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Target Management in Complex System Design Using System Norms

The problem of setting, balancing, and determining priorities of design targets among the subsystems constituting an engineering system, i.e., managing the targets, is addressed. A new norm-based benchmarking approach is proposed to relate the system-level design objectives to subsystem design targets. The proposed approach provides a systematic means of setting and balancing subsystem design targets to deliver the desired system performance and ranks the priorities of the subsystem targets. Furthermore, the use of system norms, rather than output signal norms, to quantify system and subsystem performance reduces the number of design targets in multi-input multi-output (MIMO) systems. The approach is illustrated on a vehicle example, consisting of a frame, body, and body mounts as the subsystems. [DOI: 10.1115/1.1897404]

1 Introduction

1.1 Motivation. The design of "complex systems" is a difficult, and often lengthy, process that utilizes a significant amount of resources: human, computational, etc. The term complex systems here denotes systems involving many design variables, interactions between these design variables, performance requirements from multiple disciplines, and uncertainty regarding the values of the design variables and parameters. Examples include aircraft, automobiles, photocopiers, etc. The design process for such a complex system is composed of several phases. It starts with the identification and analysis of customer requirements and ends with a final design of a product that is ready for manufacture [1]. According to Kusiak and Park, [2], 70% to 80% of the final production costs are determined during the design stage. If errors in the design process are not detected during the early design stages, they may result in flawed final products. Costly redesigns and modifications may then be necessary.

The overall design task is often decomposed into several smaller subtasks, each to be handled by a separate design team. The design then proceeds in parallel. The benefits of decomposing the overall design problem into several subproblems include [2] (i) detecting activities that can be executed in parallel, (ii) reducing the complexity of managing the design tasks, (iii) reducing design cycle times, and (iv) establishing cross-functional design teams that do not necessarily correspond to traditional organizational structures (although this is mostly a necessity and may have some drawbacks).

In order to successfully apply system decomposition and to design the resulting subsystems in parallel, it is important to develop a systematic means of setting subsystem design targets based on the desired system-level performance. This process of translating the desired system performance into design targets for the different subsystems is known as target cascading [3]. Target cascading should be performed in an efficient and consistent way to avoid iterations at later stages of the product development cycle and ensure that the different subsystems ultimately meet performance specifications upon assembly.

This motivates the present work, in which we relate the system-level objectives to the subsystem design targets. Knowing the relationship between system-level objectives and subsystem targets allows us to set and balance subsystem design targets to yield satisfactory system-level performance.

1.2 Literature Review. A system model is a "mathematical description of a relationship between externally supplied quantities (i.e., those coming from outside the system) and the dependent quantities that result from the action or effect on those external quantities" [4]. The externally supplied quantities are the "inputs" to the system and the dependent quantities are the "outputs." The design of complex engineering systems is a challenging task that requires the collaboration of many people with different areas of expertise.

The need to group these people into design teams and distribute the various design tasks among the teams motivates the need for system decomposition. The goal is to divide the overall design task into several subtasks; each small enough to be manageable by a particular team [5]. The solution obtained by solving the different subproblems should correspond to the solution of the original problem, i.e., the "truth" of the original problem should be "maintained" [6].

Several approaches exist in the literature and in industrial practice to design engineering systems. Methods based on decomposition and optimization will be reviewed briefly below. The mathematical statement of a general design problem (GDP) is [7]

$$\text{Find } \mathbf{x} \in X$$

$$\text{subject to } f(\mathbf{x},\mathbf{p}) \leqslant \mathbf{0} \tag{1}$$

In this statement, $\mathbf{x}$ represents the vector of design variables, X is the set constraints for the design variables, and $f(\mathbf{x},\mathbf{p})$ represents inequality constraints as functions of design variables $\mathbf{x}$ and design parameters $\mathbf{p}$.

The problem of decomposing a large optimal design problem into a set of smaller subproblems was addressed in Refs. [8–15]. An application of one of the problem decomposition strategies, namely optimal model-based decomposition, to a vehicle power train is illustrated in Refs. [16–18]. The problem of system decomposition is schematically illustrated in Fig. 1.

In Fig. 1, $\mathbf{Z}$ is a vector of shared design variables, $\mathbf{x}_i$ is a vector of design variables for subsystem i and together they comprise the vector $\mathbf{x}$ of (1), and $\mathbf{y}_{ij}$ is a vector of output signals to subsystem i from subsystem j. Decomposing a system into subsystems reveals interconnections between the subsystems where the output of one subsystem is used as an input to another subsystem. The number of these interconnections increases with the level of de-

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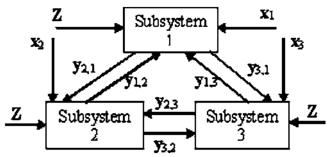

Fig. 1 Multidisciplinary design problem

composition from the system level to the subsystem and component levels. Since design targets are placed on the values of the output signals, the increase in their numbers corresponds to an increase in the number of design targets. This can result in a large increase in the number of design targets at the subsystem level for systems with several interconnections between the subsystems, such as multi-input multi-output (MIMO) systems. This increases the engineering resources required to track the status of the subsystem performance relative to the design targets. Moreover, it demands reliable methods of setting priorities among the numerous targets.

Several approaches are described in the optimization literature to optimize a large complex system, e.g., all-at-once optimization, bilevel integrated system synthesis [5,18], collaborative optimization, distributed object-based modeling environment (DOME) [19], game theoretic approaches [20], and target cascading [3]. However, a problem common to these approaches is that they assume full coordination between the subsystem design problems, i.e., that the values of each subsystem's design details and its outputs are always available to other subsystems that may require this information. This may not be the case in a real industrial environment where organizational barriers and geographical constraints may result in decisions taken by one of the design teams not being immediately and fully communicated to other design teams [21]. Two approaches used in industry to design complex systems, set-based concurrent engineering and benchmarking, are reviewed next.

1.2.1 Set-Based Concurrent Engineering. Set-based concurrent engineering was credited by Liker et al. and Sobek et al. as the reason behind Toyota's success in developing new designs in an efficient and rapid manner [22–24]. Suppliers and design teams are given performance specifications that they need to meet for their respective subsystems. Alternative subsystem designs are then explored and intersections among these "sets" of feasible subsystem designs are identified. The overall system is then obtained by recognizing which design alternatives for the different subsystems work "well" together and constitute a good overall system. This allows suppliers or design teams to work largely independently and allows them to explore several design concepts prior to committing to a design decision, which results in better overall designs.

1.2.2 Benchmarking. Benchmarking is another approach commonly used in industrial practice to set targets for the design of a new product. The idea is to quantify the performances of the subsystems of an existing comparator system that is similar to the product under development, i.e., program system, and use these as a basis for setting the performance targets for the subsystems of the new product. Benchmarking yields incremental improvements in performance. Although the literature on benchmarking is scarce, it is believed to be a commonly used approach [24].

1.3 Contributions. The number of interconnections between subsystems increases with the level of decomposition from the system level to the subsystem and component levels, causing an increase in the number of design targets. The end result is an increase in the resources required to design and verify that the design targets are met. The problems associated with designing a complex system motivate the present work in which we:

1. Express system-level objectives in terms of subsystem design targets. This allows us to set subsystem design targets to meet desired system-level performance specifications.
2. Study the trade-offs between the design targets of a single subsystem and between the design targets that influence the same system-level performance requirement (including design targets in several subsystems). This helps identify the critical subsystems and balance the system-level performance requirements among subsystems.
3. Quantify and account for the interconnections and coupling between the subsystems.

In order to set subsystem design targets that meet the required system-level objectives, we express the relationship between the system-level objectives and subsystem design targets. We use this relationship as a basis for setting design targets.

Trade-offs may exist among the design targets within a particular subsystem and/or between different subsystems. It may not be possible to simultaneously improve the performance of that subsystem with regard to different performance attributes. This introduces tradeoffs between the subsystem design targets.

In addition to trade-offs between design targets within a subsystem, several subsystems may influence the same system-level performance objective, making it possible to achieve the same level of system performance by different combinations of subsystem design targets. For example, it may be possible to relax requirements on one subsystem and tighten the required performance from another subsystem and achieve the same system-level performance. Understanding the trade-offs between subsystem targets will help avoid potential cases where designers focus on a certain subsystem target and manage to improve it, but at the expense of adversely affecting other subsystem targets, and possibly having worse overall system performance.

The process of cascading the overall system performance specifications into design targets for the different subsystems, and the overall design process in general, are complicated by the coupling between the subsystems. This coupling may take the form of shared design variables among different design teams or interconnections between subsystems, where the output of one subsystem may appear as an input to another subsystem. Methods for quantifying and accounting for interconnections are needed to enable setting subsystem targets that are consistent with the desired overall system-level specifications.

2 Problem Formulation

We assume the availability of a benchmark, or "comparator," system. The benchmark meets some system-level performance measure U and is composed of several subsystems. In the following problem formulation, we assume, without loss of generality, that the system is composed of three subsystems A, B, and C. We want to develop a new system design, termed the "program" system, which meets some different, possibly improved, system-level performance U^*.

We assume that the existing comparator and new program systems are similar in the sense that they have the same decomposition and functional dependencies of the different system and subsystem attributes on the design variables and parameters. We assume that we have full knowledge of the different subsystem attributes and their probabilities of being less than a specified upper bound for the comparator system. The design problem at hand will be to specify upper bounds for the subsystem attributes of the new program system to achieve the desired system-level objectives. In the formulation below, the subsystem design variables are assumed to be mutually exclusive, i.e., there are no shared design variables. Extension to include the case with shared design variables is currently being investigated.

Let $\mathbf{x}$ be a vector of the mean values of the design variables. The problem can then be stated formally as:

Given:

$\mathbf{x}$ = a vector of the mean values of the design variables

$\mathbf{p}$ = a vector of the mean values of the design parameters

X = the set constraints for design variables

U^* = the upper bound on the value of system attributes

$f(\mathbf{x},\mathbf{p})$ = the functional dependence of system attributes on the design variables and parameters

$f_j(\mathbf{x},\mathbf{p})$ = the functional dependence of attributes of the subsystem $j=A,B,C$ on the design variables and parameters

Find

$$U_A, U_B, U_C$$

where U_j is the upper bound on the value of attribute of the subsystem $j=A,B,C$

Such that

$$f_A(\mathbf{x},\mathbf{p}) \leq U_A$$

$$f_B(\mathbf{x},\mathbf{p}) \leq U_B \Rightarrow \mathbf{x} \in X{:}f(\mathbf{x},\mathbf{p}) \leq U^*$$

$$f_C(\mathbf{x},\mathbf{p}) \leq U_C \tag{2}$$

We want to set design targets U_j (e.g., U_A, U_B, U_C) on three subsystem-level design problems such that meeting the targets for the subsystem design problems gives a solution to the system-level problem. The process used for setting subsystem targets should give us a quantification of the tradeoffs in performance among the subsystems. This will enable us to relax design targets on one subsystem by tightening the targets on another subsystem (i.e., "balance" targets among subsystems).

3 Proposed Approach: Norm-Based Benchmarking

A new approach is proposed that aims at setting subsystem design targets to meet system-level performance requirements. The proposed norm-based benchmarking approach entails two main steps: (i) setting subsystem design targets and (ii) designing the subsystems to achieve their targets.

In order to set subsystem targets, the system-level performance is related to the performances of the different subsystems. The relationship between system-level performance and subsystems' performances is then used, together with the desired system-level performance, to set design targets on the norms of the different subsystems. The subsystem design targets are then handed over to the design teams that are responsible for designing their respective subsystems to meet their targets.

In a design problem, performance is typically expressed as a function of a system's design variables and parameters. In order to specify subsystem design targets, we need to relate system-level performance to the subsystems' performances, i.e., we want to express the system-level performance as a function of the subsystems' performances. This can be done by formulating a new design problem that approximates the original design problem, but with the subsystems' performances as design variables. The system-level performance can be expressed as a function of subsystems' performances using sensitivity analysis or other approximate means that will be discussed below. These ideas will now be discussed in more detail using the general design problem described by (1).

The system-level design problem is of the form given in (1) and repeated below

$$\text{Find } \mathbf{x} \in X$$

$$\text{subject to } f(\mathbf{x},\mathbf{p}) \leq \mathbf{0}$$

This system-level problem is decomposed into several subsystem design problems, each of the form

$$\text{Find } \mathbf{x_i} \in X_i,$$

$$\text{subject to } f_i(\mathbf{x_i},\mathbf{p_i}) \leq \mathbf{0}, \quad (i=1 \text{ to } m)$$

We then formulate a new design problem in terms of the subsystems' responses, i.e., we express the system-level response in terms of subsystems' responses. Thus, we need to find an operator $g[f_i(\mathbf{x}_i,\mathbf{p}_i)]$ that approximates the original design problem, i.e.:

$$f(\mathbf{x},\mathbf{p}) \approx g[f_i(\mathbf{x}_i,\mathbf{p}_i)], \quad (i=1 \text{ to } m) \tag{3}$$

This approach of relating system-level responses to subsystems' responses can be thought of as an alternative to response surface analysis where system-level responses are expressed in terms of simple functions of some of the design variables. In Ref. [25], the authors partition a design problem into two subproblems and use two second-order polynomials to express system-level responses in terms of some of the design variables of the two subproblems.

The preceding discussion was general and applies to different system-level attributes. However, in the following discussion, we focus on automotive systems and one particular system-level attribute, noise-vibration-harshness (NVH). In the automotive industry, one metric used to evaluate a system's NVH performance is the root mean square (rms) value of the output signal. The inputs to the subsystems in this case will be forces or displacements causing vibrations, and the outputs will be displacements or velocities at locations of interest. For a MIMO system, the rms value of the output signal is equal to the two norms of the system's transfer matrix when the system is driven by white noise. The square of the two norms of a transfer matrix is equal to the sum of the squares of the two norms of its constituent transfer functions [26]. The proposed approach is particularly well suited to such dynamic, MIMO systems.

3.1 Main Result. Sensitivity analysis is used to relate system-level objectives to the design targets of the various subsystems. This can be done using the known values of the system and subsystem performance measures of the benchmark, and linearizing the relationship between the system-level performance and the subsystem performances around these known values. Linearizing the relationship between the system-level performance and the subsystem performances is a simplifying assumption. This assumption is made to avoid having to reevaluate the sensitivities of system-level performance with respect to different values of subsystem targets. The use of sensitivity analysis to relate system-level objectives to subsystem design targets allows us to set and balance design targets for the various subsystems. Furthermore, the use of sensitivity analysis provides a measure of the influence of each subsystem on the overall system performance and can thus be used to rank subsystem design targets according to their influence.

The following proposition provides the basis for relating the system-level performance to subsystem design targets. Subsystem design targets here are assumed to be scalars relating the transfer matrices of the program subsystems to the corresponding subsystems of the benchmark. The proposition deals with a system in a general configuration.

Proposition 1. Consider the strictly proper rational matrix $G(jw,q)=C(q)[jwI-A(q)]^{-1}B(q)$, where the matrices $A(q)$, $B(q)$, and $C(q)$ have appropriate dimensions and are differentiable functions of the real vector q. Assume that all the eigenvalues of $A(q)$ have strictly negative real parts, and define $\|G(jw,q)\|_2^2 = 1/2\pi\int_0^\infty tr[G^*(jw,q)G(jw,q)]dw$.

Then

$$\frac{\partial\|G\|_2^2}{\partial q_i} = tr\left(\frac{\partial C(q)}{\partial q_i}W(q)C(q)^T + C(q)\frac{\partial W(q)}{\partial q_i}C(q)^T + C(q)W(q)\frac{\partial C(q)}{\partial q_i}\right) \quad (4)$$

where the symmetric matrix $W(q)$ is the solution of the linear matrix equation

$$A(q)W(q) + W(q)A(q)^T + B(q)B(q)^T = 0 \quad (5)$$

and $\partial W(q)/\partial q_i$ is the solution of the linear matrix equation

$$A(q)\frac{\partial W(q)}{\partial q_i} + \frac{\partial W(q)}{\partial q_i}A(q)^T + \frac{\partial B(q)}{\partial q_i}B(q)^T + B(q)\frac{\partial B(q)^T}{\partial q_i} + \frac{\partial A(q)}{\partial q_i}W(q) + W(q)\frac{\partial A(q)^T}{\partial q_i} = 0 \quad (6)$$

Proof

$$\|G\|_2^2 = tr[C(q)W(q)C(q)^T] \quad (7)$$

where $W(q)$ is given by

$$A(q)W(q) + W(q)A(q)^T + B(q)B(q)^T = 0 \quad (8)$$

From Eq. (8) and $W(q) = W(q)^T$,

$$\frac{\partial\|G\|_2^2}{\partial q_i} = tr\left(\frac{\partial C(q)}{\partial q_i}W(q)C(q)^T + C(q)\frac{\partial W(q)}{\partial q_i}C(q)^T + C(q)W(q)\frac{\partial C(q)^T}{\partial q_i}\right) \quad (9)$$

Differentiating (8) with respect to q_i yields

$$A(q)\frac{\partial W(q)}{\partial q_i} + \frac{\partial W(q)}{\partial q_i}A(q)^T + \frac{\partial B(q)}{\partial q_i}B(q)^T + B(q)\frac{\partial B(q)^T}{\partial q_i} + \frac{\partial A(q)}{\partial q_i}W(q) + W(q)\frac{\partial A(q)^T}{\partial q_i} = 0 \quad (10)$$

Equation (10) can be solved for $\partial W(q)/\partial q_i$ and the result substituted into (9) to get $\partial\|G\|_2^2/\partial q_i$.

In Proposition 1, the vector q contains the different parameters of interest when calculating the sensitivities of $\|G\|_2^2$. As we apply Proposition 1 to target cascading, vector q contains parameters representing subsystem targets. However, the same procedure would apply if q were a vector of design variables.

Several combinations of subsystem design targets can possibly result in the same system-level performance. Since the design targets for the newly developed product are expected to be close to, or show the same proportions as, the measured performances of the benchmarks' subsystems, the benchmarks can be used in selecting subsystem design targets.

Using the results of Proposition 1, we were able to find expressions for the relationship between the overall system-level objectives and subsystem design targets for subsystems in common configurations (Appendix A): series, parallel, or feedback.

Once the relationship between the system-level and subsystem performances is known, a new design problem can be formulated, with the subsystem performances as design variables. The relationship between system and subsystem performances can be used to find subsystem design targets that, if met, will result in the overall system meeting its specifications. Thus, rather than working from the subsystem and component levels and trying to improve the performances of these to reach satisfactory system-level performance, we propose starting from the desired system-level performance and working downward to find subsystem design targets that, if met, will yield this desired system-level performance. The proposed approach will now be illustrated using two examples: a simple example and a more realistic vehicle example having the same block-diagram topology.

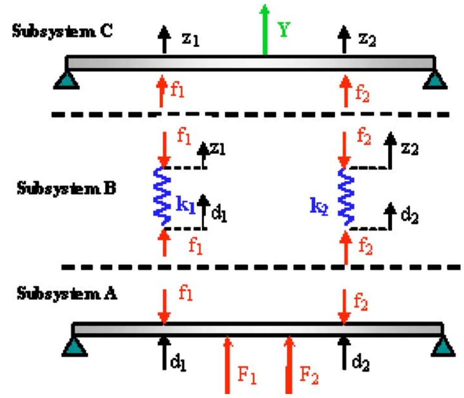

Fig. 2 Model of simple physical system after decomposition

4 Simple Example

This illustrative example features two beams connected by springs as shown in Fig. 2. The input to the system is vector **F**, consisting of forces F_1 and F_2 applied to subsystem A, and the output is the displacement Y of subsystem C. Vector **z** consists of displacements z_1 and z_2 at the springs' attachment points. We are assuming all variables in this example to be design variables, i.e., there are no design parameters. These design variables are defined and their values are specified for the comparator system in Appendix B for two distinct cases: one where subsystems A and C are weakly coupled, and the other where they are strongly coupled. In this example, functions $f_i(\mathbf{x},\mathbf{p})$, $i=A,B,C$ from (2) represent the two norms of the program subsystems, U_i, $i=A,B,C$ are obtained by scaling the two norms of the comparator subsystems and are used to set subsystem targets for the program system.

This system can be represented using the block diagram of Fig. 3. Note that even though the physical layout of subsystems A, B, C appears to be a simple tandem connection in Fig. 2, their block diagram exhibits two feedback paths as in Fig. 3. These feedback paths are due to the basic law of action/reaction and the constitutive law of linear springs. The presence of these feedback paths implies that the different subsystems are actually coupled. This coupling has to be accounted for in the target cascading process.

