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Optimization of Spatially Varying Fiber Paths for a Symmetric Laminate with a Circular Cutout under Remote Uniaxial Tension

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ABSTRACT

Minimizing the stress concentrations around cutouts in a plate is often a design problem, especially in the Aerospace industry. A problem of optimizing spatially varying fiber paths in a symmetric, linear orthotropic composite laminate with a cutout, so as to achieve minimum stress concentration under remote unidirectional tensile loading is of interest in this study. A finite element (FE) model is developed to this extent, which constraints the fiber angles while optimizing the fiber paths, proving essential in manufacturing processes. The idea to be presented could be used to derive fiber paths that would drastically reduce the Stress Concentration Factor (SCF) in a symmetric laminate by using spatially varying fibers in place of unidirectional fibers. The model is proposed for a four layer symmetric laminate, and can be easily reproduced for any number of layers. The FE model suggested would also let us use a reduced number of optimization variables, for in this case of a four layer symmetric laminate, only six optimization variables need to be defined to obtain the optimum fiber distribution to attain minimum SCF around the circular cutout. By ensuring continuity in the FE model, discrete fiber angles within each element in a single layer can be easily smoothed out to obtain the optimal fiber path.

INTRODUCTION

Effect of cutouts in infinite plates under remote tensile loading has remained as a classical problem in elasticity. The stress state in an isotropic infinite plate with a cutout under remote tensile loading creates an SCF of 3 around the cutout along the axis of loading [1]. This effect of high stress concentrations is of interest in design for fracture. Mansfield proposed to restore the stiffness of the plate with a cutout so that the cut structure has a stress state resembling the uncut structure [2], to reduce the SCF around cutouts in an isotropic plate. The problem of elevated SCFs around cutouts is utmost importance for design of composite panels as the coupling effect can produce very high values of SCF around the cutout. With the advent of composite laminates in Aerospace Industry, the problem was revisited by Senocak and Waas [3, 4]. They proposed to nullify the SCF around cutouts in orthotropic composite laminates under remote tensile loading. The study considered the bending-stretching coupling to model stiffeners around the cutouts and proposed geometries for neutral holes to eliminate the high SCF. With the development of automated manufacturing processes for orthotropic laminates, it has become much easier to steer the fiber paths in laminates as per design considerations.

The current study is interested in optimizing spatially varying fiber paths and layer thickness for a 4 layer symmetric orthotropic composite laminate to achieve reduced stress concentrations. The infinite plate is under uniform uniaxial tensile loading, assuming conditions of plane stress. The study ensures the continuity of the fibers in each layer by proposing a novel FE model that would maintain a constant fiber angle within each element, making it easy to obtain a smooth fiber path. This model can easily be reproduced for a laminate with many layers following the same procedure.

PROBLEM DEFINITION AND FORMULATION

A four layer, symmetric, linear orthotropic laminate with spatially varying fiber paths, having a circular cutout is of interest in this study. The laminate is assumed to be infinite, implying that the dimensions of the laminate is much larger when compared to the size of the cutout; and any load applied here in is considered to be a remote loading, at a distance farther away from the cutout. Further, the laminate is assumed to be in a state of plane stress, since the lateral dimensions are much larger when compared to its thickness, imposing conditions, $l, b \gg h$, where l and b are the dimensions in the lateral directions and h is the thickness, in the direction perpendicular to the plane of the paper (Figure 1). The remote loading in this case is uniaxial tension in the *x-direction* (co-ordinates placed at the center of the cutout, as in Figure 1 below).

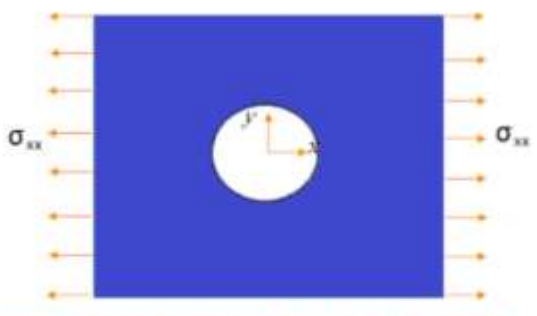

Figure 1. Infinite plate representation with placement of co-ordinate axes

This is a classical problem in elasticity, that of an infinite plate with a cutout, under remote tension, and similar symmetric boundary conditions (BCs) can be applied. In the analysis, only one quarter of the infinite plate is modeled. The BCs are $u(0,y) = v(x,0) = 0$ at $x = 0$ and $y = 0$ respectively. Symmetry of loading and geometry as well as BCs are illustrated in Firure 2 below.

