Math 412, Exam 2 - hints/solutions

1. a) False: $[x]+1$ is a zero divisor: $([x]+1)([x]+2)=0$ in $\mathbb{Q}[x]/((x+1)(x+2))$.

b) False, there is no identity element. Fore example we do not have an element e such that $2\cdot e=2$ since $2\cdot 0=0, 2\cdot 2=4, 2\cdot 4=0, 2\cdot 6=4$.

c) True. This set is closed under subtraction (Yes! You do need to check the closure under subtraction) and absorbs products.

d) False. See the solution sheet for the practice problems about ring homomorphisms and isomorphisms.

e) False. Every polynomial of degree greater or equal then 3 is reducible in $\mathbb{R}[x]$. See section 4.6.

f) True. For example $f:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}\times\mathbb{Z}\times\mathbb{Z}, f(x,y)=(x,y,0)$.

2. List of cosets:

$$(0,0)+I, (0,1)+I, (0,2)+I, (1,0)+I, (1,1)+I, (1,2)+I.$$

I leave it to you to write the addition/multiplication table, but let me just remind you that

$$((a,b)+I)+((c,d)+I)=(a+c,b+d)+I,$$
$$((a,b)+I)\cdot((c,d)+I)=(a\cdot c,b\cdot d)+I.$$

R/I is not a field, since it has zero divisors:

$$((0,1)+I)((1,0)+I)=(0,0)+I.$$

3. See solution to homework 11. The group $SL(2,\mathbb{R})$ is not cyclic, since it is not abelian (we know from the sample test that the center consists of $\pm I$).

4. We did similar problem in class when we were discussing ideals. See also an example on page 137.