Stochastic Optimal Capacity Management in Reconfigurable Manufacturing Systems

Farshid Maghami Asl, A. Galip Ulsoy (1)

NSF Engineering Research Center for Reconfigurable Manufacturing Systems, University of Michigan, Ann Arbor MI 48109-2125, USA

Abstract
This paper presents an optimal policy, based on Markov decision theory for the capacity management problem in a firm facing stochastic market demand. The firm implements a reconfigurable manufacturing system and faces a delay between the times capacity changes are ordered and the times they are delivered. Optimal policies are presented as optimal boundaries representing the optimal capacity expansion and reduction levels. To increase the robustness of the optimal policy to unexpected events, the concept of feedback control is applied to address the capacity management problem. It is shown that feedback provides sub-optimal solutions to the capacity management problem which are more robust under unexpected disturbances in market demand and unexpected events.

Keywords:
Reconfigurable Manufacturing Systems, Capacity Management, Optimization, Feedback Control

1 INTRODUCTION

Over-capacity has been a major problem in the world economy during the past decade. Reconfigurable capacity, and optimal capacity management policies, could help global economic stability. Due to customer driven economies, today's world markets are characterized by high fluctuations in market demand and the frequent arrival of new technologies and new products. To stay competitive in such markets, manufacturing companies require new types of manufacturing systems that are very responsive to global market movements; a new cost effective manufacturing system whose production capacity and/or functionality is adjustable in response to fluctuations in product demand, and which is designed to be upgradeable with new process technology needed to accommodate tighter product specifications. Reconfigurable Manufacturing Systems (RMS) [1], whose components are reconfigurable machines and reconfigurable controllers, as well as methodologies for their systematic design and diagnosis, are the cornerstones of this new manufacturing paradigm termed ``Reconfigurable Manufacturing.''

This paper introduces new approaches to optimal capacity management for a firm equipped with reconfigurable capacity and faced with uncertainties in market demand. The importance of capacity management in the US economy is depicted in Figure 1. As shown in Fig. 1(a), there was a reduction in the volatility of the US Gross Domestic Product (GDP) after 1984. According to the US Federal Reserve Bank reports, sophisticated inventory management has made important contributions to the economy's increased stability in the last decade [2]. On the other hand, over-capacity remains a significant problem in major industries. Firms tend to acquire more capacity than they need to hedge their risk of loosing market demand. Figure 1(b) shows the capacity utilization in the US economy, and its relation to economic recessions. This reinforces the importance of developing mathematical tools to improve capacity management policies and its key enabler, reconfigurable manufacturing, in today's economy.

The emphasis in this research is on manufacturing capacity, where staffed machinery is the key determinant. The goal is to study a firm implementing an RMS whose capacity is designed to be changed, and the capacity management problem would be *to make optimal decisions over time on the size of the staffed machinery to be added to (or subtracted from) the current capacity available in the firm*.

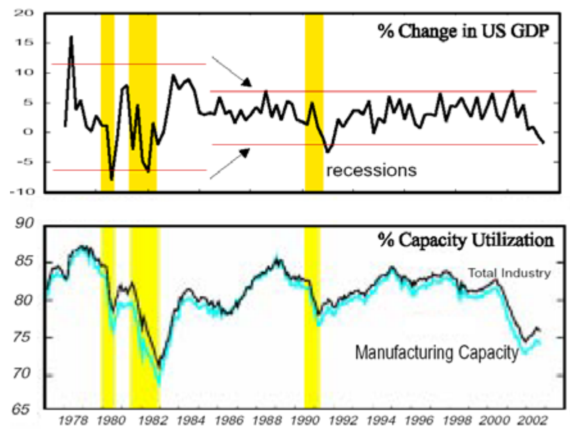

Figure 1: Percent change over time in (a) U.S. Gross Domestic Product (GDP), and (b) Capacity Utilization.

2 CAPACITY MANAGEMENT PROBLEM

2.1 Problem Formulation

Consider a capacity management problem for a firm that produces only one type of good or service over a finite *N*-unit time horizon under stochastic market demand. It is assumed that no inventory of finished good or service is allowed in this firm. Capacity management is performed by observing the current capacity and the probability distribution of the market demand at each time period, and making optimal decisions to change the capacity based on these observations for the next period. The firm can belong to an oligopoly market, but the effects of the competitors decisions on the firm's optimal policy is neglected, and it is assumed that there exists only one decision maker with perfect recall who makes the optimal decisions to manage the capacity of the firm based on the centralized information.

The market demand is stochastic with independent distributions. It can be represented by a stochastic sequence of positive independent random variables D_k with *a priori* continuous cumulative probability distribution functions $\psi_k(D_k)$. The general structure of the market demand as described above is shown in Figure 2 where $\varphi_k(D_k)$ are the probability density functions of the stochastic demand process.

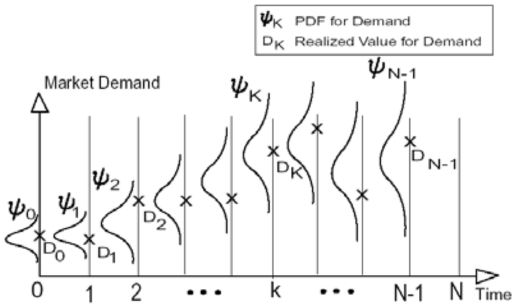

Figure 2: Distributions of the market demand.

The capacity management dynamics evolves in discrete time. It is assumed that there is a delay time from when the capacity is ordered until it can be utilized, shown by T [3]. The dynamic capacity evolution is represented by

$$\begin{aligned} y_k &= \min(C_k, D_k) \\ C_{k+1} &= C_k + X_{k-T} \end{aligned} \tag{1}$$

where C_k represents the capacity level of the firm at time k, X_k is the control input which defines the addition or removal of capacity, and y_k represents the sales of the firm. The delay time T is limited to be a multiple of the time increment, k.

2.2 Cost of Capacity Management

The production of the produced good or service costs γ_P per unit to produce, and is sold at a fixed price P per unit with $(P- \gamma_P)$ profit. Unsatisfied market demand has a penalty cost γ_S per unit. The available capacity of the firm is C_k at time k, and it takes a proportional holding or overhead cost, γ_H per unit of capacity at each time period, to maintain this level of capacity. The holding cost consists of the costs of maintenance and staffing of the capacity. Effect of all these costs at time period k, represents the one-period expected operating cost function, $G_k(C_k)$ incurring at period $[k,k+1)$ which is evaluated at time k as

$$\begin{aligned} G_k(C_k) = E\{&(\gamma_P - P)\min(C_k, D_k) \\ &+ \gamma_S \max(0, D_k - C_k) + \gamma_H C_k\} \end{aligned} \tag{2}$$

There is a cost involved with adding capacity to the firm. Addition of X units of capacity to the firm costs aX, where a is the proportional ordering cost. Decreasing capacity has a cost rX $(X<0)$ which is a return from selling extra capacity, and r is the reward of selling one unit of capacity. Effect of addition and reduction of capacity represents the management or control cost at time k, $M_k(X_k)$, which is the cost of expanding/subtracting capacity incurred at time k. At the end of the time horizon (i.e., $k=N$), the remaining capacity can be sold for a salvage value γ_N per unit of terminal capacity. The opportunity cost of money for the firm is represented by ρ and the discount factor is represented by $\beta=1/(1+\rho)$.

2.3 Optimization Problem

Given an initial capacity, the problem is to find an optimal decision sequence, or a policy that minimizes the expected discounted cost

$$\min_{X_0 \ldots X_{N-1}} \{-\beta^N \gamma_N C_N + \sum_{k=0}^{N-1} \beta^k [G_k(C_{k+1}) + M_k(X_k)]\} \tag{3}$$

3 STOCHASTIC OPTIMAL CONTROL

3.1 Optimality Equations

At each time k, the decision maker observes the current capacity C_k, and the demand distribution $\psi_k(D_k)$ and makes the decision X_k to generate the new optimal capacity level. The demand realization D_k is generated according to the given probability measure, and the operating cost G_k and control cost M_k are incurred and added to the previous costs. The terminal cost is the additional cost, which incurs at time N and it will be added to the previous costs. Assume that the firm operates at time $k+1$, and it has a minimal or optimal cost-to-go $V_{k+1}(C_{k+1})$ which represents the cost of the optimal policy to go from time $k+1$ to the terminal time N. Assuming the optimality of the cost-to-go function $V_{k+1}(C_{k+1})$, one can write the optimal cost-to-go function for the firm at time k,

$$\begin{aligned} V_k(C_k) &= \min_{X_k}\{M_k(X_k) + G_k(C_{k+1}) + \beta E V_{k+1}(C_{k+1})\} \\ V_N(C_N) &= -\gamma_N C_N \end{aligned} \tag{4}$$

where $V_N(C_N)$ represents the final salvage value of the firm's capacity at time N. Equations (4) are the optimality equations for the capacity management problem represented by stochastic dynamic programming. Based on the optimality theorem [4], a Markov policy exists and is optimal if and only if the minimum at (4) is achieved. To obtain the minimum value, it is shown that the optimal cost-to-go $V_k(C_k)$ is convex in C_k and then the functions X_k which make it minimal for $k=0,1, 2, \ldots,N-1$ are obtained [5].

The structure of the derived optimal policy for this problem is shown in Figure 3. The optimal policy is written based on two optimal thresholds. If the current capacity is lower than the lower optimal threshold L, the new capacity level should be chosen to be equal to L. If the current capacity is higher than the upper optimal threshold U, then the new capacity level should be chosen to be equal to U. And finally, for the capacity levels between L and U, the optimal decision is to maintain the current capacity (i.e., no change).

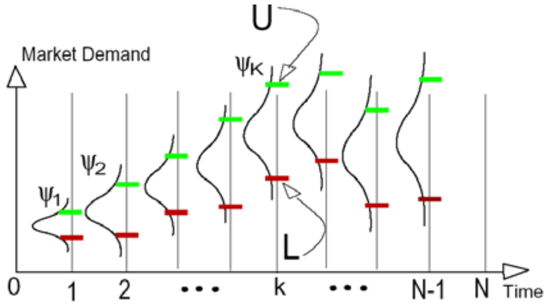

Figure 3: Optimal Capacity Management Policy.

Optimal threshold levels L and U are obtained numerically by solving Equations (4). It is also shown that there exists optimal lower and upper limits for L and U shown by L^*, and U^* and the sufficient condition for a policy to be optimal is to be located within these two limits. Thus, in numerically finding the optimal policy, one only needs to search for an optimal policy in the region between L^*, and U^* *[5].*

3.2 Numerical Results

Consider a one-year capacity management scenario for a firm facing a stochastic market demand with time increments equal to one month. The stochastic market demand distribution statistically increases exponentially for three months up to the maturity of the product or service and then declines slowly until the end of time horizon. To make the problem more realistic, it is also assumed that the uncertainty of the market demand grows with time. The simulation results for a linear cost function with a zero delay time are shown in Figure 5.

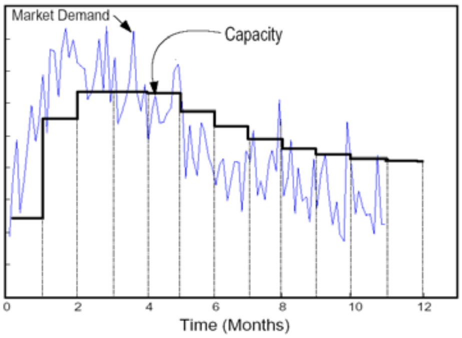

Figure 5: Simulation of the optimal policy for a firm facing stochastic market demand with known distributions and zero delay time in capacity delivery. (Parameters used for this simulation are β =0.9, a=500, r=200, P=1800, γ_P=800, γ_S =200, γ_H =350.)

Neglecting the effect of delay on capacity management not only generates non-optimal solutions, but can also cause major losses to the firm depending of the size of delay times. Figure 6 shows a five-year scenario of a firm applying capacity management with a seasonal stochastic market demand. There is an equal ten month delay time for the ordered capacity to be delivered or un-installed (*T=10* months). Neglecting the effect of this delay time can cause cases which capacity evolves in the reverse direction of the market demand as shown in the figure. It is also shown in the figure that considering the effect of delay and applying the modified optimal boundaries result optimum solutions and prevents losses to the firm due to cyclic markets.

4 CAPACITY MANAGEMENT VIA FEEDBACK

In this section, a new approach, based on the application of feedback control theory to the capacity management problem for a firm implementing reconfigurable manufacturing systems, is presented. It is shown that feedback provides sub-optimal solutions for the capacity management problem, which are more robust under unexpected system uncertainties and disturbances in the forecasts of market demand characteristics relative to the existing capacity management methods. The general capacity management problem is formulated, and sub-optimal robust solutions are obtained for this problem based on the feedback approach for both deterministic and stochastic market demand.

4.1 Reconfigurable Capacity Control

As stated before, the capacity management problem is a stochastic optimal control problem, and the most commonly applied approaches to this problem are optimization methodologies. Motivated by applying feedback in finding sub-optimal robust solutions, we consider the idea of applying a feedback control system to approach the capacity management problem. This approach will introduce sub-optimal solutions, but it will guarantee the robustness of the sub-optimal policy to changes in the model parameters and unexpected disturbances in the characteristics of the market demand. The proposed structure of the feedback reconfigurable system is shown in Figure 7 [3]. Consider the structure shown in Fig.7 and formulate the feedback policy based on an integral controller,

$$\int_0^{t_i} [D(t) - C(t)]dt = i\alpha \qquad (5)$$

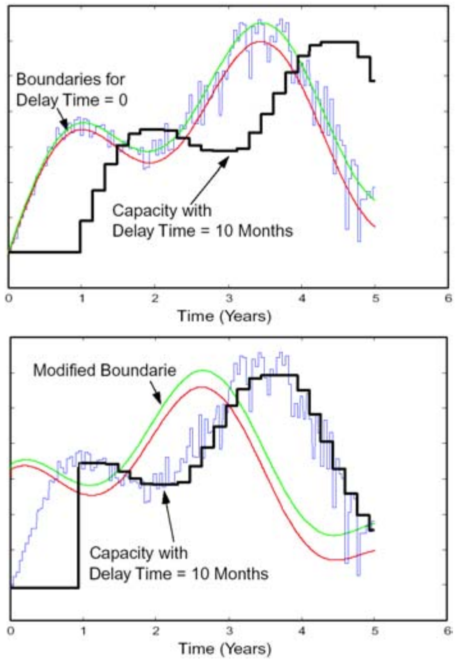

Figure: 6 Consequence of (a) neglecting the time delay in the capacity management problem, and (b) including delay.

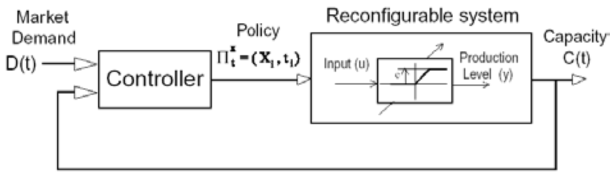

Figure 7: Feedback control system for a reconfigurable capacity management problem.

where α is the control design parameter, and can be interpreted as a threshold representing the amount of money one looses until he decide to make changes in the capacity. It needs to be chosen in the controller design stage. We consider the capacity management problem with linear deterministic demand studied in [5]. Consider a linear deterministic demand presented by $D(t)=gt$, where t represents time and g represents the rate of the linear demand. Assume that one can continue to place additional capacity in the indefinite future at the same cost. Manne in [6] studied this problem, and he considered adding uniform capacity increments of equal size, X, every time a change in the capacity level is needed. He has obtained the optimal solution for the capacity increment size, X as the one which minimizes the present value of all the future capacity costs. Consider a scenario of a manufacturing plant and assume that a linear demand with rate $g=0.1$ is expected for the next 10 years. Applying the approach presented in [6], an optimum capacity expansion policy is planned for the next 10 years of production with optimum increment size of $X=X_1$. Unexpectedly, after five years of operation, consumer's interest in the production increases and therefore the slope of market demand curve increases to $g=0.3$. The optimum value for capacity level increments will change to $X=X_2$, but the policy will continue its operation based on the predetermined plan, which is not optimal any more. Figure 8 show the diversion of Manne's policy from the optimal policy when unexpected changes happen in the market demand.

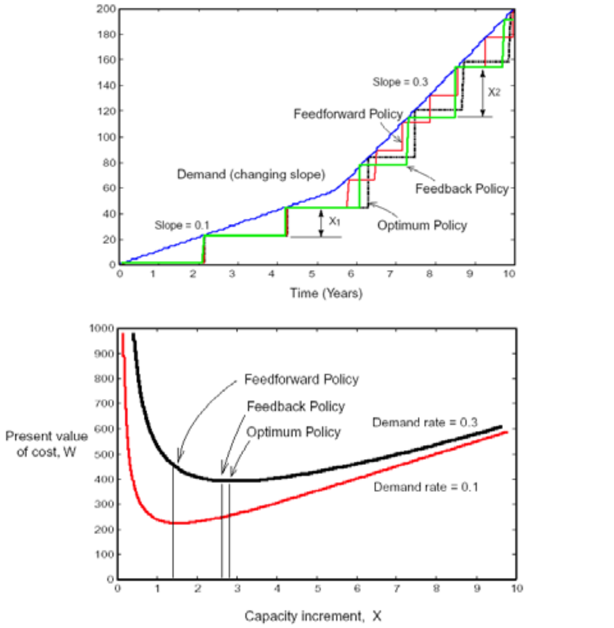

Figure 8: Diversion of Manne's policy from optimal when unexpected changes happen in the demand: (a) capacity vs. time, and (b) cost vs. capacity increment *X*.

Applying a feedback policy, and choosing the proper value for *α* in Eq.(5), the response of the feedback capacity management is a sub-optimal policy and if unexpected changes happen, it will depict a robust sub-optimal behaviour which is closer to the new optimal policy than Manne's approach as shown in Figure 8.

Manne has also suggested that if demand follows a stochastic diffusion process with respective probabilities over time, the sequential capacity problem can be re-formulated as a solvable deterministic problem. To study the case of the stochastic demand, assume that demand *D(t)* is a combination of two Poisson processes with two different arrival rates which are chosen such that the demand grows as time goes to infinity. Freidenfelds [7], motivated by Manne's work, has shown that the optimal policy for this type of stochastic demand is equivalent to having a demand growth rate of *g'* in the deterministic demand where *g'* can be obtained by the arrival rates in the market demand. We implement the feedback policy with a modified gain for capacity management for this problem. We also consider a scenario of a manufacturing system producing for six years. The market demand follows a Poisson process with certain expected value and variance. In the third year, both expected value and variance of the demand increase unexpectedly, and we study the policies from both approaches. To compare the two approaches, we use Monte Carlo simulation. The distributions of the total cost for two approaches are obtained based on the simulation and they are shown in Figure 10. As shown in the figure, the distribution of costs obtained by feedback policy shows smaller average and variance than the costs obtained by Freidenfelds' approach, which indicates more robustness for the feedback approach.

5 SUMMARY

New approaches to the capacity management problem based on Markov decision process and feedback control were presented in this paper. The simulation results for two deterministic and stochastic cases for the market demand show the advantages of the presented approaches. The feedback policy creates sub-optimal solutions, which are more robust to unexpected events. A comparison of the feedback and existing policies for three different examples are summarized in Table 1. Parameter calibration of the optimal policy is an alternative approach, which can reduce the sensitivity of the optimal policy to changes in the parameters and the market demand characteristics, but it should be noticed that parameter calibration is not as effective as feedback approach. The reason is the fact that feedback approach provides solutions regardless of the parameters and expectations of the future projections. Therefore it is less sensitive to the changes in these parameters. Further, it is much more appropriate to combine the feedback policy with the existing optimization techniques to create optimal robust solutions.

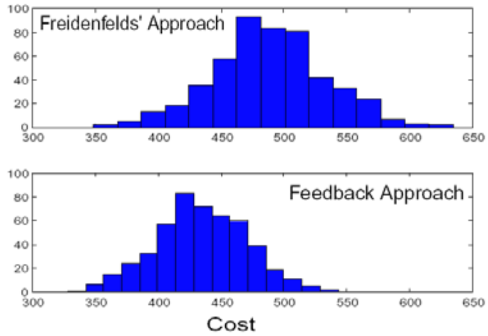

Figure 9: Cost Distributions for the two policies

Problem	***Planning***	***Feedback***
Manne's Problem (Deterministic)	Add capacity increment X whenever the demand exceeds capacity level for X units. Where, $X=\sqrt{\frac{2Ag}{Br}}$	Measure e(t)=D(t)-C(t) and its integral. Add capacity increment e(t) at time t whenever the following equation holds: $\frac{iA}{Br}=\int_0^t \{D(t)-C(t)\}dt$
Freidenfelds' Problem (Stochastic)	Add capacity increment X whenever the demand exceeds capacity level for X units. Where, $X=\sqrt{\frac{2A}{-B\ln(\theta)}}$	Measure e(t)=D(t)-C(t) and its integral. Add capacity increment e(t) at time t whenever the following equation holds: $\alpha=\int_0^t \{D(t)-C(t)\}dt$ Where α is a design variable.

Table 1: Comparison of the proposed approaches

6 ACKNOWLEDGMENTS

The authors are pleased to acknowledge the support by the National Science Foundation Engineering Research Center for Reconfigurable Manufacturing Systems under Grant No. NSF-Sub-EEC-9529125.

7 REFERENCES

[1] Koren Y., U. Heisel, F. Jovane, T. Moriwaki, G. Pritschow, A.G. Ulsoy, and H. Van Brussel, 1999, Reconfigurable Manufacturing Systems, Annals of the CIRP, 48/2:527-540.

[2] Federal Reserve Statistical Release, October 17, 2002, Industrial Production and Capacity Utilization, G 17(419)

[3] Asl Farshid. M., and A. Galip Ulsoy, 2002, Capacity Management via Feedback Control in Reconfigurable Manufacturing Systems, Proceedings of Japan-USA Symposium on Flexible Automation, Hiroshima, Japan.

[4] Kumar P. R., P. Varaiya, 1986, Stochastic Systems Estimation Identification and Adaptive Control, Prentice Hall, Englewood Cliffs, NJ.

[5] Asl Farshid M., 2002, Optimal Capacity Management in Reconfigurable Manufacturing systems, PhD Dissertation, University of Michigan, Ann Arbor, Michigan.

[6] Manne A. S., 1961, Capacity Expansion and Probabilistic Growth, Econometrica, 29/4:632-649.

[7] Freidenfeld J., 1981, Capacity Expansion: Analysis of Simple Models with Applications, Elsevier North Holland, New York.