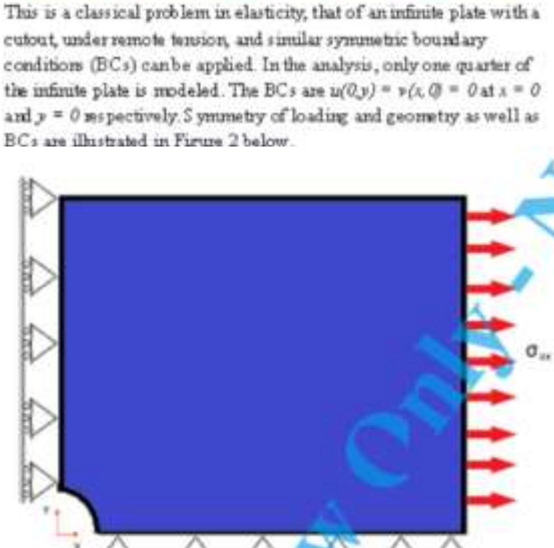

Figure 2. Representation of the Quarter Plate and Boundary Conditions

ALGORITHM

A four layer *symmetric* laminate is analyzed herein, thereby neglecting the bending-stretching coupling. Layers 1 and 2 are thus the same as layers 4 and 3 respectively (as in Figure 3), hence the subscripts 1 represent layers 1, 4 and subscript 2 represents layers 2,3 henceforth. T_i and T_f signifies initial and final fiber angles at $x = 0$ and $y = 0$ respectively. For the quarter plate under consideration T_{i1} and T_{i2} represent initial fiber angles at $x = 0$, T_{f1} and T_{f2} represent final fiber angles at $y = 0$. Hence we define the change in fiber angle in each FE along the tangential direction as $d\theta_1$ and $d\theta_2$, as follows:

$$\left.\begin{aligned} d\theta_1 &= \frac{T_{f1} - T_{i1}}{n-1} \\ d\theta_2 &= \frac{T_{f2} - T_{i2}}{n-1} \end{aligned}\right\} \qquad (1)$$

where n is the total number of FEs along the tangential direction starting from $x = 0$. If $T(m)$ represent the fiber angle within m^{th} ($1 \le m \le n$) element in the tangential direction starting from $x = 0$, then $T(m)$ can be obtained as:

$$T(m) = T_i + (m-1)d\theta \qquad (2)$$

and for each layer, it is expressed as:

$$\left.\begin{aligned} T_1(m) &= T_{i1} + (m-1)d\theta_1 \\ &\text{and} \\ T_2(m) &= T_{i2} + (m-1)d\theta_2 \end{aligned}\right\} \qquad (3)$$

thus ensuring continuity of the angles within each layer by defining a discrete variation by an angle $d\theta$ for all the fibers as it crosses each FE. As would be discussed in the next section, the fiber angle $T(m)$ specified for each element under restrictions arising from $d\theta$ would be used in the FE formulation by effectively replacing the plane stress constitutive matrix by an average of the material matrix, obtained by rotating by a fiber angle for each element before assembling the global stiffness matrix. The finite element formulation uses 4-noded quadrilateral (quad) element and is used to obtain the stresses and strains at four integration points within each element.

The average stress and strains are calculated for each direction (1,2,6 corresponding to XX, YY and XY respectively) and a strain energy density function $W(m)$ is calculated for each element, defined as:

$$W_m = \frac{1}{2} \sum_{i=1,2,6} |\sigma_i \varepsilon_i| \qquad (4)$$

The total strain energy of the structure (quarter plate being modeled) is thus obtained as:

$$W_{tot} = \sum_{m=1}^{nel} W_m \qquad (5)$$

where *nel* is the total number of elements in the solution domain.

The energy fraction E_j for each element is thus defined as: