is then realized and is privately observed by the manager. The manager subsequently issues a mandatory disclosure, $R \in \{R_H, R_L\}$, where $R_H \equiv \{x = x_H\}$ and $R_L \equiv \{x = x_L\}$. We depart from the prior literature by assuming that true earnings cannot be observed or verified by anybody, at least during the manager’s horizon, which is not unreasonable because (1) accounting involves accruals and deferrals that make the underlying earnings generating process not so transparent, and (2) the manager’s horizon is generally shorter than that of a firm. Thus we explicitly allow the manager to be able to manage the mandatory disclosure, but at some personal cost, $k$. For example, if the manager voluntarily discloses $D_H$, while the true earnings turns out to be $x_L$, the manager then has two options: he can choose not to manage the mandatory disclosure and announce $R = R_L$.

$^9$ The manager can also choose to keep silent. In our model remaining silent is equivalent to disclosing $D_L$. As in Shin (1994), it can also be verified that the manager will never find it optimal to disclose only the upper bound of the earnings, that is, earnings is at most $x_H$ or $x_L$. 

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or he can choose to incur a cost $k$ and engage in earnings management that increases the reported earnings to $R_H$. We use $m(D, x, R) \in \{0,1\}$ to denote the manager’s earnings management strategies with $m(D, x, R) = 0$ (or 1) denoting no earnings management (or earnings management). We also assume that the cost of downward earnings management is also $k$, but the manager will never find it optimal to involve in downward earnings management.

The manager chooses $D$, $R$, and $m$ to maximize $\alpha P(D) + (1 - \alpha)P(R, D; m) - km$, that is, a weighted average of the selling price of the firm conditional on the two disclosures, less the (possible) earnings management costs.\(^{10}\) We interpret the manipulation cost $k$ as representing accounting discretion with higher $k$ representing lower accounting discretion allowed.\(^{11}\)

The timeline of the model is summarized as follows.

**Date 0:** The underlying state $i \in \{H, L\}$ is realized, and the realization of $\tilde{s}$ is then determined based on equation (1). The manager observes the signal $\tilde{s}$.

**Date 1:** At some anticipated date, the manager (possibly) makes a voluntary disclosure, $D$, regarding the firms’ future true earnings. The market prices the firm at $P(D)$.

**Date 2:** The firm’s true earnings number is realized and observed by the manager. The manager then issues a mandatory disclosure of the firm’s earnings, $R$, that is subject to potential management, $m$, by the manager. The market prices the firm at $P(R; D, M)$. The manager chooses $D$, $R$, and $m$ to maximize $\alpha P(D) + (1 - \alpha)P(R, D; m) - km$.

**Date 3:** The firm’s true earnings are revealed and consumed.

In the next section, we solve for the equilibrium (pure) voluntary and mandatory disclosure strategies of the manager and show that granting no discretion or some degree of discretion over the mandatory disclosure has no effect on the informativeness of the voluntary disclosure. Appendix A summarizes the notations used in this paper.

### 3. Equilibria in the basic model

\(^{10}\) Throughout the paper, we use the upper case letter $P$ to represent the price response and the lower case letter $p$ to represent probability.

\(^{11}\) Gao and Jiang (2015) use a similar approach to model discretion.
Denote $\Omega$ as manager’s information set before making a voluntary disclosure. From the discussion above the manager observes either $x_i$ or $\bar{s}$. Let $E[.]$ and $p(.)$ be the expectation and probability operators respectively. The Bayesian-Nash equilibrium of this game is defined as follows. We focus on pure strategy equilibrium unless otherwise stated, with the off-equilibrium beliefs satisfying the criteria in Cho and Kreps (1987).

**Definition:** A Bayesian-Nash equilibrium contains the manager’s voluntary disclosure strategy, $D(\Omega)$, mandatory disclosure strategy, $R(D, x)$, earnings management strategy $m(D, x, R) \in \{0,1\}$, and the market’s pricing rules at the voluntary and mandatory disclosure dates, $P(D, R; m)$ and $P(D, R; m)$ respectively, such that:

(i) $D(\Omega), R(D, x), \text{and } m(D, x, R)$ maximize the manager’s expected payoff, $\alpha \hat{P}(D) + (1 - \alpha)\hat{P}(D, R; m) - km(D, x, R)$ where $\hat{P}(D)$ and $\hat{P}(D, R; m)$ are the manager’s conjectures about the investors’ pricing rules in response to the voluntary and mandatory disclosures, respectively;

(ii) The risk-neutral investor sets up $P(D) = E[\bar{x} | D, \hat{R}(D, x); \hat{m}(D, x, R)]$ and $P(D; R, m) = E[\bar{x} | D, R, \hat{m}(D, x, R)]$ after observing $D$ and $R$, respectively, where $\hat{R}(D, x)$ and $\hat{m}(D, x, R)$ are the investors’ conjectures about the manager’s mandatory disclosure and earnings management strategy, respectively;

(iii) $P(D, R; m) = \hat{P}(D, R; m), \hat{R}(D, x) = R(D, x), \text{and } \hat{m}(D, x, R) = m(D, x, R)$. i.e., in equilibrium, the manager’s conjectured pricing rules, and the market’s conjectured mandatory disclosure and earnings management strategies, are consistent with the actual pricing rules of the market, and the actual mandatory disclosure and earnings management strategies of the manager;

(iv) The conditional distribution of $R$ on $x$ is consistent with the manager’s mandatory disclosure strategy, $R(D, x)$, and his earnings management strategy $m(D, x, R)$, i.e., $p(R = R_H | R(D, x_H) = R_H, m = 0) = 1$ and $p(R = R_L | R(D, x_L) = R_L, m = 0) = 1 \forall D, m; \text{and, } p(R = R_H | R(D, x_L) = R_H, m = 1) = 1$ and $p(R = R_L | R(D, x_H) = R_H, m = 1) = 0 \forall D$. 


We first examine the case when $k$ is large enough so that virtually no discretion is granted over mandatory disclosures.  

**Proposition 1:** In any equilibria where $m=0$, voluntary disclosure is completely uninformative, i.e., $P(D_H) = P(D_L)$.

**Proof:** All proofs are in Appendix B.

The intuition of Proposition 1 is straightforward. When $m = 0$, $P(R,D;0) = R$, which is independent of $D$. The manager therefore chooses $D$ to maximize $\alpha P(D)$. Thus, if $D$ is informative, i.e., $P(D_H) \neq P(D_L)$, then the manager will choose the voluntary disclosure that results in a higher price, regardless of his private information about the state. Thus the voluntary disclosure has no information content.

Proposition 1 is counterintuitive in the sense that not allowing earnings management results in completely uninformative voluntary disclosure in a market setting, a result that is in striking contrast to the notion of the disciplinary role of mandatory disclosure demonstrated in the prior literature (e.g., Gigler and Hemmer 1998, 2001). The reason is that the works of Gigler and Hemmer focus on a contracting setting where there is an endogenous cost of mandatory disclosure being inconsistent with voluntary disclosure. In our basic setting, even though a low mandatory disclosure results in low price response, this price response is independent of the prior voluntary disclosure because the manager has to truthfully report the actual realization of the true earnings. Thus the disciplining role of mandatory disclosure over its preceding voluntary disclosure is absent here, rendering voluntary disclosure uninformative.

A naturally question is that will this result change when managers are allowed to manage their mandatory disclosure? The next proposition shows that in a pure exchange economy, even if discretion over mandatory disclosure is allowed, there is still no credibility to voluntary disclosures.

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12 A sufficient condition is $k > x_H$. 

Proposition 2: In any equilibria, voluntary disclosure is completely uninformative.

The underlying intuition is as follows. Suppose a High type manager (i.e., a manager who observes \( s = h \)) prefers disclosing \( D_H \) and managing subsequent mandatory disclosure if the state turns out to be low, to disclosing \( D_L \) and manipulate. This implies that \( P(D_H) \) must be sufficiently larger than \( P(D_L) \). However, if \( P(D_H) \) is sufficiently larger than \( P(D_L) \), a Low type manager (i.e., \( s = l \)) will find it beneficial to disclose \( D_H \) and do not manage reported earnings. As a result, voluntary disclosure loses its information role.

The driving force of the intuition underlying Proposition 2 is the lack of real consequences, i.e., disclosing \( D_H \) or \( D_L \) has not effect on firms’ true earnings. Thus, whether to disclose \( D_H \) or \( D_L \) depends crucially on the difference between \( P(D_H) \) and \( P(D_L) \), which is independent of whether or not to manage mandatory reports, which further depends on the amount of discretion allowed, \( k \). A Low type manager can still disclose \( P(D_H) \) and choose not to manage reported earnings (if \( k \) is too large) without getting penalized because the true earnings is not affected by \( s \).

We now move to an extended model that incorporates investment. We show that, in this case, allowing a limited amount of reporting discretion can sometimes unambiguously improve not only the informativeness of voluntary disclosure but also the real investment efficiency.

4. The model with real effects

There are two states of nature, denoted by \( H \) and \( L \), respectively, with equal prior probability. The firm is endowed with an asset-in-place that generates underlying unmanaged true earnings of \( x_i \) in state \( i \), where \( i = H \) or \( L \). Without loss of generality, we assume that \( x_H > x_L \equiv 0 \). The firm is also endowed with a growth option. The growth option requires a fixed investment of \( I > 0 \) and generates a payoff depending on the underlying state of nature, to be discussed in more detail below. The manager observes a noisy signal \( \hat{s} \in \{h, l\} \) that is informative of the underlying state in the following manner:

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p(s = h| i = H) = p(s = l| i = L) = q \in \left(\frac{1}{2}, 1\right).
\] (2)
We assume that the manager cannot credibly convey $s$ to the public directly due to their non-verifiable nature. Other than the manager’s observed signal and, later on, the realized state of nature, everything else is common knowledge.

After observing the private signal, the manager could choose to invest an amount of $I$ in the growth option immediately. The payoff of the growth option realizes after the manager leaves the firm. The payoff, if the option is exercised immediately and the underlying state is $H$, will be $x_H + \delta - I$. However, it is $x_L - I$ (or simply $-I$ since $x_L \equiv 0$) if the option is exercised immediately and the underlying state is $L$. The parameter $\delta > 0$ represents the importance of exercising the growth option in a more timely fashion.\(^{13}\) This assumption is meant to capture the fact that exercising the growth option in a less timely manner will result in lower payoffs due to, e.g., actions taken by competitors. The underlying state and thus the true earnings number from the asset-in-place is then realized and is privately observed by the manager. The manager subsequently issues a mandatory disclosure, $R \in \{R_H, R_L\}$, where $R_H \equiv \{x = x_H\}$ and $R_L \equiv \{x = x_L\}$. If the manager did not exercise the growth option immediately after observing the private signal, he can choose to let investors exercise the growth option after he leaves the firm and the true state is revealed to everybody. The payoff of the growth option, if not exercised immediately after voluntary disclosure, is $x_i - I$ for $i = H, L$. To make the problem interesting, we assume that $x_H > I > 0 = x_L$, i.e., if it is left to the investors to exercise the growth option, it is optimal to invest when the true state is $H$ and not if the true state is $L$. We denote manager’s decision of whether to exercise growth option immediately or not as $D \in \{D_H, D_L\}$ with $D_H$ representing exercising growth option immediately (i.e., disclosing that the state is good) and $D_L$ representing not exercising growth option immediately (i.e., disclosing that the state is bad). This denotation allows us to conveniently interpret $D_H$ and $D_L$ as voluntary disclosure in the same fashion as in sections 2 and 3. We again explicitly allow the manager to be able to manage the mandatory disclosure at some personal cost $k$. The earnings management technology is the same as modeled in section 2 and is thus not repeated. The manager chooses $R, m$ and $D$ to maximize the same objective function,

\(^{13}\) One can think of this payoff of the growth option as an expectation of an uncertain payoff from the growth option in the future rather than literally as a certain payoff. None of the results will change because of risk-neutrality.
\[ \alpha P(D) + (1-\alpha)P(R, m, D) - km \] The manager then leaves the firm. The true state and, correspondingly, the value of the firm will be revealed to public after the manager leaves.

Finally, we introduce some assumptions on the parameters to make the problem interesting. We assume that for a High type manager (i.e., \( s = h \)), it is more efficient to exercise the growth option immediately, i.e.,
\[ q[(2x_H + \delta) - I] + (1-q)(-I) > q(2x_H - I), \]

or, equivalently,
\[ q\delta > (1-q)I \quad (3) \]

Similarly, for a Low type manager (i.e., \( s = l \)), it is more efficient to not exercise the growth option immediately, i.e.,
\[ (1-q)[(2x_H + \delta) - I] + q(-I) < (1-q)(2x_H - I), \]

or, equivalently,
\[ qI > (1-q)\delta \quad (4) \]

When equations (3) and (4) are satisfied, it is straightforward to show that letting a High type manager to always exercise the growth option immediately and letting a Low type manager to always wait to exercise the growth option till the mandatory disclosure is released or delegate the exercise to investors after the true state is revealed to public is the ex-ante most efficient investment policy. We will later explore whether allowing some degree of reporting discretion will lead us to this most efficient investment policy.

One new key assumption in this setup is that the underlying state determines both the payoff of the asset-in-place and that of the growth option. Similar assumption has been adopted in Gao and Liang (2013) to capture the correlation between the value of the asset-in-place and that of the growth option.

The timeline of the model is summarized as follows.
**Date 0:** The underlying state $i \in \{H, L\}$ and the realizations of $\bar{s}$ are then determined based on equation (1).

**Date 1:** The manager observes his private information and makes the decision $D$ about whether to exercise the growth option immediately. The market prices the firm at $P(D)$.

**Date 2:** The true underlying state is realized and observed by the manager. The manager then issues a mandatory disclosure of the firm’s earnings, $R$, that is subject to potential management, $m$, by the manager. The market prices the firm at $P(R, D; m)$. The manager chooses $R$, $m$ and $D$ to maximize $\alpha P(D) + (1-\alpha)P(R, D; m) - km(D, x, R)$ and leaves the firm afterwards.

**Date 3:** The underlying state and, correspondingly, firm’s true value are revealed. If the growth option is not exercised, investors choose whether to exercise this growth option.

We now solve for the pure strategy equilibrium. We first look at the case where no earnings management is allowed, or equivalently, the earnings management cost, $k$ is prohibitively high, and show that when $\alpha$ is sufficiently large, i.e., when the manager is sufficiently myopic, his investment decisions will not be efficient in equilibrium.

**Proposition 3:** Suppose $k > (1 - \alpha)(2x_H + \delta)$. If $\alpha < \frac{q(1-q)\delta}{(2q-1)(2x_H+\delta)}$, there is an equilibrium where a Low type manager invests after mandatory disclosure, a High type manager invests immediately after (the anticipated) voluntary disclosure, and there is no earnings management.

If either $\delta > 1$ and $\frac{q(1-q)\delta}{(q-\frac{1}{2})(2x_H+\delta)} < \alpha < \frac{\delta+1}{2x_H+\delta}$, or, $\frac{1}{2}(2x_H + \delta) > (1-q)(2x_H + 1)$ and $\alpha > \max\left(\frac{\delta+1}{2x_H+\delta}, \frac{q(1-q)\delta}{(q-\frac{1}{2})(2x_H+\delta+2f)}\right)$, there is an equilibrium where both High and Low types invest immediately after voluntary disclosure and there is no earnings management.

Proposition 3 is intuitive. When real effects are taken into account, early investment bears a cost. This is because making early investment when the state is low reduces firm value, and since the manager cannot manage the mandatory disclosure, investors price the firm really low after bad news mandatory disclosure. Thus, when the manager is not too myopic, he will
choose the efficient investment strategy based on his private information. However, when the manager is sufficiently myopic, i.e., he cares too much about $P(D)$, the cost associated with early investment (i.e., the drop in price when low earnings is reported) for a Low type manager, is more than offset by the increase in price in response to a favorable voluntary disclosure. As a result, a Low type manager will mimic the voluntary disclosure and investment strategies of a High type manager to increase $P(D)$. This leads to overinvestment by a Low type manager and impaired investment efficiency in general.

The next proposition shows that allowing an intermediate degree of reporting discretion can potentially restore the equilibrium investment efficiency.

**Proposition 4** When reporting discretion is allowed, there are two pure strategy equilibria:

1) Both High and Low types disclose $D_H$ and engage in earnings management when the true earnings number is $x_L$ if:

$$\frac{1}{2}(2x_H + \delta) - I > q(2x_H - I), \text{ and,}$$

$$k < \min\left(\frac{1}{2}(1 - \alpha)(2x_H + \delta), \alpha \frac{(1-q)(2x_H + \delta - (1-q)I)}{1-q} + (1 - \alpha)q(2x_H - I)\right).$$

2) A High type manager discloses $D_H$ and engages in earnings management when the true earnings number is $x_L$, and a Low type manager discloses $D_L$ and does not engage in earnings management if:

$$q > \frac{2x_H - I}{2x_H + \delta},$$

$$\max\left(0, \frac{2(1-q)x_H - q\delta}{q(2x_H - I)}\right) < \alpha < \frac{q(1-q)\delta + (1-q)^2 2x_H}{q^2(2x_H + \delta)}, \text{ and,}$$

$$\frac{2q-1}{q} (2x_H - I) + \frac{1}{q}[q\delta - (1 - q)I] < k < \min\left(\frac{\alpha}{1-q}[(2q - 1)(2x_H - I)] + \frac{1}{1-q}[q\delta - (1 - q)I], (1 - \alpha)q(2x_H + \delta)\right).$$
The first equilibrium in Proposition 4 suggests that when too much reporting discretion is allowed, both High and Low types managers make favorable voluntary disclosures and invest in the growth option early irrespective of their private information. A Low type manager is able to mimic the voluntary disclosure and investment strategies of a High type because it is beneficial for him to inflate the price at the voluntary disclosure, and incur a relatively low cost to manage his mandatory disclosure to cover up the bad news when necessary. Note that since the expected earnings management cost for a Low type manager increases with respect to $q$, this equilibrium requires that $q$ cannot be too high. In the second equilibrium of Proposition 4, an intermediate level of reporting discretion makes a High type manager’s expected earnings management to be less costly than that of a Low type manager, and thus enable him to separate himself out from a Low type by issuing a optimistic voluntary disclosure. Note that this equilibrium requires that 1) the manager cannot be too myopic, i.e., $\alpha$ cannot be too large, and 2) the manager’s private signal has to be sufficiently precise, i.e., $q$ has to be sufficiently large.

Given Propositions 3 and 4, one may wonder whether the restrictions are too strict such that there are no certain parameters of $\alpha, q, I, \delta$ that results in some degree of discretion maximizing investment efficiency. The following corollary shows that there exist such parameters of $\alpha, q, I, \delta$.

**Corollary 1** When $q > \max\left(\frac{1}{2x_H+\delta} - I, \frac{2x_H-I}{2x_H+\delta}\right)$ and $\max\left(\frac{q(1-q)\delta}{(2q-1)(2x_H+\delta)}, \frac{2(1-q)x_H-q\delta}{q(2x_H-I)}\right) < \alpha < \frac{q(1-q)\delta+(1-q)^22x_H}{q^2(2x_H+\delta)}$, allowing some degree of reporting discretion strictly improves investment efficiency.

Corollary 1 thus implies that some degree of reporting discretion is desirable in the sense of improving the informativeness of the voluntary disclosure. In other words, granting the manager some discretion over what he can report in his mandatory disclosure would enable him to convey to the market more of his private information in a more timely fashion and thus facilitate efficient investment decisions. For example, when $q$ is large, the manager who observes $s = h$ infers a sufficiently optimistic distribution of the earnings (i.e., skewed to the

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14 It is shown in the proof that the parameter space is non-empty.
larger value) from his private signal. He can then choose to forecast a good earnings number and exercise growth option immediately to convey this information, and in the case that the low earnings is realized (a low likelihood event), he will be able to potentially cover himself by using the granted discretion to manage earnings upward so that the reported earnings will not fall short of the forecast. Investors anticipate the manager’s behavior and rationally interpret the manager’s disclosure as conveying that the firm’s prospect is good and price the firm accordingly. Such pricing results in manager with good private information investing immediately but manager with bad private information refraining from making immediate investments, leading to more efficient investment decision.

Previously, we mentioned that giving the manager the option to manage reported earnings might lead him to exercise this option for his own benefit. However, it is exactly this self-interested earnings management that sometimes enables the manager to incorporate more private information in his voluntary disclosure. Thus (opportunistic) earnings management can have a hidden benefit of improving the informativeness of the more timely voluntary disclosure and thus investment efficiency, a message that we try to convey. Beyer et al. (2010) document that approximately 66% of the accounting-based information that investors use is provided by firms’ voluntary disclosures, while mandatory disclosures account for less than 12%. Therefore voluntary disclosure has become a major (and dominant) source of financial information that the capital market relies upon, and the informativeness of voluntary disclosure is, and indeed shown in our paper, to be an important efficiency metric. We believe the message that earnings management may help to improve the informativeness of voluntary disclosure and investment efficiency has significant policy implications. To the best of our knowledge, this paper is one of the few analytical papers to provide justification for allowing managers some discretion in financial reporting in the context of the interaction between mandatory and voluntary disclosures.

5. **Empirical Implications**

The results of this paper offer insights into some empirical findings and yield additional empirical implications that could be tested.
First, in an attempt to investigate whether accounting discretion is explained by managerial opportunism or efficient contracting, Bowen et al. (2008) provide empirical evidence that shareholders benefit from earnings management resulting from the latitude allowed by GAAP. They conjecture that this might be due to managers incorporating private information into accounting reports to signal competence or future performance. Our model suggests that the discretion imbedded in GAAP enables managers to convey more private information about firm performance and improve investment efficiency through more timely voluntary disclosure.

Second, we show that some degree of earnings management can result in a more informative voluntary disclosure, despite a decreased (or “more skeptical”) price response to the (high) voluntary disclosure. We are not aware of any empirical papers that directly test this hypothesis. Rogers and Stocken (2005) document that managers’ willingness to bias their forecasts decreases with the market’s ability to detect misrepresentation and that investors’ response to management forecasts is consistent with their ability to identify predictable forecast biases in the management forecasts. This result comports with our story in the sense that investors are more skeptical toward management forecasts that are perceived to be inflated.

Most of the literature that explores the relationship between voluntary and mandatory disclosures focuses on the relationships between certain attributes of voluntary earnings forecasts and reported earnings. Kasznik (1999) hypothesizes and finds that managers make income-increasing accounting decisions when earnings would otherwise be below management forecasts on the premise that earnings forecast errors impose costs on managers (such as investors’ perception of their (in)ability to anticipate changes in the economic environment), but he finds no evidence that underestimated earnings are associated with income-decreasing discretionary accruals. We confirm the findings in Kasznik (1999) that managers use earnings management to avoid disagreement between their voluntary and mandatory disclosures. However, our results go beyond showing a relationship between voluntary and mandatory disclosures. We further demonstrate that, under certain conditions, there exists a positive

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15 One possible exception is Kim (2012). He finds that managers with more accounting flexibility choose to issue more specific earnings forecasts before their firms’ seasoned equity offerings. However, he also finds that forecasts are more optimistic, which may or may not increase price informativeness, and his setting is restricted to firms that issue seasoned equities.
relationship between the degree of discretion allowed in managing reported earnings and the price informativeness of earnings forecasts.

Third, Corollary 1 provides conditions where some degree of discretion will result in higher investment efficiency. Specifically, when \( q \) is sufficiently high, some discretion results in higher investment efficiency. Note that in our model \( q \) can be viewed as a proxy for the degree of information asymmetry between manager and outside investors. Thus the first prediction is that this effect will be stronger for firms with higher degree of information asymmetry. In addition, Corollary 1 also predicts that the effect will be present for manager with \( \alpha \) that are large but not sufficiently large. Since in our model \( \alpha \) represents the percentage of short-term investors, we would expect our results to be stronger for firms with an intermediate percentage of transient investors. Finally, our propositions indicate that this effect will be present for firms with growth options and firms operating in more competitive industries when investing early will have more advantages. Thus, we expect our results to be stronger for growth firms and firms operating in industries that are more competitive.

6. Conclusions

We investigate the interaction between the voluntary disclosure, investment decisions and the subsequent mandatory disclosure of value-relevant information. We assume that the manager receives a private signal that helps him to assess the distribution of true earnings. The manager has the option of issuing a voluntary disclosure of the firm’s earnings and making investment decisions after observing the signal. Then he has to make a mandatory disclosure of the earnings after he observes the realization of the true state. The manager’s objective is a weighted average of the short-term price response to voluntary disclosure and the long-term price response to mandatory disclosure. We show that, in equilibrium, allowing the manager some discretion over the mandatory financial report may enable him to incorporate private information into his more timely voluntary disclosure and thus enhances the price informativeness of the voluntary disclosure, which in turn improves investment efficiency. In addition, too much discretion is undesirable because it induces the manager to always voluntarily disclose high earnings numbers, choose suboptimal real actions and ignore his private information. Thus there is a hidden benefit to granting the manager some degree of
discretion over his mandatory disclosure, and there is a maximal amount of such discretion that should be tolerated, consistent with the discretion imbedded in GAAP.

Our finding that voluntary and mandatory disclosures complement each other in the sense that the primarily backward-looking and less timely mandatory disclosure allows the manager to disclose voluntarily (and somewhat credibly) forward-looking and timely private information echoes the long-standing view in the accounting literature that the voluntary disclosure serves as a source of new information, while the mandatory disclosure plays an important confirmatory (or disciplinary) role that makes the voluntary disclosure credible and informative (e.g., Sansing 1992; Stocken 2000; Dutta and Gigler 2002). However, we derive this intuition in a completely different setting, and the result that granting some but not too much discretion over the mandatory disclosure has the potential of enabling managers to incorporate more of their private information in the “unrestricted” voluntary disclosure, which results in real effects, is new.

Our results also have policy implications. From a regulator’s perspective, tolerating some degree of earnings management by allowing some discretion in the accounting standards may be desirable if the informativeness of voluntary disclosures is an important goal. In our model it is an important goal as more informative voluntary disclosure leads to more efficient investment decisions. Given that voluntary disclosure is a major source of firms’ information and accounts for about 85% of the firm-provided accounting-based information that investors use (Beyer et. al. 2010), enhancing the informativeness of voluntary disclosures seems to be essential for efficiently allocating the scarce resources.
REFERENCES:


Appendix A: A summary of notations used in the paper

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning of the notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{i, i} = H, L$</td>
<td>True Earnings which depends on the underlying state $i$. We sometimes also use $x$ to refer to true earnings in general.</td>
</tr>
<tr>
<td>$s \in {h, l}$</td>
<td>A noisy signal about the true earnings when $\sigma(i)$ cannot be observed.</td>
</tr>
<tr>
<td>$q$</td>
<td>The informativeness of $s$ about true earnings, as defined in equation (2).</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Manager’s information set before voluntary disclosure.</td>
</tr>
<tr>
<td>$D_{i, i} = H, L$</td>
<td>Voluntary disclosure of true earnings number being at least $x_{i}$. We sometimes also use $D$ to refer to voluntary disclosure in general.</td>
</tr>
<tr>
<td>$R(D, x)$</td>
<td>Manager’s mandatory disclosure strategy, which is a function of his voluntary disclosure strategy $D$ and true earnings number $x$.</td>
</tr>
<tr>
<td>$\hat{R}(D, x)$</td>
<td>Manager’s mandatory disclosure strategy and his earnings management strategy jointly determine the distribution of the realized mandatory disclosure.</td>
</tr>
<tr>
<td>$m(D, R, x) \in {0, 1}$</td>
<td>Manager’s earnings management decision as a function of his voluntary disclosure strategy, mandatory disclosure strategy and true earnings. $m(D, R, x) = 0(1)$ represents no earnings management (earnings management).</td>
</tr>
<tr>
<td>$\hat{m}(D, R, x) \in {0, 1}$</td>
<td>Investors’ conjecture about manager’s earnings management decision.</td>
</tr>
<tr>
<td>$R_{i, i} = H, L$</td>
<td>Mandatory disclosure. We sometimes also use $R$ to refer to realization of mandatory mandatory disclosure.</td>
</tr>
</tbody>
</table>
disclosure in general.

<table>
<thead>
<tr>
<th>$P(D, R; m)$</th>
<th>Price response when the voluntary disclosure is $D$, the realization of mandatory disclosure is $R$ and the manager’s manipulation strategy is $m$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}(D, R; m)$</td>
<td>Manager’s conjecture about investor’s price response.</td>
</tr>
</tbody>
</table>

Furthermore, in general we use $p$ to denote probability, $P$ to denote price, and $E$ to denote expectation.
Appendix B: Technical Details Including Proofs

For simplicity in the subsequent proofs we refer to manager observing \( s = l \) as type 1 and manager observing \( s = h \) as type 2.

**Proof of Proposition 1:**

Recall that manager’s objective function is \( \alpha P(D) + (1 - \alpha) P(R; D, m) - km \).

When \( k \) is sufficiently large so that \( m = 0 \) in equilibrium, \( P(R; D, 0) = R \) and is thus independent of \( D \). This implies that the manager will be choosing \( D \) to maximize \( \alpha P(D) \), or, equivalently, \( P(D) \). Thus, suppose \( D \) is informative in the sense that \( P(D_H) > P(D_L) \), then all types of managers will choose \( D_H \), but then \( D_H \) cannot be informative, resulting in contradiction. The case where \( P(D_H) < P(D_L) \) can be proved analogously.

Therefore in equilibrium voluntary disclosure is completely uninformative and Proposition 1 is thus proved. *Q.E.D.*

**Proof of Proposition 2:**

When \( k \) is not too large, manager may choose to manipulate \( R \). Recall that manager’s objective function is \( \alpha P(D) + (1 - \alpha) P(R; D, m) - km \).

We want to show that even allowing manipulation, there can be no equilibrium where voluntary disclosure is informative. We prove by contradiction.

Suppose there is an equilibrium where \( D \) is informative. There are two possibilities: one, type 1 chooses \( D_L \) and type 2 chooses \( D_H \) and two, type 1 chooses \( D_H \) and type 2 chooses \( D_L \). We will
prove in detail that the first possibility cannot be equilibrium as the proof for the second possibility is essentially the same. It then follows that \( P(D_H) = qx_H \) and \( P(D_L) = (1 - q)x_H \).

From proposition 1 informative \( D \) cannot be equilibrium when neither types manipulate. We thus now consider the other three cases: both types manipulate; only type 1 manipulates and only type 2 manipulates.

**Case 1: Both types manipulate.**

In this case \( P(R_H; D_H, 1) = qx_H = P(D_H) \) and \( P(R_H; D_L, 1) = (1 - q)x_H = P(D_L) \) as \( R_H \) is completely uninformative. The off-equilibrium belief will be \( P(R_L; D_H, 1) = P(R_L; D_L, 1) = 0 \) as observing \( R_L \) will be interpreted as no earnings manipulation and the true earnings being \( x_L \).

This strategy, however, cannot be optimal. To see this, note that for type 1, disclosing \( D_L \) and manipulating results in \( \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - k = P(D_L) - k \). Deviating to disclosing \( D_H \) and manipulating, however, results in \( \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k = P(D_H) - k \), which is higher.

**Case 2: Only type 1 manipulates.**

In this case \( P(R_H; D_H, 0) = x_H > P(D_H) \), \( P(R_L; D_H, 0) = 0 \) and \( P(R_H; D_L, 1) = (1 - q)x_H = P(D_L) \) as \( R_H \) is completely uninformative followed by \( D_L \). The off-equilibrium belief will be \( P(R_L; D_L, 1) = 0 \) as observing \( R_L \) will be interpreted as no earnings manipulation and the true earnings being \( x_L \).

This strategy, however, cannot be optimal. To see this, note that for type 1, disclosing \( D_L \) and manipulating results in \( \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - k = P(D_L) - k \). Deviating to
disclosing $D_H$ and manipulating, however, results in $\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k >\alpha P(D_H) + (1 - \alpha)P(D_H) - k = P(D_H) - k$, which is higher.

**Case 3: Only type 2 manipulates.**

In this case $P(R_H; D_H, 1) = qx_H = P(D_H)$ as $R_H$ is completely uninformative followed by $D_H$, $P(R_H; D_L, 0) = x_H > P(D_L)$ and $P(R_L; D_L, 0) = 0$. The off-equilibrium belief will be $P(R_L; D_L, 1) = 0$ as observing $R_L$ will be interpreted as no earnings manipulation and the true earnings being $x_L$.

This strategy, however, cannot be optimal. To see this, note that for type 1, disclosing $D_L$ and not manipulating results in $\alpha P(D_L) + (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0)] = \alpha(1 - q)x_H + (1 - \alpha)(1 - q)x_H$. Deviating to disclosing $D_H$ and not manipulating results in $\alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) + qP(R_L; D_H, 1)] = \alpha qx_H + (1 - \alpha)(1 - q)qx_H$.

For disclosing $D_L$ and not manipulating to be optimal for type 1, we need $\alpha(1 - q)x_H + (1 - \alpha)(1 - q)x_H > \alpha qx_H + (1 - \alpha)(1 - q)x_H$, or, equivalently, $\alpha(2q - 1)x_H < (1 - \alpha)(1 - q)^2 x_H$.

Now consider type 2. Disclosing $D_H$ and manipulating results in $\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq = \alpha qx_H + (1 - \alpha)qx_H - kq$. Deviating to disclosing $D_L$ and manipulating, meanwhile, results in $\alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 0) - kq = \alpha(1 - q)x_H + (1 - \alpha)x_H - kq$, which is larger as the difference is $(1 - \alpha)(1 - q)x_H > \alpha(2q - 1)x_H$. As a summary, type 1 discloses $D_L$ and type 2 discloses $D_H$ cannot be optimal. Proposition 2 is thus proved.

Q.E.D.
Proof of Proposition 3:

There are only four possible cases when there is no manipulation in equilibrium, 1) managers of both types choose to invest immediately after the voluntary disclosure; 2) type 1 managers choose to invest immediately but type 2 managers choose to invest after the mandatory disclosure; 3) managers of both types choose to invest after the mandatory disclosure; 4) managers of type 1 choose to invest after the mandatory disclosure and managers of type 2 choose to invest immediately after the voluntary disclosure. We will show below that the second case cannot be equilibrium whereas the rest three cases will be equilibrium strategy under certain conditions.

In addition, for ease of notation, we denote managers who choose to invest immediately after the voluntary disclosure as \(D_H\) and managers who choose to invest after the mandatory disclosure as \(D_L\). Since the timing of managers’ investment decisions are observable, it doesn’t matter what exactly managers say at the voluntary disclosure date. For example, even though managers can be disclosing \(D_L\), if they are investing immediately, investors will treat the manager as if they are disclosing \(D_H\).