The results of the sensitivity analysis are given in Table 1 for two cases: weakly and strongly coupled subsystems. Weak coupling occurs when the infinity norm of the subsystems in a feedback loop is much less than unity. For the weakly coupled case, all subsystems have nearly equal influence on the overall system performance. For the strongly coupled case, subsystems A and C have nearly equal priorities, and thus equal engineering resources should be assigned to improving these two subsystems. However, subsystem B has a much lower priority, so changes in this subsystem are not expected to have a great influence on the overall system performance. Furthermore, the negative sign observed with the design target for subsystem B means that the system-level performance actually improves when subsystem B's design target worsens, resulting in the use of a stiffer spring. This largely counterintuitive result is due to significant coupling between the subsystems, and helps explain why the design of strongly coupled subsystems is a difficult task.

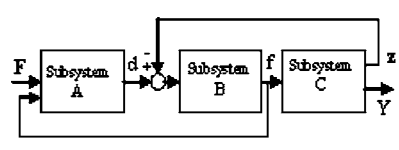

Fig. 3 System with feedback loops

Table 1 Results of the sensitivity analysis

	Weakly coupled subsystems	Strongly coupled subsystems
$\frac{\partial\|G\|_2^2}{\partial\alpha}$	0.6776757×10^{-6}	0.4393×10^{-4}
$\frac{\partial\|G\|_2^2}{\partial\beta}$	0.6652135×10^{-6}	-0.0176×10^{-4}
$\frac{\partial\|G\|_2^2}{\partial\gamma}$	0.6727637×10^{-6}	0.4515×10^{-4}

Proposition 1 was used to evaluate the sensitivities of the system-level performance with respect to performance levels of the subsystems. The results of the sensitivity analysis were used to determine design targets for the subsystems (see Tables 2 and 3). Note that this amounts to a linear approximation of the function g of Eq. (3). For the case with strongly coupled subsystems, the design targets for subsystems A and C are based on an 8% reduction in their 2-norms compared to the benchmark, identified as the comparator system. The sensitivity of subsystem B was negative, so it was decided not to change its design target. The system-level response as a function of α and its sensitivity with respect to α are plotted in Fig. 4. These plots show robustness with respect to subsystem targets (the plots for β and γ show similar results and were omitted).

This simple example was also solved using the results for common configurations assuming the subsystems have a tandem connection (i.e., neglecting the feedback loops). The assumption that the feedback loops can be neglected was verified by checking the magnitudes of the feedback loops and ensuring that they are much less than unity. The performance requirements on subsystem A of the program system were relaxed and the target chosen resulted in a performance level that is poorer than that of the benchmark subsystem A. However, by tightening the requirements on subsystem B, the desired overall system performance is achieved and the program system still outperforms the benchmark system.

5 Vehicle Example

The vehicle example used consists of three multi-input-multi-output (MIMO) subsystems; the frame, body mounts, and body. These three subsystems model the path through which different inputs, such as road inputs and power-train vibration, are transferred to the passengers in a lightweight truck. The design goal in this case is to reduce the vibration transferred to the vehicle passengers. The two norm, which is proportional to the rms norm, of the velocities at certain measurement points in the vehicle body is used as a measure of vibration intensity.

Inputs to the frame subsystem include the suspension forces induced by road surface inputs and the vibrations produced by the different components of the vehicle drive train. A full-vehicle model, generated through a combination of finite element model-

Table 2 Target setting using proposed approach (strongly coupled subsystems)

		Comparator (C)	Program (P)	
Performance Targets	Subsystem A	$\|A^*\|_2=0.0153$	$\|A\|_2=0.014$	P>C
	Subsystem B	$\|B^*\|_\infty=892$	$\|B\|_\infty=892$	P=C
	Subsystem C	$\|C^*\|_2=0.0103$	$\|C\|_2=0.0095$	P<C
Weak-coupling Conditions	Feedback loop 1	$\|A^*B^*\|_\infty=3225$	$\|AB\|_\infty=3081$	C, P$\gg$1
	Feedback loop 2	$\|B^*C^*\|_\infty=3012$	$\|BC\|_\infty=2877$	C, P$\gg$1
System Output	Y	0.0067	0.0062	P<C

Table 3 Target setting using proposed approach (weakly coupled subsystems)

		Comparator (C)	Program (P)		
Performance Targets	Subsystem A	$\|A^*\|_\infty=0.0106$	$\|A\|_\infty=0.0111$	P>C	$\alpha=1.05$
	Subsystem B	$\|B^*\|_\infty=50$	$\|B\|_\infty=45$	P<C	$\beta=0.9$
	Subsystem C	$\|C^*\|_\infty=0.0075$	$\|C\|_\infty=0.0075$	P=C	$\gamma=1$
Weak-coupling Conditions	Feedback loop 1	$\|A^*B^*\|_\infty=0.5929$	$\|AB\|_\infty=0.5001$	C, P$\ll$1	
	Feedback loop 2	$\|B^*C^*\|_\infty=0.3763$	$\|BC\|_\infty=0.3387$	C, P$\ll$1	
System Output	Y	0.00223	0.00215	P<C	

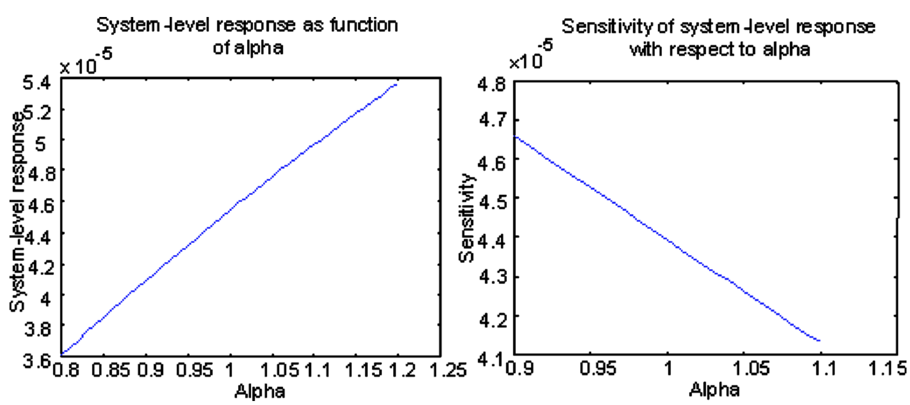

Fig. 4 System-level response and its sensitivity with respect to α

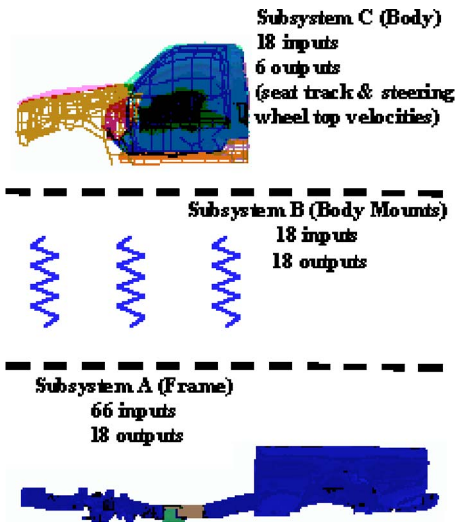

Fig. 5 Models of vehicle subsystems

ing and component-mode synthesis, is used to obtain the frame inputs. Inputs in three coordinate directions are transferred to the frame at each of 22 attachment points, resulting in 66 inputs. The outputs of the frame subsystem are the displacements in three coordinate directions at each of the six body mount locations (18 outputs). The frame outputs serve as inputs to the body mounts subsystem. This subsystem is composed of six springs, each of which has stiffness in three coordinate directions. The outputs of the body mounts subsystem are the 18 spring forces, which in turn are transferred to the body subsystem at the body mount attachment points. The spring forces excite vibrations of the vehicle body. The intensity of these vibrations is evaluated by measuring the three components of velocity at two locations: the steering column and seat track. Hence, the body subsystem has six outputs. The system performance measures, in this case, are the two norms of the velocities at the different body measurement locations. The three vehicle subsystems considered in this example are shown in Figs. 5–8.

The interconnections between the three subsystems are of the general feedback configuration shown in Fig. 3. To apply the proposed approach, the transfer matrices for the three vehicle subsystems need to be constructed. The transfer functions are obtained using the frequency responses from different input locations to different output, or attachment point, locations.

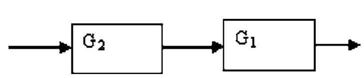

Fig. 6 Series connection

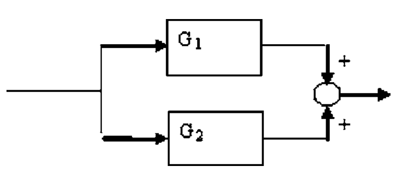

Fig. 7 Parallel connection

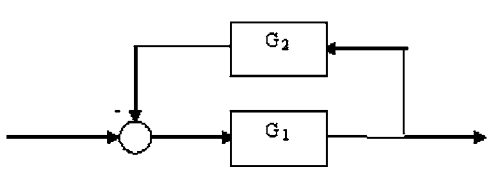

Fig. 8 Feedback connection

Assuming the program vehicle is similar to the benchmark vehicle, e.g., both are body-on-frame light pick-up trucks, the program vehicle subsystem targets are scalar functions of frequency that multiply the subsystem transfer matrices of the existing benchmark vehicle. If the different vehicle subsystems satisfy certain weak-coupling conditions, i.e., the infinity norm of the subsystems in each feedback loop in Fig. 3 is much less than unity, then the scalars multiplying the different subsystems can be selected to guarantee better performance in the program vehicle. The new program vehicle is guaranteed to outperform the comparator vehicle if these scalars are chosen such that the infinity norm of their product is less than unity.

State-space realizations for the three vehicle subsystems were obtained using modal data from a finite element model of the vehicle. The first 50 modes, up to frequencies of 43.7 and 64.2 Hz for the body and frame, respectively, were used to construct the state-space matrices. The three subsystems were assumed to be in the form:

$$x = Ax + Bu$$

$$y = Cx + Du$$

where

A = block diagonal with blocks in the form $\begin{bmatrix} 0 & 1 \\ -w_n^2 & -2\xi w_n \end{bmatrix}$
B = block column with blocks in the form $\begin{bmatrix} 0 \\ \nu(q) \end{bmatrix}$
C = block row with blocks in the form $[0 \ \nu(r)]$
w_n = natural frequency for the mode corresponding to nth block
ξ = modal damping ratio (ξ=0.03)
$\nu(o)$ = modal displacements at input locations
$\nu(r)$ = modal displacements at output locations

The results obtained using the state-space realizations of the vehicle subsystems are presented in Table 4. The results in Table 4 show that the program system (P) outperforms the comparator system (C) in spite of the weak-coupling conditions not being met. This is not unexpected, since the weak-coupling conditions are sufficient rather than necessary conditions to guarantee that the program system outperforms the comparator. To verify that, indeed, reducing the norms of the subsystems reduces the norm of the overall system, the sensitivities of the overall system with respect to the scalars relating the norms of the program and benchmark subsystems were calculated. The results of the sensitivity analysis are presented in Table 5. These results were evaluated numerically using finite differencing due to the size of the problem, which made analytical calculations based on Proposition 1 quite difficult.

6 Conclusions and Future Work

The proposed norm-based benchmarking method assumes, and makes use of, the existence of a benchmark, or "comparator," product with adequate performance that is similar to the product being developed. Thus, the approach is suitable for design tasks involving incremental improvement to existing products.

Using the proposed approach, it is possible to systematically balance system targets among the different subsystems for the case of weakly coupled subsystems. In the example at hand, the performance requirements on subsystem A of the program system were relaxed. The target chosen for that subsystem resulted in a performance level that is poorer than that of the corresponding subsystem in the comparator system. However, by tightening the requirements on subsystem B, the desired overall system performance is achieved, and the program system still outperforms the comparator system.

For strongly coupled subsystems, a new procedure for calculating the sensitivities of the two norm of an overall system with

Table 4 Target setting using proposed approach for vehicle example

		Comparator (C)	Program (P)		
Performance Targets	Subsystem A (Frame)	$\|A^*\|_2 = 141.567$	$\|A\|_2 = 134.489$	P < C	$\alpha = 0.95$
	Subsystem B (Body mounts)	$\|B^*\|_\infty = 3810$	$\|B\|_\infty = 3810$	P = C	$\beta = 1$
	Subsystem C (Body)	$\|C^*\|_2 = 10.895$	$\|C\|_2 = 10.350$	P < C	$\gamma = 0.95$
Weak-coupling Conditions	Feedback loop 1	$\|A^*B^*\|_\infty = 635.74$	$\|AB\|_\infty = 603.95$	C, P > 1	
	Feedback loop 2	$\|B^*C^*\|_\infty = 277.03$	$\|BC\|_\infty = 263.18$	C, P > 1	
System Output	Y	6736.1	6320.2	P < C	

respect to changes in the design targets of the constituent subsystems is proposed. The constituent subsystems can be connected in serial, parallel, feedback, or arbitrary configurations.

Calculating the sensitivities with respect to subsystem targets rather than design variables, as is traditionally done in the design literature, is a key component of the proposed approach. The ability to set and rank subsystem targets facilitates the systems engineering approach to product development. This enables product teams to outsource the tasks of designing components to their suppliers. Using the proposed procedure, engineering managers can assign more resources to improving subsystems that are expected to have a greater influence on the overall system performance, thus enabling a more time- and cost-efficient product development process.

In the present work, the program subsystems were assumed to be related to the comparator subsystems using constant scaling factors, independent of frequency. Relaxing this assumption and using matrix functions of frequency will allow us to shape the transfer functions of the program subsystems differently. This is a subject of future work. Also, methods to account for the capabilities of the different design teams when assigning subsystem targets are the subject of future work. Thus far, we have focused on NVH attributes in the results pertaining to sensitivity analysis. Extensions to handle other attributes, and possibly several attributes simultaneously, will provide a valuable extension of the present work, and is currently in progress. In addition, we are investigating the application of this approach to probabilistic design.

Appendix A

Proposition 2. For two subsystems connected in series, parallel, or feedback configurations, the system-level performance can be related to the subsystems' performances using the following equations. Assume that G_1 and G_2 are two subsystems having the state-space representations $G_1 = \left[\begin{smallmatrix} A_1 & \alpha B_1 \\ C_1 & 0 \end{smallmatrix}\right]$ and $G_1 = \left[\begin{smallmatrix} A_2 & \beta B_2 \\ C_2 & 0 \end{smallmatrix}\right]$.

Series Connection:

Table 5 Results of the sensitivity analysis for vehicle example

	Strongly coupled subsystems
$\dfrac{\partial\|G\|_2^2}{\partial\alpha}$	7.072×10^7
$\dfrac{\partial\|G\|_2^2}{\partial\beta}$	2.451×10^7
$\dfrac{\partial\|G\|_2^2}{\partial\gamma}$	4.458×10^7

$$G_1G_2 = \begin{bmatrix} A_1 & \alpha B_1 \\ C_1 & 0 \end{bmatrix}\begin{bmatrix} A_2 & \beta B_2 \\ C_2 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & \alpha B_1C_2 & 0 \\ 0 & A_2 & \beta B_2 \\ C_1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_2 & 0 & \beta B_2 \\ \alpha B_1C_2 & A_1 & 0 \\ 0 & C_1 & 0 \end{bmatrix}$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\alpha} = \frac{2}{\alpha}C_1W_1(\alpha,\beta)C_1^T = 2\|G_1G_2\|_2^2 \quad \text{(A1)}$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\beta} = \frac{2}{\beta}C_1W_1(\alpha,\beta)C_1^T = 2\|G_1G_2\|_2^2 \quad \text{(A2)}$$

Parallel connection:

$$G_1 + G_2 = \begin{bmatrix} A_1 & \alpha B_1 \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} A_2 & \beta B_2 \\ C_2 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & \alpha B_1 \\ 0 & A_2 & \beta B_2 \\ C_1 & C_2 & 0 \end{bmatrix}$$

$$\frac{\partial\|G_1 + G_2\|_2^2}{\partial\alpha} = \|G_1 + G_2\|_2^2 + \|G_1\|_2^2 - \|G_2\|_2^2 \quad \text{(A3)}$$

$$\frac{\partial\|G_1 + G_2\|_2^2}{\partial\beta} = \|G_1 + G_2\|_2^2 + \|G_2\|_2^2 - \|G_1\|_2^2 \quad \text{(A4)}$$

Feedback connection:

$$(I + G_1G_2)^{-1}G_1 = \begin{bmatrix} A_1 & \alpha B_1C_2 & 0 \\ 0 & A_2 & \beta B_2 \\ C_1 & 0 & I \end{bmatrix}^{-1}\begin{bmatrix} A_1 & \alpha B_1 \\ C_1 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & \alpha B_1C_2 & \alpha B_1 \\ -\beta B_2C_1 & A_2 & 0 \\ C_1 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial\|(I + G_1G_2)^{-1}G_1\|_2^2}{\partial\alpha} = 2tr(C_1W_1C_1^T) \quad \text{(A5)}$$

where

$$\begin{bmatrix} A_1 & B_1C_2 \\ -B_2C_1 & A_2 \end{bmatrix}\begin{bmatrix} W_1 & W_2 \\ W_2^T & W_4 \end{bmatrix} + \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_4 \end{bmatrix}\begin{bmatrix} A_1^T & -(B_2C_1)^T \\ (B_1C_2)^T & A_2^T \end{bmatrix} + \begin{bmatrix} B_1B_1^T & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\frac{\partial\|(I + G_1G_2)^{-1}G_1\|_2^2}{\partial\beta} = tr\left(C_1\frac{\partial W_1}{\partial\beta}C_1^T\right) \quad \text{(A6)}$$

where

$$A_1\frac{\partial W_1}{\partial\beta}+\frac{\partial W_1}{\partial\beta}A_1^T+B_1C_2W_2^T+W_2(B_1C_2)^T=0$$

Proof:

Series Connection:

$$G_1G_2=\begin{bmatrix}A_1 & \alpha B_1\\ C_1 & 0\end{bmatrix}\begin{bmatrix}A_2 & \beta B_2\\ C_2 & 0\end{bmatrix}=\begin{bmatrix}A_1 & \alpha B_1C_2 & 0\\ 0 & A_2 & \beta B_2\\ C_1 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}A_2 & 0 & \beta B_2\\ \alpha B_1C_2 & A_1 & 0\\ 0 & C_1 & 0\end{bmatrix}$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\alpha}=tr\left\{[C_1\quad 0]\frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix}C_1^T\\ 0\end{bmatrix}\right\}=tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\alpha}C_1^T\right)$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\beta}=tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\beta}C_1^T\right)$$

where

$$W(\alpha,\beta)=\begin{bmatrix}W_1(\alpha,\beta) & W_2(\alpha,\beta)\\ W_2^T(\alpha,\beta) & W_4(\alpha,\beta)\end{bmatrix}$$

with

$$\begin{bmatrix}A_1 & \alpha B_1C_2\\ 0 & A_2\end{bmatrix}W(\alpha,\beta)+W(\alpha,\beta)\begin{bmatrix}A_1^T & 0\\ \alpha C_2^TB_1^T & A_2^T\end{bmatrix}+\begin{bmatrix}0\\ \beta B_2\end{bmatrix}\times[0\quad \beta B_2^T]=0$$

$$\begin{bmatrix}0 & B_1C_2\\ 0 & 0\end{bmatrix}W(\alpha,\beta)+\begin{bmatrix}A_1 & \alpha B_1C_2\\ 0 & A_2\end{bmatrix}\frac{\partial W(\alpha,\beta)}{\partial\alpha}+W(\alpha,\beta)\times\begin{bmatrix}0 & 0\\ C_2^TB_1^T & 0\end{bmatrix}+\frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix}A_1^T & 0\\ \alpha C_2^TB_1^T & A_2^T\end{bmatrix}=0$$

Solving these equations

$$\frac{\partial W_1(\alpha,\beta)}{\partial\alpha}=\frac{2}{\alpha}W_1(\alpha,\beta)$$

$$\frac{\partial W_1(\alpha,\beta)}{\partial\beta}=\frac{2}{\beta}W_1(\alpha,\beta)$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\alpha}=\frac{2}{\alpha}C_1W_1(\alpha,\beta)C_1^T=2\|G_1G_2\|_2^2$$

$$\frac{\partial\|G_1G_2\|_2^2}{\partial\beta}=\frac{2}{\beta}C_1W_1(\alpha,\beta)C_1^T=2\|G_1G_2\|_2^2$$

Parallel connection:

$$G_1+G_2=\begin{bmatrix}A_1 & \alpha B_1\\ C_1 & 0\end{bmatrix}+\begin{bmatrix}A_2 & \beta B_2\\ C_2 & 0\end{bmatrix}=\begin{bmatrix}A_1 & 0 & \alpha B_1\\ 0 & A_2 & \beta B_2\\ C_1 & C_2 & 0\end{bmatrix}$$

$$\frac{\partial\|G_1+G_2\|_2^2}{\partial\alpha}=tr\left\{[C_1\quad C_2]\frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix}C_1^T\\ C_2^T\end{bmatrix}\right\}$$
$$=tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\alpha}C_1^T+C_1\frac{\partial W_2(\alpha,\beta)}{\partial\alpha}C_2^T+C_2\frac{\partial W_2^T(\alpha,\beta)}{\partial\alpha}C_1^T+C_2\frac{\partial W_4(\alpha,\beta)}{\partial\alpha}C_2^T\right)$$

$$\frac{\partial\|G_1+G_2\|_2^2}{\partial\beta}=tr\left\{[C_1\quad C_2]\frac{\partial W(\alpha,\beta)}{\partial\beta}\begin{bmatrix}C_1^T\\ C_2^T\end{bmatrix}\right\}$$
$$=tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\beta}C_1^T+C_1\frac{\partial W_2(\alpha,\beta)}{\partial\beta}C_2^T+C_2\frac{\partial W_2^T(\alpha,\beta)}{\partial\beta}C_1^T+C_4\frac{\partial W_4(\alpha,\beta)}{\partial\beta}C_2^T\right)$$

where

$$W(\alpha,\beta)=\begin{bmatrix}W_1(\alpha,\beta) & W_2(\alpha,\beta)\\ W_2^T(\alpha,\beta) & W_4(\alpha,\beta)\end{bmatrix}$$

with

$$\text{with}\begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}W(\alpha,\beta)+W(\alpha,\beta)\begin{bmatrix}A_1^T & 0\\ 0 & A_2^T\end{bmatrix}+\begin{bmatrix}\alpha B_1\\ \beta B_2\end{bmatrix}\times[\alpha B_1^T\quad \beta B_2^T]=0$$

$$\begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}\frac{\partial W(\alpha,\beta)}{\partial\alpha}+\frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix}A_1^T & 0\\ 0 & A_2^T\end{bmatrix}+\begin{bmatrix}2\alpha B_1B_1^T & \beta B_1B_2^T\\ \beta B_2B_1^T & 0\end{bmatrix}=0$$

$$\begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}\frac{\partial W(\alpha,\beta)}{\partial\beta}+\frac{\partial W(\alpha,\beta)}{\partial\beta}\begin{bmatrix}A_1^T & 0\\ 0 & A_2^T\end{bmatrix}+\begin{bmatrix}0 & \alpha B_1B_2^T\\ \alpha B_2B_1^T & 2\beta B_2B_2^T\end{bmatrix}=0$$

Solving these equations:

$$\frac{\partial W_1(\alpha,\beta)}{\partial\alpha}=\frac{2}{\alpha}W_1(\alpha,\beta)$$

$$\frac{\partial W_2(\alpha,\beta)}{\partial\alpha}=\frac{1}{\alpha}W_2(\alpha,\beta)$$

$$\frac{\partial W_4(\alpha,\beta)}{\partial\beta}=\frac{2}{\beta}W_4(\alpha,\beta)$$

$$\frac{\partial W_2(\alpha,\beta)}{\partial\beta}=\frac{1}{\beta}W_2(\alpha,\beta)$$

$$\frac{\partial\|G_1+G_2\|_2^2}{\partial\alpha}=\|G_1+G_2\|_2^2+\|G_1\|_2^2-\|G_2\|_2^2$$

$$\frac{\partial\|G_1+G_2\|_2^2}{\partial\beta}=\|G_1+G_2\|_2^2+\|G_2\|_2^2-\|G_1\|_2^2$$

Feedback connection:

$$(I+G_1G_2)^{-1}G_1=\begin{bmatrix}A_1 & \alpha B_1C_2 & 0\\ 0 & A_2 & \beta B_2\\ C_1 & 0 & I\end{bmatrix}^{-1}\begin{bmatrix}A_1 & \alpha B_1\\ C_1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}A_1 & \alpha B_1C_2 & \alpha B_1\\ -\beta B_2C_1 & A_2 & 0\\ C_1 & 0 & 0\end{bmatrix}$$

$$\frac{\partial\|(I+G_1G_2)^{-1}G_1\|_2^2}{\partial\alpha}=tr\left\{[C_1\quad 0]\frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix}C_1^T\\ 0\end{bmatrix}\right\}$$
$$=tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\alpha}C_1^T\right)$$

Table 6 Values for design variables and parameters for weakly and strongly coupled subsystems (SI units are used)

	Weakly coupled subsystems	Strongly coupled subsystems
ρ (kg/m^3)	7810	7810
ξ	0.2	0.2
A_A (m^2)	0.0001897	0.0001897
A_C (m^2)	0.0001897	0.0001897
E (N/m^2)	200×10^9	200×10^9
I_A (m^4)	878×10^{-12}	878×10^{-12}
I_C (m^4)	995×10^{-12}	995×10^{-12}
k_1 (N/m)	8.9214	892.1404
k_2 (N/m)	8.9214	892.1404
L (m)	2.7	2.7

$$\frac{\partial\|(I+G_1G_2)^{-1}G_1\|_2^2}{\partial\beta} = tr\left\{[C_1 \quad 0]\frac{\partial W(\alpha,\beta)}{\partial\beta}\begin{bmatrix} C_1^T \\ 0\end{bmatrix}\right\} = tr\left(C_1\frac{\partial W_1(\alpha,\beta)}{\partial\beta}C_1^T\right)$$

where

$$W(\alpha,\beta) = \begin{bmatrix} W_1(\alpha,\beta) & W_2(\alpha,\beta) \\ W_2^T(\alpha,\beta) & W_4(\alpha,\beta)\end{bmatrix}$$

with

$$\begin{bmatrix} A_1 & \alpha B_1C_2 \\ -\beta B_2C_1 & A_2\end{bmatrix}W(\alpha,\beta) + W(\alpha,\beta)\begin{bmatrix} A_1^T & -\beta(B_2C_1)^T \\ \alpha(B_1C_2)^T & A_2^T\end{bmatrix} + \begin{bmatrix}\alpha^2B_1B_1^T & 0 \\ 0 & 0\end{bmatrix} = 0$$

$$\begin{bmatrix} A_1 & \alpha B_1C_2 \\ -\beta B_2C_1 & A_2\end{bmatrix}\frac{\partial W(\alpha,\beta)}{\partial\alpha} + \frac{\partial W(\alpha,\beta)}{\partial\alpha}\begin{bmatrix} A_1^T & -\beta(B_2C_1)^T \\ \alpha(B_1C_2)^T & A_2^T\end{bmatrix} + \begin{bmatrix} 0 & B_1C_2 \\ 0 & 0\end{bmatrix}W(\alpha,\beta) + W(\alpha,\beta)\begin{bmatrix} 0 & 0 \\ (B_1C_2)^T & 0\end{bmatrix} + \begin{bmatrix} 2\alpha B_1B_1^T & 0 \\ 0 & 0\end{bmatrix} = 0$$

$$\begin{bmatrix} A_1 & \alpha B_1C_2 \\ -\beta B_2C_1 & A_2\end{bmatrix}\frac{\partial W(\alpha,\beta)}{\partial\beta} + \frac{\partial W(\alpha,\beta)}{\partial\beta}\begin{bmatrix} A_1^T & -\beta(B_2C_1)^T \\ \alpha(B_1C_2)^T & A_2^T\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -B_2C_1 & 0\end{bmatrix}W(\alpha,\beta) + W(\alpha,\beta)\begin{bmatrix} 0 & -(B_2C_1)^T \\ 0 & 0\end{bmatrix} = 0$$

Solving these equations:

$$\frac{\partial W_1(\alpha,\beta)}{\partial\alpha} = \frac{2}{\alpha}W_1(\alpha,\beta)$$

$$\frac{\partial W_2(\alpha,\beta)}{\partial\beta} = W_2$$

$$\frac{\partial\|(I+G_1G_2)^{-1}G_1\|_2^2}{\partial\alpha} = 2tr(C_1W_1C_1^T)$$

$$\frac{\partial\|(I+G_1G_2)^{-1}G_1\|_2^2}{\partial\beta} = tr\left(C_1\frac{\partial W_1}{\partial\beta}C_1^T\right)$$

with

$$A_1\frac{\partial W_1}{\partial\beta} + \frac{\partial W_1}{\partial\beta}A_1^T + B_1C_2W_2^T + W_2(B_1C_2)^T = 0$$

Appendix B

The design variables in this problem are assumed to be the mean values of the spring stiffnesses k_1 and k_2, and the moments of inertia I_A and I_C. The areas A_A and A_C are assumed to be known functions of the moments of inertia. The length L density ρ, damping ratio ξ, and modulus of elasticity E are assumed to be known design parameters. To ensure that the two beams remain connected, lower bounds on the values of the stiffnesses of the springs are used (see Table 6).

References

[1] Belhe, U., and Kusiak, A., 1997, "Dynamic Scheduling of Design Activities With Resource Constraints," IEEE Trans. Syst. Man Cybern. **27**(1), pp. 105–111.

[2] Kusiak, A., and Park, K., 1990, "Concurrent Engineering: Decomposition of Design Activities," *Proc.: Rensselaer's Second International Conference on Computer Integrated Manufacturing*, RPI, Troy, NY, pp. 557–563.

[3] Kim, H., 2001, Target Cascading in Optimal System Design, Ph.D. thesis, University of Michigan, Ann Arbor.

[4] Bay, J. S., 1999, *Fundamentals of Linear State Space Systems*, McGraw-Hill, Boston.

[5] Sobieszczanski-Sobieski, J., Agte, J. S., and Sandusky Jr., R. R., 2000, "Bi-level Integrated System Synthesis," AIAA J. **38**(1), pp. 164–172.

[6] Medland, A. J., and Mullineux, G., 2000, "A Decomposition Strategy for Conceptual Design," J. Eng. Design **11**(1), pp. 3–16.

[7] Altus, S., Kroo, I., and Gage, P., 1996, "A Genetic Algorithm for Scheduling and Decomposition of Multidisciplinary Design Problems," J. Mech. Des. **118**(4), pp. 486–489.

[8] Krishnamachari, R., and Papalambros, P., 1997, "Hierarchical Decomposition Synthesis in Optimal Systems Design," J. Mech. Des. **119**(4), pp. 448–457.

[9] Courtois, P., 1985, "On Time and Space Decomposition of Complex Structures," Communications of the Association for Computing Machinery, 28(6), pp. 590–603.

[10] Krishnamachari, R., and Papalambros, P., 1997, "Optimal Hierarchical Decomposition Synthesis Using Integer Programming," J. Mech. Des. **119**(4), pp. 440–447.

[11] Kusiak, A., and Larson, N., 1995, "Decomposition and Representation Methods in Mechanical Design," J. Mech. Des. **117B**(3), pp. 17–24.

[12] Kusiak, A., and Wang, J., 1993, "Decomposition of the Design Process," J. Mech. Des. **115**(4), pp. 687–695.

[13] Michelena, N., and Papalambros, P., 1997, "A Hypergraph Framework for Optimal Model-Based Decomposition of Design Problems," Computational Optimization and Applications 8(2), pp. 173–196.

[14] Michelena, N., Park, H. A., Papalambros, P., and Kulkarni, D., 1999, "Hierarchical Overlapping Coordination for Large-Scale Optimization by Decomposition," AIAA J. **37**(7), pp. 890–896.

[15] Sage, A., and Rouse, W., 1999, *Handbook of Systems Engineering and Management*, Wiley, New York.

[16] Michelena, N. F., and Papalambros, P. Y., 1995, "Optimal Model-Based Decomposition of Powertrain System Design," J. Mech. Des. **117**(4), pp. 499–505.

[17] Wagner, T., 1993, A General Decomposition Methodology for Optimal System Design, Ph.D. thesis, University of Michigan, Ann Arbor.

[18] Balling, R. J., and Sobieszczanski-Sobieski, J., 1996, "Optimization of Coupled Systems: A Critical Overview of Approaches," AIAA J. **34**(1), pp. 6–17.

[19] Senin, N., Wallace, D., and Borland, N., 2003, "Distributed Object-Based Modeling in Design Simulation Workplace," J. Mech. Des. **125**(1), pp. 2–13.

[20] Rao, J. R. J., Badhrinath, K., Pakala, R., and Mistree, F., 1997, "A Study of Conflict Using Models of Multiplayer Games," Eng. Optimiz. **28**(1–2), pp. 63–94.

[21] Lewis, K., and Mistree, F., 1997, "Modeling Interactions in Multidisciplinary Design: A Game Theoretic Approach," AIAA J. **35**(8), pp. 1387–1392.

[22] Liker, J. K., Sobek II, D. K., Ward, A. C., and Cristiano, J. J., 1996, "Involving Suppliers in Product Development in the United States and Japan: Evidence for Set-Based Concurrent Engineering," IEEE Trans. Eng. Manage. **43**(2), pp. 165–178.

[23] Sobek II, D. K., Liker, J. K., and Ward, A. C., 1998, "Another Look at How Toyota Integrates Product Development," IEEE Eng. Manage. Rev. **26**(4), pp. 69–77.

[24] Rothamer, D., Ehlinger, H., Jones, A., and Jakubisin, M., 2001, "Developing World-Class Customer Fit Objectives, Utilizing Competitive Benchmarking Techniques," SAE 2001-01-3063.

[25] Koch, P., Marvris, D., and Mistree, F., 2000, "Partitioned, Multilevel Response Surfaces for Modeling Complex Systems," AIAA J. **38**(5), pp. 875–881.

[26] Boyd, S. P., and Barratt, C. H., 1991, *Linear Controller Design*, Prentice Hall, Englewood Cliffs.