Optimization of Spatially Varying Fiber Paths for a Symmetric Laminate with a Circular Cutout under Remote Uniaxial Tension

Pinar Acar, Avinashk S. Vijayachandran, and Veera Sundararaghavan
University of Michigan

Anthony Waas
University of Washington

Mostafa Rassian
Boeing

ABSTRACT

Minimizing the stress concentrations around cutouts in a plate is often a design problem, especially in the Aerospace industry. A problem of optimizing spatially varying fiber paths in a symmetric, linear orthotropic composite laminate with a cutout, so as to achieve minimum stress concentration under remote uniaxial tensile loading is of interest in this study. A finite element (FE) model is developed to this extent, which constraints the fiber angles while optimizing the fiber paths, proving essential in manufacturing processes. The idea to be presented could be used to design fiber paths that would drastically reduce the stress concentration factors (SCF) in a symmetric laminate by using spatially varying fibers in place of unidirectional fibers. The model is proposed for a four layer symmetric laminate, and can be easily reproduced for any number of layers. The FE model suggested would also let us use a reduced number of optimization variables, for in this case of a four layer symmetric laminate, only six optimization variables need to be defined to obtain the optimum fiber distribution to achieve minimum SCF around the circular cutout. By ensuring continuity in the FE model, discrete fiber angles within each element in a single layer can be easily smoothed out to obtain the optimal fiber path.

INTRODUCTION

Effect of cutouts in infinite plates under remote tensile loading has remained as a classical problem in elasticity. The stress state in an isotropic infinite plate with a cutout under a remote tensile loading creates an SCF of 3 around the cutout along the axis of loading [1]. This effect of high stress concentrations is of interest in design for fracture. Mindell proposed to increase the stiffness of the plate with a cutout so that the cutout thus has a stress state resembling the uncut structure [2], to reduce the SCF around cutouts in an isotropic plate. The problem of elevated SCF around cutouts is utmost importance for design of composite laminates in Aerospace Industry, the problem was revisited by Samoz and Waas [3, 4]. They proposed to multiply the SCF around cutouts in orthotropic composite laminates under remote tensile loading. The study considered the bending-stretching coupling to model stiffness around the cutouts and proposed geometries for neutral holes to eliminate the high SCF. With the development of automated manufacturing processes for orthotropic laminates, it has become much easier to steer the fiber paths in laminates as per design considerations.

The current study is interested in optimizing spatially varying fiber paths and layer thickness for a 4-layer symmetric orthotropic composite laminate to achieve reduced stress concentrations. The infinite plate is under uniform uniaxial tensile loading, assuming conditions of plane stress. The study ensures the continuity of the fiber in each layer by proposing a novel FE model that would maintain a constant fiber angle within each element, making it easy to obtain a smooth fiber path. The model can easily be reproduced for a laminate with many layers following the same procedure.

PROBLEM DEFINITION AND FORMULATION

A four layer, symmetric, linear orthotropic laminate with spatially varying fiber paths, having a circular cutout of interest in this study. The laminate is assumed to be infinite, implying that the dimensions of the laminate is much larger when compared to the size of the cutout, and any load applied here is considered to be a remote loading, at a distance farther away from the cutout. Further, the laminate is assumed to be in a state of plane stress, since the lateral
dimensions are much larger when compared to its thickness, imposing conditions, \( l \gg b \), where \( l \) and \( b \) are the dimensions in the lateral directions and \( h \) is the thickness, in the direction perpendicular to the plane of the paper (Figure 1). The remote loading in this case is uniaxial tension in the \( x \)-direction (coordinates placed at the center of the cutout, as in Figure 1 below).

\[
\begin{align*}
\sigma_{xx} & \quad \text{(1)} \\
\sigma_{yy} & \quad \text{(2)}
\end{align*}
\]

\[
d\theta_1 = \frac{T_{11} - T_{12}}{n - 1} \\
d\theta_2 = \frac{T_{21} - T_{22}}{n - 1}
\]

where \( n \) is the total number of FEs along the tangential direction starting from \( s = 0 \). If \( T(m) \) represent the fiber angle within the \( \alpha \) element in the tangential direction starting from \( s = 0 \), then \( T(m) \) can be obtained as:

\[
\begin{align*}
T(m) &= T_1 + (m - 1)d\theta_1 \\
T(m) &= T_2 + (m - 1)d\theta_2
\end{align*}
\]

Thus ensuring continuity of the angles within each layer by defining a discrete variation by an angle \( d\theta \) for all the fibers as it crosses each FE. As would be discussed in the next section, the fiber angle \( T(m) \) is specified for each element under restrictions arising from \( d\theta \) would be used in the FE formulation by effectively replacing the plane stress constitutive matrix by an average of the material matrix, obtained by rotating by a fiber angle for each element before assembling the global stiffness matrix. The finite element formulation uses 4-node quadrilateral (quad) element and is used to obtain the stresses and strains at four integration points within each element.

The average stress and strains are calculated for each direction (1,2,6 corresponding to \( XX \), \( YY \), and \( XY \) respectively) and a strain energy density function \( W(m) \) is calculated for each element, defined as:

\[
W_{m} = \frac{1}{2} \sum_{i=1,2,6} \left| \sigma_{m,i} \right|
\]

The total strain energy of the structure (quarter plate being modeled) is thus obtained as:

\[
W_{tot} = \sum_{m=1}^{\text{end}} W_{m}
\]

where \( \text{end} \) is the total number of elements in the solution domain.

The energy fraction \( \delta \) for each element is thus defined as: