Solutions/ Hints to homework 14

April 16, 2008

section 7.5

2. a)We have four cosets: K , $Kr_1 = \{r_1, t\} = Kt$, $Kr_2 = \{r_2, h\} = Kh$ and $Kr_3 = \{r_3, d\} = Kd$.

b) We have two right cosets: K and (for example) Kt.

3. a) 4 b) 3 c) 1 d)4 e)6

Let's do b): $H =< 3 >= \{3, 6, 9, 0\}$, i.e. the order of H is 4. Therefore the index (by the Lagrange Theorem) is $\frac{12}{4} = 3$.

Hint for d): show first that H is generated by 4.

12. Assume that G is not cyclic. Let $x \in G$ be a nonidentity element. By the Lagrange Thm. the order of x must divide the order of G. We also know that the order of x it can not be equal to 1 or 25. The only choice left is that the order of x is 5.

section 7.6

3. a) We did this one in class.

b) We have two left cosets: H and $r_1H = r_3H = dH = tH = \{r_1, r_3, d, t\}$ and two right cosets: H and $Hr_1 = Hr_3 = Hd = H_t = \{r_1, r_3, d, t\}$.

6. We did this one in class as well.

section 7.7

1. We did this one in class.

4. The element $(3, 2)$ has order 4 (check it!). Therefore the quotient group will have order $\frac{16}{4} = 4$. To show that the quotient group G/N is isomorphic to $\mathbb{Z}_4$ is enough to find an element of order 4 in G/N. One could take for example $(1, 1)N$.

15. a) Consider a coset $\frac{p}{q}\mathbb{Z}$. Note that $q \cdot \frac{p}{q}\mathbb{Z} = p\mathbb{Z} = \mathbb{Z}$, i.e. the order of $\frac{p}{q}\mathbb{Z}$ in the quotient group $\mathbb{Q}/\mathbb{Z}$ is less or equal than q.

b) Let q be an arbitrary positive integer. Show that $\frac{1}{q}\mathbb{Z}$ has order q in the quotient group $\mathbb{Q}/\mathbb{Z}$.