Case 1: Managers of both types disclose \(D_H\)

In this case, \(P(D_H) = \frac{1}{2}(2x_H + \delta) - I\) and \(P(R_H; D_H, 0) = 2x_H + \delta - I\) and \(P(R_L; D_H, 1) = -I\). For the off-equilibrium belief regarding \(D_L\), it can be type 1 or type 2 depending on which type benefits most from deviation.

For type 1, disclosing \(D_H\) results in \(\alpha P(D_H) + (1 - \alpha)[qP(R_L; D_H, 0) + (1 - q)(R_H; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)qI\)
If he deviates to disclosing $D_L$ and the off-equilibrium belief is that type 1 discloses $D_L$, this results in $\alpha P(D_L) + (1 - \alpha)[qP(R_L; D_L, 0) + (1 - q)P(R_H; D_L, 0)] = \alpha(1 - q)(2x_H - I) + (1 - \alpha)(1 - q)(2x_H - I)$.

For type 2, disclosing $D_H$ results in $\alpha P(D_H) + (1 - \alpha)[qP(R_H; D_L, 0) + (1 - q)P(R_L; D_H, 0)] = \alpha(1 - q)(2x_H - I) + (1 - \alpha)(1 - q)l$.

If he deviates to disclosing $D_L$ and the off-equilibrium belief is that type 2 discloses $D_L$, this results in $\alpha P(D_L) + (1 - \alpha)[qP(R_H; D_L, 0) + (1 - q)P(R_L; D_L, 0)] = \alpha P(D_L) = (1 - q)(2x_H - I)$.

The difference of the gain in payoff by deviating to $D_L$ between type 2 and type 1 can be calculated to be $(2q - 1)[-(1 - \alpha)(\delta + I) + \alpha(2x_H - I)]$. Thus, this difference is increasing in $\alpha$. When $\alpha < \frac{\delta + I}{2x_H + \delta}$, type 1 gains more by deviating to $D_L$ and the off-equilibrium belief will be $P(D_L) = (1 - q)(2x_H - I)$, $P(R_H; D_L, 0) = 2x_H - I$ and $P(R_L; D_L, 0) = 0$. When $\alpha > \frac{\delta + I}{2x_H + \delta}$, type 2 gains more by deviating to $D_L$ and the off-equilibrium belief will be $P(D_L) = (1 - q)(2x_H - I)$, $P(R_H; D_L, 0) = 2x_H - I$ and $P(R_L; D_L, 0) = 0$.

**Subcase 1.1 When $\alpha < \frac{\delta + I}{2x_H + \delta}$**

We first look at the subcase when $\alpha < \frac{\delta + I}{2x_H + \delta}$. In this subcase $P(D_L) = (1 - q)(2x_H - I)$.

For type 1, disclosing $D_H$ and not manipulating results in the payoff of $\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)ql$. 
Switching to $D_H$ and manipulating results in the payoff of $\alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - kq = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - kq$.

Switching to $D_L$ and not manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)((1 - q)P(x_H; D_L, 0) + qP(x_L; D_L, 0)) = \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I)$.

Switching to $D_L$ and manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - kq = \alpha P(D_L) + (1 - \alpha)(2x_H - I) - kq$.

Thus, for type 1, disclosing $D_H$ is optimal if

\[
\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)q l > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - kq, \tag{B.1}
\]

\[
\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)q l > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I), \tag{B.2}
\]

and

\[
\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)q l > \alpha P(D_L) + (1 - \alpha)(2x_H - I) - kq \tag{B.3}
\]

Equation (A.1) can be reduced to $kq > (1 - \alpha)q(2x_H + \delta) > (1 - \alpha)q(2x_H - I)$, implying the right hand side of (A.2) is larger than that of (A.3). Equation (A.3) can thus be ignored and the conditions can be reduced to

$k > (1 - \alpha)(2x_H + \delta)$ and

\[
\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)q l > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I).
\]
For managers of type 2, disclosing $D_H$ and not manipulating results in the payoff of $\alpha P(D_H) + (1 - \alpha)[qP(x_H; D_H, 0) + (1 - q)P(x_L; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l$.

Switching to $D_H$ and manipulating results in the payoff of

$$\alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - k(1 - q) = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - k(1 - q)$$

Switching to $D_L$ and not manipulating results in the payoff of

$$\alpha P(D_L) + (1 - \alpha)[qP(x_H; D_L, 0) + (1 - q)P(x_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)q(2x_H - l).$$

Switching to $D_L$ and manipulating results in the payoff of

$$\alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - k(1 - q) = \alpha P(D_L) + (1 - \alpha)(2x_H - l) - k(1 - q)$$

Thus, for type 2, disclosing $D_H$ and not manipulating is optimal if

$$\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - k(1 - q),$$

$$\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)q(2x_H - l),$$

$$\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)q(2x_H - l),$$

(B.4)
and

\[ \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)(2x_H - l) - k(1 - q). \] (B.6)

Again, the first inequality can be reduced to

\[ k(1 - q) > (1 - \alpha)(2x_H + \delta) > (1 - \alpha)(1 - q)(2x_H - l), \]

implying the right hand side of the second inequality is larger than that of the third inequality. The third inequality can thus be ignored and the conditions can be reduced to

\[ k > (1 - \alpha)(2x_H + \delta) \] and

\[ \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)q(2x_H - l). \]

Thus, for both types, disclosing \( D_H \) is optimal if

\[ k > (1 - \alpha)(2x_H + \delta), \] (B.7)

and

\[ \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - l) - (1 - \alpha)q(l) > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) \] (B.8)

and

\[ \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)q(2x_H - l). \] (B.9)
It can be shown that the left hand side of (A.9) is larger than the left hand side of (A.8) by a greater amount than the difference of the right hand sides of the two inequalities, satisfying of (A.8) implies satisfaction of (A.9), i.e.,

\[ \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)q l > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I), \]

which is equivalent to

\[ \alpha[P(D_H) - P(D_L)] > (1 - \alpha)[ql - (1 - q)\delta], \]

which can be further reduced to

\[ \frac{(2q-1)\alpha}{2}(2x_H + \delta) > ql - (1 - q)\delta. \]

Note that we also need \( \alpha < \frac{\delta + l}{2x_H + \delta} \), implying that we need \( \frac{ql - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta)} < \frac{\delta + l}{2x_H + \delta} \), or, equivalently, \( \delta > I \).

As a summary, for this subcase to be equilibrium, we need \( \delta > I, \frac{ql - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta)} < \alpha < \frac{\delta + l}{2x_H + \delta} \) and \( k > (1 - \alpha)(2x_H + \delta) \).

**Subcase 1.2 When \( \alpha > \frac{\delta + l}{2x_H + \delta} \).**

We first look at the subcase when \( \alpha > \frac{\delta + l}{2x_H + \delta} \). In this subcase \( P(D_L) = q(2x_H - I) \). However, other than the expression of \( P(D_L) \), everything else is the same. Thus we can follow essentially the same procedure as above and get that for both types, disclosing \( D_H \) is optimal if

\[ k > (1 - \alpha)(2x_H + \delta) \]

\[ \alpha[P(D_H) - P(D_L)] > (1 - \alpha)[ql - (1 - q)\delta]. \]
which can be further reduced to
\[
\frac{(2q-1)\alpha}{2} (2x_H + \delta + 2l) > ql - (1 - q)\delta.
\]

Note that we also need \(\alpha > \frac{\delta + l}{2x_H + \delta}\), implying that we need \(\alpha > \max\left(\frac{\delta + l}{2x_H + \delta}, \frac{q\delta - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta + 2l)}\right)\).

Note that we also need \(\frac{q\delta - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta + 2l)} < 1\), which is equivalent to \(\frac{1}{2} (2x_H + \delta) > (1 - q)(2x_H + l)\).

As a summary, for this subcase to be equilibrium, we need \(\frac{1}{2} (2x_H + \delta) > (1 - q)(2x_H + l)\),
\[
\alpha > \max\left(\frac{\delta + l}{2x_H + \delta}, \frac{q\delta - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta + 2l)}\right), \text{ and } k > (1 - \alpha)(2x_H + \delta).
\]

Overall, for managers of both types to disclose \(D_H\), we need \(k > (1 - \alpha)(2x_H + \delta)\) and either
\[
\delta > l, \quad \frac{q\delta - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta)} < \alpha < \frac{\delta + l}{2x_H + \delta} \text{ or } \frac{1}{2} (2x_H + \delta) > (1 - q)(2x_H + l),
\]
\[
\alpha > \max\left(\frac{\delta + l}{2x_H + \delta}, \frac{q\delta - (1 - q)\delta}{(q - \frac{1}{2})(2x_H + \delta + 2l)}\right).
\]

**Case 2: Managers of both types disclose \(D_L\)**

We now look at the second case, where managers of both types choose to invest after mandatory disclosure, denoted as they both voluntarily disclose \(D_L\). In this case \(D_L\) has no information content and \(P(D_L) = \frac{1}{2} (x_H + x_H - l)\), \(P(R_H; D_L, 0) = x_H + x_H - l\) and \(P(R_L; D_L, 0) = 0\). The off-equilibrium beliefs are set to be \(P(R_H; D_H, 0) = x_H + x_H + \delta - l\) and \(P(R_L; D_H, 0) = -l\). To determine \(P(D_H)\) we again need to check with type benefits more from deviating to \(D_H\).
For type 1, disclosing $D_L$ generates a payoff of $\alpha P(D_L) + (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I)$. Disclosing $D_H$ and setting the off-equilibrium belief to be that $D_H$ comes from type 1 generates a payoff of $\alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 0) + qP(R_L; D_L, 0)] = \alpha[(1 - q)(2x_H + \delta) - I] + (1 - \alpha)(1 - q)(2x_H + \delta)$. The gain in deviating is thus $\alpha[(1 - q)(2x_H + \delta) - I] + (1 - \alpha)(1 - q)(2x_H + \delta) - \alpha P(D_L) - (1 - \alpha)(1 - q)(2x_H - I)$.

For type 2, disclosing $D_L$ generates a payoff of $\alpha P(D_L) + (1 - \alpha)[qP(R_H; D_L, 0) + (1 - q)P(R_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)q(2x_H - I)$. Disclosing $D_H$ and setting the off-equilibrium belief to be that $D_H$ comes from type 2 generates a payoff of $\alpha P(D_H) + (1 - \alpha)[qP(R_H; D_H, 0) + (1 - q)P(R_L; D_L, 0)] = \alpha[q(2x_H + \delta) - I] + (1 - \alpha)q(2x_H + \delta)$. The gain for deviating is thus $\alpha[q(2x_H + \delta) - I] + (1 - \alpha)q(2x_H + \delta) - \alpha P(D_L) - (1 - \alpha)q(2x_H - I)$.

The difference in the gain for deviating between type 2 and type 1 is thus $(2q - 1)[(2x_H + \delta) - (1 - \alpha)(2x_H - I)] > 0$. Thus, the off-equilibrium belief will always be that $D_H$ comes from type 2 and $P(D_H) = q(2x_H + \delta) - I$. Note that since $q\delta > (1 - q)I$, $P(D_H) = q(2x_H + \delta) - I > q(2x_H - I) > \frac{1}{2}(2x_H - I) = P(D_L)$.

For managers of type 2, disclosing $D_L$ and not manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)[qP(x_H; D_L, 0) + (1 - q)P(x_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)q(2x_H - I)$.

Disclosing $D_L$ and manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - k(1 - q) = \alpha P(D_L) + (1 - \alpha)(2x_H - I) - k(1 - q)$.
Switching to $D_H$ and not manipulating results in the payoff of

$$\alpha P(D_H) + (1 - \alpha)[qP(x_H; D_H, 0) + (1 - q)P(x_L; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)l.$$  

Switching to $D_H$ and manipulating results in the payoff of

$$\alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - k(1 - q) = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - k(1 - q).$$

Disclosing $D_L$ and not manipulating is thus optimal for type 2 if

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)(2x_H - I) - k(1 - q),$$

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)l \text{ and}$$

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - k(1 - q).$$

Since $P(D_H) > P(D_L)$, the right hand side of the first inequality is smaller than the third and thus the first inequality be ignored. This results in

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - k(1 - q)$$

and

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)l.$$  

For managers of type 1, disclosing $D_L$ and not manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)[(1 - q)P(x_H; D_L, 0) + qP(x_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I)$.  

Disclosing $D_L$ and manipulating results in the payoff of $\alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - kq = \alpha P(D_L) + (1 - \alpha)(2x_H - I) - kq$.  

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Switching to $D_H$ and not manipulating results in the payoff of $\alpha P(D_H) + (1 - \alpha)\left[(1 - q)P(x_H; D_H, 0) + qP(x_L; D_H, 0)\right] = \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - l) - (1 - \alpha)qI$.

Switching to $D_H$ and manipulating results in the payoff of $\alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - kq = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - kq$.

Disclosing $D_L$ and not manipulating is thus optimal for type 1 if

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(2x_H - l) - kq,$$  \hspace{1em} (B.10)

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H + \delta - l) - (1 - \alpha)qI$$  \hspace{1em} (B.11)

and

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(2x_H + \delta - l) - kq.\hspace{1em} (B.12)$$

Again since $P(D_H) > P(D_L)$, the right hand side of (B.10) is smaller than (B.12) and can thus be ignored. This results in

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(2x_H + \delta - l) - kq \text{ and}$$

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H + \delta - l) - (1 - \alpha)qI.$$

Thus, for both types, disclosing $D_L$ is optimal if

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - l) > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - k(1 - q); \hspace{1em} (B.13)$$

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - kq; \hspace{1em} (B.14)$$

$$\alpha P(D_L) + (1 - \alpha)q(2x_H - l) > \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l$$
and

\[ \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I) > \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)qI \]

(B.16)

First look at equations (B.13) and (B.14), which results in

\[ k > \frac{1}{1-q} [\alpha P(D_H) - \alpha P(D_L) + (1 - \alpha)\delta] + (1 - \alpha)(2x_H - I) \]

and

\[ k > \frac{1}{q} [\alpha P(D_H) - \alpha P(D_L) + (1 - \alpha)\delta] + (1 - \alpha)(2x_H - I) \]

Since \( q > \frac{1}{2} \), \( \frac{1}{1-q} [\alpha P(D_H) - \alpha P(D_L) + (1 - \alpha)\delta] + (1 - \alpha)(2x_H - I) > \frac{1}{q} [\alpha P(D_H) - \alpha P(D_L) + (1 - \alpha)\delta] + (1 - \alpha)(2x_H - I) \). Thus, to satisfy equation (B.13) and (B.14), it is sufficient that \( k > \frac{1}{1-q} [\alpha P(D_H) - \alpha P(D_L) + (1 - \alpha)\delta] + (1 - \alpha)(2x_H - I) \), or, equivalently,

\[ k > (1 - \alpha)(2x_H + \delta - I) + \frac{q\delta - \alpha(1-q)\delta}{1-q} + \alpha \frac{2q-1}{2(1-q)} (2x_H - I). \]

Again since \( q > \frac{1}{2} \), it can be shown that the left hand side of (B.15) is larger than the left hand side of (B.16) by a greater amount than the difference of the right hand sides, thus to satisfy (B.15) and (B.16), we need

\[ \alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)I, \]

which is equivalent to

\[ \alpha [P(D_H) - P(D_L)] + (1 - \alpha)q \delta < (1 - \alpha)(1 - q)I. \]
Note that since $q\delta > (1 - q)I$ and $P(D_H) > P(D_L)$, this inequality cannot be satisfied. Thus this case is impossible.

**Case 3: Managers of type 1 disclose $D_H$ and managers of type 2 disclose $D_L$.**

In this case $P(D_H) = (1 - q)(x_H + x_H + \delta) - I$ and $P(D_L) = q(x_H + x_H - I)$. We also have $P(x_H; D_H, 0) = x_H + x_H + \delta - I$ and $P(x_L; D_H, 0) = -I$; $P(x_H; D_L, 0) = x_H + x_H - I$ and $P(x_L; D_L, 0) = 0$.

This strategy, however, cannot be an equilibrium as for type 1, for type 1, disclosing $D_H$ and not manipulating generates payoff of $
alpha P(D_H) + (1 - \alpha)(1 - q)P(x_H; D_H, 0) + qP(x_L; D_H, 0) = \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)qI$
whereas disclosing $D_L$ and not manipulating generates payoff of $\alpha P(D_L) + (1 - \alpha)(1 - q)P(x_H; D_L, 0) + qP(x_L; D_L, 0) = \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I)$.

Thus, for type 1, disclosing $D_H$ and not manipulating is optimal only if $\alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)qI > \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I)$, or, equivalently, $\alpha P(D_H) - \alpha P(D_L) > (1 - \alpha)qI - (1 - \alpha)(1 - q)\delta$.

For type 2, disclosing $D_H$ and not manipulating generates payoff of $\alpha P(D_H) + (1 - \alpha)[qP(x_H; D_H, 0) + (1 - q)P(x_L; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)I$ whereas disclosing $D_L$ and not manipulating generates payoff of $\alpha P(D_L) + (1 - \alpha)[qP(x_H; D_L, 0) + (1 - q)P(x_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)q(2x_H - I)$.

For type 2, disclosing $D_L$ and not manipulating is optimal only if $\alpha P(D_L) + (1 - \alpha)q(2x_H - I) > \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)I$, or, equivalently, $\alpha[P(D_H) - P(D_L)] < (1 - \alpha)(1 - q)I - (1 - \alpha)q\delta$. Thus, we need both $\alpha[P(D_H) - P(D_L)] >$
\[(1 - \alpha)ql - (1 - \alpha)(1 - q)\delta \text{ and } \alpha[P(D_H) - P(D_L)] < (1 - \alpha)(1 - q)l - (1 - \alpha)q\delta.\]

However, this is impossible. Thus, this case cannot be equilibrium.

**Case 4: Managers of type 1 disclose \(D_L\) and managers of type 2 disclose \(D_H\).**

In this case \(P(D_H) = q(x_H + x_H + \delta) - l\) and \(P(D_L) = (1 - q)(x_H + x_H - l)\). We also have 
\[P(x_H; D_H, 0) = x_H + x_H + \delta - l\text{ and } P(x_L; D_H, 0) = -l; P(x_H; D_L, 0) = x_H + x_H - l\text{ and } \]
\[P(x_L; D_L, 0) = -l.\] Note that since \(q\delta > (1 - q)l\), \(P(D_H) = q(x_H + x_H + \delta) - l > q(2x_H - l) > (1 - q)(2x_H - l) = P(D_L)\).

For managers of type 2, disclosing \(D_H\) and not manipulating results in \(\alpha P(D_H) + (1 - \alpha)[qP(x_H; D_H, 0) + (1 - q)P(x_L; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l.\)

Deviating to disclosing \(D_H\) and manipulating results in
\[\alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - k(1 - q) = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - k(1 - q).\]

Deviating to disclosing \(D_L\) and not manipulating results in \(\alpha P(D_L) + (1 - \alpha)[qP(x_H; D_L, 0) + (1 - q)P(x_L; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)q(2x_H - l).\)

Deviating to disclosing \(D_L\) and manipulating results in \(\alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - k(1 - q) = \alpha P(D_L) + (1 - \alpha)(2x_H - l) - k(1 - q).\)

For disclosing \(D_H\) and not manipulating to be optimal, we need
\[\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - l) - (1 - \alpha)(1 - q)l > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - k(1 - q)\]  \(\text{(B.17)}\)
\[
\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)q(2x_H - I),
\]

(B.18)

and

\[
\alpha P(D_H) + (1 - \alpha)q(2x_H + \delta - I) - (1 - \alpha)(1 - q)l > \alpha P(D_L) + (1 - \alpha)(2x_H - I) - k(1 - q)
\]

(B.19)

Since \( P(D_H) > P(D_L) \), the right hand side of (B.19) is smaller than that of (B.17) and thus can be ignored. Solving those inequalities result in

\[ k > (1 - \alpha)(2x_H + \delta) \text{ and } \]

\[ \alpha(2q - 1)(2x_H - I) > (1 - q)l - q\delta \]

Note that since \((1 - q)l - q\delta < 0\) and \(2x_H - I > 0\), \(\alpha(2q - 1)(2x_H - I) > (1 - q)l - q\delta\)

is automatically satisfied. Thus the conditions can be reduced to \( k > (1 - \alpha)(2x_H + \delta) \).

For managers of type 1, disclosing \( D_L \) and not manipulating results in \( \alpha P(D_L) + (1 - \alpha)[(1 - q)P(x_H; D_L, 0) + qP(x_H; D_L, 0)] = \alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - I). \)

Deviating to \( D_L \) and manipulating results in \( \alpha P(D_L) + (1 - \alpha)P(x_H; D_L, 0) - kq = \alpha P(D_L) + (1 - \alpha)(2x_H - I) - kq. \)

Deviating to \( D_H \) and not manipulating results in \( \alpha P(D_H) + (1 - \alpha)[(1 - q)P(x_H; D_H, 0) + qP(x_L; D_H, 0)] = \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - I) - (1 - \alpha)qI. \)

Deviating to \( D_H \) and manipulating results in \( \alpha P(D_H) + (1 - \alpha)P(x_H; D_H, 0) - kq = \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - I) - kq \).
For disclosing $D_L$ to be optimal, we need

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_L) + (1 - \alpha)(2x_H - l) - kq, \quad (B.20)$$

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_H) + (1 - \alpha)(1 - q)(2x_H + \delta - l) - (1 - \alpha)ql$$

(B.21)

and

$$\alpha P(D_L) + (1 - \alpha)(1 - q)(2x_H - l) > \alpha P(D_H) + (1 - \alpha)(2x_H + \delta - l) - kq. \quad (B.22)$$

Since $P(D_H) > P(D_L)$, the right hand side of equation (B.22) is larger than (B.20). Thus (B.20) can be ignored, which gives us

$$kq > \alpha[(2q - 1)(2x_H + \delta) + (1 - q)\delta - ql] + (1 - \alpha)[q(2x_H + \delta) + (1 - q)\delta - ql]$$

and

$$\alpha(2q - 1)(2x_H + \delta) < ql - (1 - q)\delta.$$

When $\alpha(2q - 1)(2x_H + \delta) < ql - (1 - q)\delta$, $\alpha[(2q - 1)(2x_H + \delta) + (1 - q)\delta - ql] + (1 - \alpha)[q(2x_H + \delta) + (1 - q)\delta - ql] < (1 - \alpha)[ql - (1 - q)\delta] + (1 - \alpha)q(2x_H + \delta) - (1 - \alpha)[ql - (1 - q)\delta] = (1 - \alpha)q(2x_H + \delta).$ Thus, $k > (1 - \alpha)(2x_H + \delta)$ implies this inequality.

As a summary, for this strategy to be equilibrium, we need

$k > (1 - \alpha)(2x_H + \delta)$ and $\alpha < \frac{q(1 - q)\delta}{(2q - 1)(2x_H + \delta)}$. Thus, $\alpha$ cannot be too large. Note that

$$\frac{q(1 - q)\delta}{(2q - 1)(2x_H + \delta)} < \frac{q(1 - q)\delta}{(q - \frac{3}{2})(2x_H + \delta)}.$$ Thus, when $k > (1 - \alpha)(2x_H + \delta)$ and $\alpha < \frac{q(1 - q)\delta}{(2q - 1)(2x_H + \delta)}$, there is an equilibrium where type 1 invests after mandatory disclosure, type 2 invests immediately.
before voluntary disclosure and there is no manipulation. When either $\delta > I$, $\frac{ql-(1-q)\delta}{(q-\frac{1}{2})(2xH+\delta)} < \alpha < \frac{\delta+1}{2xH+\delta}$ or $\frac{1}{2}(2xH + \delta) > (1 - q)(2xH + I)$, $\alpha > \max \left( \frac{\delta+1}{2xH+\delta}, \frac{ql-(1-q)\delta}{(q-\frac{1}{2})(2xH+\delta+2q)} \right)$ and $k > (1 - \alpha)(2xH + \delta)$, there is an equilibrium where both type invests immediately after voluntary disclosure and there is no manipulation.

Proposition 3 is thus proved. Q.E.D.

**Proof of Proposition 4:**

When $k$ is not so large managers may manipulate their mandatory disclosures, i.e., may choose $m = 1$. From Proposition 3 we know that the managers will all invest immediately if $\alpha$ is sufficiently large. We now focus on, whether under those parameter space, allowing discretion will improve the investment efficiency, i.e., result in managers of type 1 to invest immediately and managers of type 2 to not invest immediately. We first characterize the full set of pure strategy equilibria when managers may manipulate their mandatory disclosures.

We divide the discussion into four categories based on each type’s investment strategy (i.e., both types invest early; both types invest late; type 1 invests early but type 2 invests late; type 1 invests late but type 2 invests early). In each category we divide into three cases based on each type’s possible manipulation strategy (i.e., both types manipulate; only type 1 manipulates and only type 2 manipulates).

**Subcategory 1: managers of both types disclose $D_H$.**

Under this conjecture, $P(D_H) = \frac{1}{2} (2xH + \delta) - I$ as $D_H$ is completely uninformative.

*Case 1.1: Both types manipulate.*
In this case $P(R_H; D_H, 1) = \frac{1}{2}(2x_H + \delta) - I = P(D_H)$ as $D_H$ is completely uninformative. For the off-equilibrium beliefs, $P(R_L; D_H, 1) = -I$ and $P(R_L; D_L, 1) = 0$ as $R_L$ will be interpreted as no earnings management. To calculate $P(D_L)$ we again need to calculate which type is more likely to deviate to $D_L$, assuming that they always manipulate earnings.

For managers of type 1, disclosing $D_H$ and manipulating results in $\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq = P(D_H) - kq$. Deviating to $D_L$ and manipulating results in $\alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - kq = P(D_L) - kq = (1 - q)(2x_H - I) - kq$ when the off-equilibrium belief is that type 1 deviates to $D_L$.

For managers of type 2, disclosing $D_H$ and manipulating results in $\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) = P(D_H) - k(1 - q)$. Deviating to $D_L$ and manipulating results in $\alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - k(1 - q) = P(D_L) - kq = q(2x_H - I) - k(1 - q)$ when the off-equilibrium belief is that type 2 deviates to $D_L$.

Thus, the gain from deviating to $D_L$ for type 2 minus the gain from deviating to $D_L$ for type 1 would be $q(2x_H - I) - (1 - q)(2x_H - I) = (2q - 1)(2x_H - I) > 0$. Therefore, the off-equilibrium belief will be that $D_L$ comes from type 2. Thus $P(D_L) = q(2x_H - I)$.

For managers of type 2, disclosing $D_H$ and manipulating results in $P(D_H) - k(1 - q)$.

Disclosing $D_H$ and not manipulating results in $\alpha P(D_H) + (1 - \alpha)qP(R_H; D_H, 1) + (1 - q)P(R_L; D_H, 1) = [1 - (1 - \alpha)(1 - q)]P(D_H) - (1 - \alpha)(1 - q)I$.

Disclosing $D_L$ and manipulating results in $P(D_L) - k(1 - q)$. 
Disclosing $D_L$ and not manipulating results in $\alpha P(D_L) + (1 - \alpha)[q P(R_H; D_L, 1) + (1 - q) P(R_L; D_L, 1)] = [1 - (1 - \alpha)(1 - q)]P(D_L)$.

For type 2, disclosing $D_H$ and manipulating is optimal if

$$P(D_H) - k(1 - q) > [1 - (1 - \alpha)(1 - q)]P(D_H) - (1 - \alpha)(1 - q)I,$$

$$P(D_H) - k(1 - q) > P(D_L) - k(1 - q) \text{ and}$$

$$P(D_H) - k(1 - q) > [1 - (1 - \alpha)(1 - q)]P(D_L).$$

For type 1, disclosing $D_H$ and manipulating results in $P(D_H) - kq$.

Disclosing $D_H$ and not manipulating results in $\alpha P(D_H) + (1 - \alpha)[(1 - q) P(R_H; D_H, 1) + q P(R_L; D_H, 1)] = [1 - (1 - \alpha)q]P(D_H) - (1 - \alpha)qI$.

Disclosing $D_L$ and manipulating results in $P(D_L) - kq$.

Disclosing $D_L$ and not manipulating results in $\alpha P(D_L) + (1 - \alpha)[(1 - q) P(R_H; D_L, 1) + q P(R_L; D_L, 1)] = [1 - (1 - \alpha)q]P(D_L)$.

For type 2, disclosing $D_H$ and manipulating is optimal if

$$P(D_H) - kq > [1 - (1 - \alpha)q]P(D_H) - (1 - \alpha)qI,$$

$$P(D_H) - kq > P(D_L) - kq \text{ and}$$

$$P(D_H) - kq > [1 - (1 - \alpha)q]P(D_L).$$

The inequalities can be reduced to

$$k < (1 - \alpha)P(D_H) + (1 - \alpha)I = \frac{1}{2} (1 - \alpha) (2x_H + \delta),$$
\[ P(D_H) > P(D_L), \text{ which are equivalent to } \frac{1}{2}(2x_H + \delta) - l > q(2x_H - l) \]

\[ k < \frac{P(D_H) - P(D_L)}{1-q} + (1 - \alpha)P(D_L), \text{ which are equivalent to } k < \frac{\left(\frac{1}{2} - q\right)2x_H + \delta - (1-q)l}{1-q} + (1 - \alpha)q(2x_H - l) \]

Thus, it is optimal for both types to disclose \( D_H \) and manipulate if

\[ \frac{1}{2}(2x_H + \delta) - l > q(2x_H - l) \text{ and } k < \min\left(\frac{1}{2}(1 - \alpha + \beta)(2x_H + \delta), \alpha \frac{\left(\frac{1}{2} - q\right)2x_H + \delta - (1-q)l}{1-q} + (1 - \alpha)q(2x_H - l)\right) \]

\[ (1 - \alpha)q(2x_H - l) \]

**Case 1.2: Only type 1 manipulate.**

In this case \( P(R_L; D_H, 1) = -l \). For the off equilibrium beliefs, \( P(R_L; D_L, 1) = 0 \) as \( R_L \) is inferred as no earnings management.

For type 1, disclosing \( D_H \) and manipulating generates a payoff of

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq \text{ whereas disclosing } D_H \text{ and not manipulating generates a} \]

payoff of

\[ \alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) + qP(R_L; D_H, 1)] = \alpha P(D_H) + (1 - \alpha)(1 - q)P(R_H; D_H, 1) - (1 - \alpha)l. \]

Thus, we need

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq > \alpha P(D_H) + (1 - \alpha)(1 - q)P(R_H; D_H, 1) - (1 - \alpha)ql, \]

or, equivalently, \( k < P(R_H; D_H, 1) + (1 - \alpha)l. \)

For type 2, we can similarly calculate that disclosing \( D_H \) and manipulating generates a payoff of

\[ P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) \text{ whereas disclosing } D_H \text{ and not manipulating} \]

generates a payoff of

\[ \alpha P(D_H) + (1 - \alpha)qP(R_H; D_H, 1) - (1 - \alpha)(1 - q)l. \]

For disclosing \( D_H \) and not manipulating to be optimal, we need \( P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) < \)
\[ \alpha P(D_H) + (1 - \alpha)qP(R_H; D_H, 1) - (1 - \alpha)(1 - q)l, \] or, equivalently, \[ k > P(R_H; D_H, 1) + (1 - \alpha)l, \] which is a contradiction. Thus, this case is impossible.

**Case 1.3: Only type 2 manipulate.**

Again in this case \( P(R_L; D_H, 1) = -l \). For the off equilibrium beliefs, \( P(R_L; D_L, 1) = 0 \) as \( R_L \) is inferred as no earnings management.

For managers of type 2, disclosing \( D_H \) and manipulating results in \( \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) \).

Disclosing \( D_H \) and not manipulating results in \( \alpha P(D_H) + (1 - \alpha)[qP(R_H; D_H, 1) + (1 - q)P(R_L; D_H, 1)] = \alpha P(D_H) + (1 - \alpha)qP(R_H; D_H, 1) - (1 - \alpha)(1 - q)l \).

For type 2, disclosing \( D_H \) and manipulating is optimal only if

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) > \alpha P(D_H) + (1 - \alpha)qP(R_H; D_H, 1) - (1 - \alpha)(1 - q)l, \] which is equivalent to \( k < (1 - \alpha)P(R_H; D_H, 1) + (1 - \alpha)l \).

For type 1, disclosing \( D_H \) and manipulating results in \( \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq \).

Disclosing \( D_H \) and not manipulating results in \( \alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) + qP(R_L; D_H, 1)] = \alpha P(D_H) + (1 - \alpha)(1 - q)P(R_H; D_H, 1) - (1 - \alpha)ql \).

Thus for type 1, disclosing \( D_H \) and not manipulating is optimal only if

\[ P(D_H) - kq < \alpha P(D_H) + (1 - \alpha)(1 - q)P(R_H; D_H, 1) - (1 - \alpha)ql, \] which is equivalent to \( k > (1 - \alpha)P(R_H; D_H, 1) + (1 - \alpha)l \), again a contradiction. Thus, this case is impossible.
As a summary, for this subcategory, it is optimal for both types to disclose \(D_H\) and manipulate if
\[
\frac{1}{2}(2x_H + \delta) - I > q(2x_H - I)
\] and
\[
k < \min\left(\frac{1}{2}(1 - \alpha + \beta)(2x_H + \delta), \alpha \frac{(1-q)^2 x_H + \delta}{1-q} + (1-\alpha)q(2x_H - I)\right).
\]

**Subcategory 2: managers of both types disclose \(D_L\).**

Under this conjecture, \(P(D_L) = \frac{1}{2}(2x_H - I)\) as \(D_L\) is completely uninformative. We again compare the relative gain in payoffs from deviating to \(D_H\) to determine the off-equilibrium belief for \(D_H\) in each case.

**Case 2.1: Both types manipulate.**

In this case \(P(R_H; D_L, 1) = \frac{1}{2}(2x_H - I) = P(D_L)\).

By disclosing \(D_L\) and manipulating, type 2’s payoff is
\[
\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - k(1 - q) = P(D_L) - k(1 - q).
\]
Deviating to \(D_H\) and manipulating results in a payoff of
\[
\alpha P(D_H) + (1 - \alpha) P(R_H; D_H, 1) - k(1 - q) = P(D_H) - k(1 - q) = q(2x_H + \delta) - I - k(1 - q)
\]
if the off-equilibrium belief is that type 2 deviates to \(D_H\).

By disclosing \(D_L\) and manipulating, type 1’s payoff is
\[
\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - kq = P(D_L) - kq.
\]
Deviating to \(D_H\) and manipulating results in a payoff of
\[
\alpha P(R_H; D_H, 1) - kq = P(D_H) - kq = (1 - q)(2x_H + \delta) - I - kq
\]
if the off-equilibrium belief is that type 1 deviates to \(D_H\) and manipulates.
The gain from deviating to $D_H$ for type 2 minus the gain from deviating to $D_H$ for type 1 is

$$(2q - 1)(2x_H + \delta) > 0.$$ Thus, the off-equilibrium belief will be $P(D_H) = q(2x_H + \delta) - I > q(2x_H - I) > \frac{1}{2}(2x_H - I) = P(D_L)$ where the first inequality is because $q \delta > (1 - q)I$.

Note, however, this case cannot be an equilibrium as for type 2 manager, since deviating to $D_H$ and manipulate generates a payoff of $\alpha P(D_H) + (1 - \alpha) P(R_H; D_H, 1) - k(1 - q) = P(D_H) - k(1 - q)$, which is greater than disclosing $D_L$ and manipulating, which generates a payoff of $\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - k(1 - q) = P(D_L) - k(1 - q)$ as $P(D_H) > P(D_L)$.

**Case 2.2: Type 1 manipulates but type 2 does not.**

In this case we have $P(R_H; D_L, 1) = \frac{1}{2}\left[1\left(2x_H - I\right) + (2x_H - I)\right] = \frac{3}{4}(2x_H - I)$.

By disclosing $D_L$ and manipulating, type 2’s payoff is $\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - k(1 - q)$. Deviating to $D_H$ and not manipulating results in a payoff of $\alpha P(D_H) + (1 - \alpha)[qP(R_H; D_H, 0) + (1 - q)P(R_L; D_H, 0)] = \alpha[q(2x_H + \delta) - I] + (1 - \alpha)[q(2x_H + \delta - I) + (1 - q)(-I)] = q(2x_H + \delta) - I$ if the off-equilibrium belief is that type 2 deviates to $D_H$ and does not manipulate.

By disclosing $D_L$ and manipulating, type 1’s payoff is $\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - kq = P(D_L) - kq$. Deviating to $D_H$ and manipulating results in a payoff of $\alpha P(D_H) + (1 - \alpha) P(R_H; D_H, 1) - kq = P(D_H) - kq = (1 - q)(2x_H + \delta) - I - kq$ if the off-equilibrium belief is that type 1 deviates to $D_H$ and manipulates.

The gain from deviating to $D_H$ for type 2 minus the gain from deviating to $D_H$ for type 1 is $(2q - 1)(2x_H + \delta) + kq > 0$. Thus, the off-equilibrium belief will be $P(D_H) = q(2x_H + \delta) - I > q(2x_H - I) > \frac{1}{2}(2x_H - I) = P(D_L)$ where the first inequality is because $q \delta > (1 - q)I$. 

Note, however, this case cannot be an equilibrium as for type 2 manager, since deviating to $D_H$ and manipulate generates a payoff of $\alpha P(D_H) + (1 - \alpha) P(R_H; D_H, 1) - k(1 - q) = P(D_H) - k(1 - q)$, which is greater than disclosing $D_L$ and manipulating, which generates a payoff of $\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - k(1 - q) = P(D_L) - k(1 - q)$ as $P(D_H) > P(D_L)$.
\[\delta - I > q(2x_H - I) > \frac{1}{2}(2x_H - I) = P(D_L)\] where the first inequality is because \(q\delta > (1 - q)I\). We also have \(P(R_H; D_H, 0) = 2x_H + \delta - I\) and \(P(R_L; D_H, 0) = -I\).

Given this off-equilibrium belief, however, type 1, by deviating to \(D_H\) and manipulating will get a payoff of \(aP(D_H) + (1 - a)P(R_H; D_H, 0) - kq\), which is higher than the payoff of disclosing \(D_L\) and manipulating, \(aP(D_L) + (1 - a)P(R_H; D_L, 0) - kq\) as \(P(D_H) > P(D_L)\) and \(P(R_H; D_H, 0) = 2x_H + \delta - I > P(R_H; D_L, 1) = \frac{3}{4}(2x_H - I)\). Thus, this case is impossible.

**Case 2.3: Type 1 does not manipulate but type 2 does.**

In this case we again have \(P(R_H; D_L, 1) = \frac{3}{4}(2x_H - I)\). This case is also impossible.

To see this, note that for type 2 manager, disclosing \(D_L\) and manipulating results in the payoff of \(aP(D_L) + (1 - a)P(R_H; D_L, 1) - k(1 - q)\). Disclosing \(D_L\) and not manipulating results in the payoff of \(aP(D_L) + (1 - a)[qP(R_H; D_L, 1) + (1 - q)P(R_L; D_L, 1)] = aP(D_L) + (1 - a)qP(R_H; D_L, 1)\). For disclosing \(D_L\) and manipulating to be optimal, we need \(aP(D_L) + (1 - a)P(R_H; D_L, 1) - k(1 - q) > aP(D_L) + (1 - a)qP(R_H; D_L, 1)\), or, equivalently, \(k < (1 - a)P(R_H; D_L, 1)\). For type 1 manager, disclosing \(D_L\) and manipulating results in the payoff of \(aP(D_L) + (1 - a)P(R_H; D_L, 1) - kq\). Disclosing \(D_L\) and not manipulating results in the payoff of \(aP(D_L) + (1 - a)[(1 - q)P(R_H; D_L, 1) + qP(R_L; D_L, 1)] = aP(D_L) + (1 - a)(1 - q)P(R_H; D_L, 1)\). For disclosing \(D_L\) and not manipulating to be optimal, we need \(aP(D_L) + (1 - a)P(R_H; D_L, 1) - kq < aP(D_L) + (1 - a)(1 - q)P(R_H; D_L, 1)\), or, equivalently, \(k > (1 - a)P(R_H; D_L, 1)\). This contradicts with the requirement that \(k < (1 - a)P(R_H; D_L, 1)\). Thus this case is impossible.

As a summary, this subcategory is impossible.
Subcategory 3: Type 1 discloses $D_H$ and type 2 discloses $D_L$.

Under this scenario, $P(D_H) = (1 - q)(2x_H + \delta) - I$ and $P(D_L) = q(2x_H - I)$.

Note that since $(1 - q)\delta < qI$, $P(D_H) < (1 - q)(2x_H - I) < q(2x_H - I) = P(D_L)$.

**Case 3.1: Both type manipulates**

In this case, $P(R_H; D_L, 1) = P(D_L)$ and $P(R_H; D_H, 1) = (1 - q)(2x_H + \delta) - I = P(D_H)$. For type 1, disclosing $D_H$ and manipulating results in the payoff of 

$$\alpha P(D_H) + (1 - \alpha) P(R_H; D_L, 1) - kq = P(D_H) - kq.$$

Disclosing $D_L$ and manipulating results in the payoff of 

$$\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - kq = P(D_L) - kq.$$  

Similarly, for type 2, disclosing $D_H$ and manipulating results in $P(D_H) - k(1 - q)$ whereas disclosing $D_L$ and manipulating results in $P(D_L) - k(1 - q)$. Since $P(D_L) < P(D_H)$, type 2 has an incentive to deviate to disclosing $D_H$ and manipulating. Thus, this case cannot be optimal.

**Case 3.2: Only type 1 manipulates**

In this case, $P(R_H; D_L, 0) = 2x_H - I$, $P(R_H; D_H, 1) = P(D_H)$, $P(R_L; D_L, 0) = 0$ and the off-equilibrium belief $P(R_L; D_H, 1) = -I$.

Note that this case is impossible as for type 1, disclosing $D_H$ and manipulating results in the payoff of 

$$\alpha P(D_H) + (1 - \alpha) P(R_H; D_H, 1) - kq = P(D_H) - kq.$$  

By deviating to disclosing $D_L$ and manipulating, the payoff is $\alpha P(D_L) + (1 - \alpha) P(R_H; D_L, 1) - kq = \alpha P(D_L) + (1 - \alpha)(2x_H - I) - kq > \alpha P(D_L) + (1 - \alpha) P(D_H) - kq = P(D_L) - kq > P(D_H) - kq$. Thus, this case is impossible.

**Case 3.3: Only type 2 manipulates**
In this case, \( P(R_H; D_L, 1) = q(2x_H - I) = P(D_L) \), \( P(R_H; D_H, 0) = 2x_H + \delta - I \), \( P(R_L; D_H, 0) = -I \) and the off-equilibrium belief \( P(R_L; D_L, 1) = 0 \).

We now show that this case is impossible as well. Note that for type 1, disclosing \( D_H \) and not manipulating results in
\[
\alpha P(D_H) + (1 - \alpha)[qP(R_L; D_H, 0) + (1 - q)P(R_H; D_H, 0)] = \\
\alpha P(D_H) + (1 - \alpha)P(D_H) = P(D_H).
\]
Disclosing \( D_L \) and not manipulating results in
\[
\alpha P(D_L) + (1 - \alpha)[qP(R_L; D_L, 1) + (1 - q)P(R_H; D_L, 1)] = [1 - q(1 - \alpha)]P(D_L).
\]
Thus, for disclosing \( D_H \) and not manipulating to be optimal, we need \( P(D_H) > [1 - q(1 - \alpha)]P(D_L) \).

Now consider type 2, disclosing \( D_H \) and manipulating results in
\[
\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 0) - kq = \alpha P(D_H) + (1 - \alpha)[P(D_H) + q(2x_H + \delta)] - kq = \\
P(D_H) + (1 - \alpha)q(2x_H + \delta) - kq > P(D_H) + (1 - \alpha)P(D_L) - kq, \text{ whereas the last expression is the payoff from disclosing } D_L \text{ and manipulating. Thus, this case is impossible.}
\]

As a summary, subcategory 3 is impossible.

**Subcategory 4: Type 1 discloses \( D_L \) and type 2 discloses \( D_H \).**

**Case 4.1: Both types manipulate**

When both types manipulate, \( P(D_H) = q(x_H + x_H + \delta) - I \) and \( P(D_L) = (1 - q)(x_H + x_H - I) \); \( P(R_H; D_H, 1) = q(x_H + x_H + \delta) - I \) and the off-equilibrium belief for \( P(R_L; D_H, 1) = -I \); \( P(R_H; D_L, 1) = (1 - q)(2x_H - I) \) and the off-equilibrium belief is \( P(R_L; D_H, 1) = 0 \).

This strategy cannot be an equilibrium strategy, however. To see this, note that for managers of type 1, the payoff will be \( \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - kq = (1 - q)(2x_H - I) - kq \). If he chooses to disclose \( D_H \) and manipulate, the payoff will be
\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq = q(2x_H + \delta) - I - kq > q(2x_H - I) - kq > (1 - q)(2x_H - I) - kq. \]

**Case 4.2: Only type 1 manipulates**

In this case, \( P(D_H) = q(x_H + x_H + \delta) - I \) and \( P(D_L) = (1 - q)(x_H + x_H - I); \)
\( P(R_H; D_H, 1) = 2x_H + \delta - I \) and \( P(R_H; D_H, 1) = -I; P(R_H; D_L, 1) = (1 - q)(2x_H - I) \) and the off-equilibrium-belief is \( P(R_L; D_L, 1) = 0. \)

This strategy, however, cannot be an equilibrium strategy. To see this, note that for managers of type 1, the payoff will be \( \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 1) - kq = (1 - q)(2x_H - I) - kq. \) If he discloses \( D_H \) and manipulates, the payoff will be \( \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq = \alpha[q(2x_H + \delta) - I] + (1 - \alpha)(2x_H + \delta - I) - kq > q(2x_H + \delta) - I - kq > (1 - q)(2x_H - I) - kq, \) as shown above.

