Math 412, Exam 2

1. (30 points) True or false? Explain!

a) The inverse of $[x]+1$ in the quotient ring $\mathbb{Q}[x]/((x+1)(x+2))$ is equal to $[x]+2$.

b) The set $\{0, 2, 4, 6\}$ in $\mathbb{Z}_8$, $a \star b = ab$ is a group.

c) The set $\{0, 2, 4, 6\}$ is an ideal in $\mathbb{Z}_8$.

d) The rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are isomorphic.

e) The polynomial $x^4 + 1$ is irreducible in $\mathbb{R}[x]$.

f) There exists a non trivial (i.e. non-zero) ring homomorphism $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

2. (30 points) The set $I = 2\mathbb{Z} \times 3\mathbb{Z}$ is an ideal in $R = \mathbb{Z} \times \mathbb{Z}$. (You do not need to prove it). Write a (non repetitive) list of all cosets. Write the addition and multiplication table for the quotient ring R/I. Is R/I a field?

3. (30 points) Show that $SL(2,\mathbb{R})$ (2 by 2 matrices with real entries and determinant equal to 1) is a group. Is it a cyclic group?

4. (10 points) (a) Show that the ideal I generated by 3 and x in $\mathbb{Z}[x]$ consists of the polynomials in $\mathbb{Z}[x]$ whose constant terms are divisible by 3.

(b) Show I is not a principal ideal. (Hint: assume that I is generated by a polynomial $p(x) \in \mathbb{Z}[x]$. Then $3 \in (p(x))$ and $x \in (p(x))$. Reach a contradiction).