**Case 4.3: Only type 2 manipulates**

In this case, \( P(D_H) = q(x_H + x_H + \delta) - I \) and \( P(D_L) = (1 - q)(x_H + x_H - I); \)
\( P(R_H; D_H, 1) = q(x_H + x_H + \delta) - I \) and the off-equilibrium belief is \( P(R_L; D_H, 1) = -I; \)
\( P(R_H; D_L, 0) = 2x_H - I \) and \( P(R_L; D_L, 0) = -I. \)

For managers of type 2, disclosing \( D_H \) and managing results in the expected payoff of
\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q). \]

If he chooses to not manipulate, then the expected payoff is
\[ \alpha P(D_H) + (1 - \alpha)[qP(R_H; D_H, 1) + (1 - q)P(R_L; D_H, 1)]. \]

If he chooses to disclose \( D_L \) but manipulate, then the expected payoff will be
\[ \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 0) - k(1 - q) \]

If he chooses to disclose \( D_L \) but not manipulate, then the expected payoff will be

\[ \alpha P(D_L) + (1 - \alpha)[qP(R_H; D_L, 0) + (1 - q)P(R_L; D_L, 0)] \]

For manipulation and disclosing \( D_H \) to dominate for type 2, we need

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) > \alpha P(D_H) + (1 - \alpha)[qP(R_H; D_H, 1) + (1 - q)P(R_L; D_H, 1)], \]

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) > \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 0) - k(1 - q), \]

and

\[ \alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - k(1 - q) > \alpha P(D_L) + (1 - \alpha)[qP(R_H; D_L, 0) + (1 - q)P(R_L; D_L, 0)], \]

which can be reduced to

\[ k < (1 - \alpha)q(2x_H + \delta), \] (B.23)

\[ [q(1 + \alpha) - 1](2x_H - I) > (1 - q)I - q\delta, \] (B.24)

and

\[ k < \frac{\alpha}{1 - q}[(2q - 1)(2x_H - I)] + \frac{1}{1 - q}[q\delta - (1 - q)I] \] (B.25)

For managers of type 1, not manipulating and disclosing \( D_L \) results in the expected payoff of

\[ \alpha P(D_L) + (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0)] \]

If he chooses to manipulate, then the expected payoff is

\[ \alpha P(D_L) + (1 - \alpha)P(R_H; D_L, 0) - kq \]
If he chooses to disclose $D_H$ but manipulate, then the expected payoff will be

$$\alpha P(D_H) + (1 - \alpha)P(R_H; D_H, 1) - kq$$

If he chooses to disclose $D_H$ but not manipulate, then the expected payoff will be

$$\alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) + qP(R_L; D_H, 1)]$$

For not manipulating and disclosing $D_L$ to dominate, we need

$$\alpha P(D_L) + (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0)] > \alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) + qP(R_L; D_H, 1)]$$

, and

$$\alpha P(D_L) + (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0)] > \alpha P(D_H) + (1 - \alpha)[(1 - q)P(R_H; D_H, 1) - kq$$

, which can be reduced to

$$k > (1 - \alpha)(2x_H - I), \quad (B.26)$$

$$[(1 - q)^2 - \alpha q^2](2x_H + \delta) > (1 - q)\delta - qI \quad (B.27)$$

and

$$k > \frac{2q-1}{q} (2x_H - I) + \frac{1}{q} [q\delta - (1 - q) I]. \quad (B.28)$$
Note that \((1 - \alpha)(2x_H - l) - \left\{ \frac{2q-1}{q} (2x_H - l) + \frac{1}{q} [q\delta - (1 - q)l] \right\} = -\frac{1}{q} [(q(1 + \alpha) - 1)(2x_H - l) - (1 - q)l + q\delta] < 0\) where the last inequality is from equation (B.24).

Also note that \([(1 - q)^2 - \alpha q^2](2x_H + \delta) > (1 - q)\delta - qI\) implies that \(\frac{2q-1}{q} (2x_H - l) + \frac{1}{q} [q\delta - (1 - q)l] < (1 - \alpha)q(2x_H + \delta)\). In addition, \([q(1 + \alpha) - 1](2x_H - l) > (1 - q)l - q\delta\) implies that \(\frac{2q-1}{q} (2x_H - l) + \frac{1}{q} [q\delta - (1 - q)l] < \frac{\alpha}{1-q} [(2q - 1)(2x_H - l)] + \frac{1}{1-q} [q\delta - (1 - q)l]\).

As a summary, the conditions to sustain this equilibrium will be

\[
[q(1 + \alpha) - 1](2x_H - l) > (1 - q)l - q\delta, \quad \text{(B.29)}
\]

\[
[(1 - q)^2 - \alpha q^2](2x_H + \delta) > (1 - q)\delta - qI, \quad \text{(B.30)}
\]

\[
\frac{2q-1}{q} (2x_H - l) + \frac{1}{q} [q\delta - (1 - q)l] < k < \min\left( \frac{\alpha}{1-q} [(2q - 1)(2x_H - l)] + \frac{1}{1-q} [q\delta - (1 - q)l], (1 - \alpha)q(2x_H + \delta) \right). \quad \text{(B.31)}
\]

Also note that equations (B.29) and (B.30) come from

\[
\alpha[P(D_H) - P(D_L)] > (1 - \alpha)[P(R_H; D_L, 0) - P(R_H; D_H, 1)]
\]

and

\[
\alpha[P(D_H) - P(D_L)] < (1 - \alpha)[(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0) - (1 - q)P(R_H; D_H, 0) - qP(R_L; D_H, 0)].
\]
Thus, we need \((1 - \alpha)(1 - q)P(R_H; D_L, 0) + qP(R_L; D_L, 0) - (1 - q)P(R_H; D_H, 0) - qP(R_L; D_H, 0)\) > \((1 - \alpha)\left[P(R_H; D_L, 0) - P(R_H; D_H, 1)\right]\), or, equivalently, \((1 - q)2x_H < I + q\delta\), i.e., \(q > \frac{2x_H - l}{2x_H + \delta}\).

As a summary, the only possible equilibria are 1) both types disclose \(D_H\) and manipulate if

\[
\frac{1}{2}(2x_H + \delta) - I > q(2x_H - I) \quad \text{and} \quad k < \min\left(\frac{1}{2}(1 - \alpha)(2x_H + \delta), \frac{\alpha}{1-q}\right) + \alpha \frac{(1-q)^2 x_H + \delta}{2} \frac{I}{1-q} \quad \Rightarrow \\
(1 - \alpha)q(2x_H - I)\).
\]

2) managers of type 2 disclose \(D_H\) and manipulate when the true earnings number is not \(x_H\) and that managers of type 1 disclose \(D_L\) and does not manipulate. This equilibrium is possible if the following conditions are satisfied:

\[
q > \frac{2x_H - l}{2x_H + \delta},
\]

\[
[q(1 + \alpha) - 1](2x_H - I) > (1 - q)I - q\delta,
\]

\[
[(1 - q)^2 - \alpha q^2](2x_H + \delta) > (1 - q)\delta - qI
\]

\[
\frac{2q-1}{q}(2x_H - I) + \frac{1}{q}[q\delta - (1 - q)I] < k < \min\left(\frac{\alpha}{1-q}[(2q - 1)(2x_H - I)] + \frac{1}{1-q}[q\delta - (1 - q)I], (1 - \alpha)q(2x_H + \delta)\right)
\]

Q.E.D.

Proof of Corollary 1:
We now show that there are certain parameters of $\alpha$, $\beta$ and $q$ such that the equilibrium without discretion is that managers of all types disclose $D_H$ or $D_L$ whereas managers’ disclosures will separate when discretion is allowed.

First note from proposition 4 that when $\alpha > \frac{ql - (1-q)\delta}{(2q-1)(2x_H + \delta)}$, the equilibrium without discretion cannot be completely informative voluntary disclosure. Thus, so long as we can establish that the equilibrium with discretion results in completely informative voluntary disclosure then we are done showing that some degree of discretion maximizes investment efficiency.

Secondly, when $q$ is sufficiently large so that $\frac{1}{2} (2x_H + \delta) - I \leq q(2x_H - I)$, the equilibrium where both types disclose $D_H$ and manipulate will not exist, which requires $q > \frac{\frac{1}{2}(2x_H + \delta) - I}{2x_H - I}$.

Third, for the equilibrium with discretion to have the desirable investment rules, we need $[q(1 + \alpha) - 1](2x_H - I) > (1 - q)l - q\delta$ and $[(1 - q)^2 - \alpha q^2](2x_H + \delta) > (1 - q)\delta - ql$, or, equivalently, $[\alpha q^2 - (1 - q)^2](2x_H + \delta) < qI - (1 - q)\delta$.

As mentioned above, those conditions require $q > \frac{2x_H - I}{2x_H + \delta}$. Thus, when

$$q > \max\left(\frac{\frac{1}{2}(2x_H + \delta) - I}{2x_H - I}, \frac{2x_H - I}{2x_H + \delta}\right),$$

we have $\frac{2(1-q)x_H-q\delta}{q(2x_H-l)} < \alpha < \frac{ql-q(1-q)\delta+(1-q)^22x_H}{q^2(2x_H+\delta)}$. Since

$$\frac{2(1-q)x_H-q\delta}{q(2x_H-l)}$$

may be smaller than zero but $\frac{ql-q(1-q)\delta+(1-q)^22x_H}{q^2(2x_H+\delta)}$ is smaller than 1 as

$$\frac{ql-q(1-q)\delta+(1-q)^22x_H}{q^2(2x_H+\delta)} = \frac{qI-(1-q)\delta+(1-q)^2(2x_H+\delta)}{q^2(2x_H+\delta)} < \frac{\frac{2q-1}{q}(2x_H+\delta)}{q^2(2x_H+\delta)} <$$

$$\frac{(2q-1)(2x_H+\delta)+(1-q)^2(2x_H+\delta)}{q^2(2x_H+\delta)} = 1,$$

we have this equilibrium as the unique equilibrium when

$$q > \max\left(\frac{\frac{1}{2}(2x_H + \delta) - I}{2x_H - I}, \frac{2x_H - I}{2x_H + \delta}\right) \text{ and } \max\left(0, \frac{2(1-q)x_H-q\delta}{q(2x_H-l)}\right) < \alpha < \frac{ql-q(1-q)\delta+(1-q)^22x_H}{q^2(2x_H+\delta)}.$$
Note that this strategy will be the unique equilibrium when \( q > \frac{q_1 - (1-q)\delta}{(2q-1)(2x_H+\delta)} \). Thus, we need

\[
\frac{q_1-q(1-q)\delta+(1-q)^2x_H}{q^2(2x_H+\delta)} > \frac{q_1-(1-q)\delta}{(2q-1)(2x_H+\delta)},
\]
which is equivalent to \( 2x_H + \delta > ql - (1-q)\delta \). This will always be satisfied as \( 2x_H + \delta > l > ql - (1-q)\delta \). Thus when

\[
q > \max\left(\frac{(2x_H+\delta)-l}{2x_H-l}, \frac{2x_H-l}{2x_H+\delta}\right) \quad \text{and} \quad \max\left(\frac{q_1-(1-q)\delta}{(2q-1)(2x_H+\delta)}, \frac{2(1-q)x_H-q\delta}{q(2x_H-l)}\right) < \alpha < \frac{q_1-q(1-q)\delta+(1-q)^2x_H}{q^2(2x_H+\delta)},
\]

allowing some degree of discretion, i.e.,

\[
\frac{2q-1}{q} (2x_H - l) + \frac{1}{q} [q\delta - (1-q)l] < k < \min\left(\frac{\alpha}{1-q} [(2q-1)(2x_H - l)] + \frac{1}{1-q} [q\delta - (1-q)l], (1-\alpha)q(2x_H + \delta)\right),
\]

improves investment efficiency.

\textit{Q.E.D.}