PETRI NET BASED DESIGN AND ANALYSIS OF LOGIC CONTROLLERS

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Outline
• Motivation / Viewpoint
• Signal Interpreted Petri Net (SIPN)
• Correctness Analysis
• Transparency Analysis
• Implementation
MOTIVATION

Background: Challenges in industrial Controller Design

- High Complexity
- Reuse and Modification of Software
- More or less unique specification for each controller

Objective: Application of formal Methods

- Transparency and Intuition
- Use of Control Theory and Software Engineering Concepts

Limit: Formal Area

- “In this game we’re playing, we can’t win. Some kinds of failure are better than other kinds, that's all” George Orwell, 1984
VARYING VIEWS AND COMBINED VIEW

Control Theory
- SIPN as formalization of the informal problem description
- Question: Is the formal specification complete and consistent
- Implementational aspects are of interest (IEC 1131-3)

Software Engineering
- SIPN Control Algorithm as Application Software (ANSI/IEEE 610)
- Question: Software Quality (ISO/IEC 9126)
- Implementational aspects are of minor interest

Control Theory + Software Engineering
- SIPN as formal problem description AND as application software
- Question: Completeness, Consistency, AND Quality (Transparency)
- Implementational aspects are of interest (IEC 1131-3)

Pre-Condition: Implemented Software follows Formal Specification exactly
LIMIT

Formal Analysis can NEVER answer the question of SENSE

- Analogy: Word-processor

The same?

Task performed by Assistant
Controlled Process

Assistant reads Text
Code-Generation

Correctly Spelled Text
+ Readability Measure
Formally Correct SIPN + Transparency Measure

Style-Check
Transparency Analysis

Correctly Spelled Text
Formally Correct SIPN

Sentences are correct

Text with Typos
Incorrect SIPN

Idea of Prof.
Informal Specification

Word-processor
SIPN-Editor
SIPN Based Control Algorithms

Signal Interpreted Petri Net (SIPN) [Frey and Litz]

- Functions of Input Signal at Transitions
- Output Signals at Places
- Forced firing
- Synchronous firing
- Iterative firing

\[ T_1: \text{Start Button pressed} \]
\[ \varphi(T_1) = i_4 \land \neg i_1 \land \neg i_2 \]

\[ T_2: \text{Filled & Temp. low} \]
\[ \varphi(T_2) = i_2 \land \neg i_3 \]

\[ T_3: \text{Filled & Temp. OK} \]
\[ \varphi(T_3) = i_3 \]

\[ T_4: \text{Tank is empty} \]
\[ \varphi(T_4) = \neg i_1 \land \neg i_2 \land \neg i_4 \]

\[ T_5: \text{Filled & Temp. OK} \]
\[ \varphi(T_5) = i_2 \land i_3 \]

\[ P_1: \text{Stand By} \]
\[ \omega(p_1) = (0, 0, 0, 0) \]

\[ P_2: \text{Filling} \]
\[ \omega(p_2) = (1, 0, -, 0) \]

\[ P_3: \text{Heating} \]
\[ \omega(p_3) = (0, 0, -, 1) \]

\[ P_4: \text{Emptying} \]
\[ \omega(p_4) = (0, 1, -, 0) \]

\[ P_5: \text{Stirring} \]
\[ \omega(p_5): o_3 = 1 \]
**DYNAMIC SYNCHRONIZATION (DS)**

**Definition**

Two transitions $t_1$ and $t_2$ form a dynamic synchronization if the firing of $t_1$ implies the simultaneous firing of $t_2$.

**Classification**

- Full (always synchronized)
- Partial (synchronized under special constraints)

**Validity of PN Analysis for SIPN under full DS**

- Safety of the underlying PN is sufficient for the safety of the SIPN but not necessary.
- Liveness of the underlying PN is necessary but not sufficient for the liveness of the SIPN.
- Reversibility is neither necessary nor sufficient.
EXAMPLE 1: SAFETY & REVERSIBILITY

a) SIPN

\[ \phi(t_1) = i_1 \land i_2 \]

\[ \omega(p_1) = (0, 1) \]

\[ \varphi(t_2) = -i_1 \]

\[ \omega(p_2) = (-, 0) \]

\[ \omega(p_3) = (1, -) \]

\[ \varphi(t_3) = i_1 \]

\[ \omega(p_4) = (1, 0) \]

\[ \varphi(t_4) = -i_2 \]

b) RG\textsubscript{PN}

\[ m_0 = (1, 0, 0, 0) \]

\[ t_1 \]

\[ m_1 = (0, 1, 1, 0) \]

\[ t_2 \]

\[ m_2 = (0, 0, 2, 0) \]

\[ t_3 \]

\[ m_3 = (0, 0, 1, 0) \]

\[ t_4 : i_2 = 0 \]

\[ \omega(p_4) = (1, 0) \]

\[ \omega(p_4) = (1, 0) \]

\[ \varphi(t_1) = i_1 \land i_2 = 1 \]

\[ \varphi(t_4) = -i_2 \]

• PN is not safe SIPN is (not sufficient)
• PN is not reversible SIPN is (not necessary)
• Both are not live
EXAMPLE 2: LIVENESS & REVERSIBILITY

- PN is live SIPN not (not sufficient)
- PN is reversible SIPN not (not sufficient)
CRITERIA FOR CORRECTNESS

Unambiguity
Every control algorithm has to be defined unambiguously. This criterion can be subdivided into four sub-criteria:

- Determinism
- Termination
- Defined output
- Unambiguous output

Liveness
When a transition or a set of transitions is not live then part of the control algorithm doesn’t work anymore.

Reversibility
Reversibility guarantees that the described controller reaches its initial state again.
CRITERION 1a: DETERMINISM

Determinism: A control algorithm has to be deterministic. If it was not, its behavior in a given situation would depend on implementational aspects.

→ The algorithm is deterministic if the firing conditions at every branching in $RGSIPN$ are disjoint.
**CRITERION 1b: TERMINATION**

Termination: In a cycle of a logic control algorithm, at least one marking must be stable. A cycle without stable marking leads to an algorithm that does not terminate.

→ *The algorithm terminates if there is no self-loop at any state in $RG_{SIPN}$.***

![Diagram of SIPN and RG_SIPN](image)
CRITERION 1c and 1d: OUTPUT

Defined and unambiguous output: There has to be a specification for the value of every output signal at every reachable marking. If two places marked at the same time assign different values to an output signal, a contradictory output setting results.

→ Undefined and contradictory outputs can be directly read from the output functions in $\text{RG}_{\text{SIPN}}$.

\[
\begin{align*}
\omega(p_1) &= (0,0,1) \\
\varphi(t_1) &= i_1 \\
\omega(p_2) &= (0,0,-) \\
\varphi(t_2) &= -i_1 \\
\omega(p_3) &= (1,0,-)
\end{align*}
\]

\[
\begin{align*}
\varphi(t_1) &= i_1 \\
\varphi(t_2) &= -i_1
\end{align*}
\]
WHAT IS TRANSPARENCY?

Definition of Transparency [Frey and Litz SMC99]:

• At any time it must be easy and clear to see what the controller does in the moment and what it will do in the next step.

• At any time there must be the possibility to reinterpret the algorithm. This means the aim of the control algorithm must be recognizable.

Kind of criteria we seek (Lord Kelvin):

“When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind.”
TRANSPARENCY CRITERIA (4 OF 8)

Comments

• There should be a comment at every place and at every transition.

No trivial Input

• Defined input signals should influence the controller.

Directionality

• The control flow should follow one preferred direction.

No redundant Output

• There is redundant information if several activated places set an output signal to the same value. This hinders understanding.
**TRANSPARENCY: EXAMPLE**

Transparency = 1.00 (optimal)  
Transparency = 0.26 (worst case is 0)

Net1

- **p1: Stand By**  
  $\omega(p_1) = (0, 0, 0, 0)$

Net2

- **p1**: Start Button pressed  
  $\varphi(T_1) = i_4 \land \neg i_1 \land \neg i_2$

- **p2**: Filling  
  $\omega(P_2) = (1, 0, -, 0)$

- **p3**: Heating  
  $\omega(P_3) = (0, 0, -, 1)$

- **p4**: Emptying  
  $\omega(P_4) = (0, 1, -, 0)$

- **p5**: Stirring  
  $\omega(P_5) = (0, 0, 1, 1, 0, 1)$

- **p7**: Stand By  
  $\omega(p_7) = (-,-,-,-,0,1)$

- **p8**: Filling & Temp. OK  
  $\omega(p_8) = (0,1,1,0,1)$

- **p5**: Filled & Temp. OK  
  $\varphi(T_5) = i_2 \land i_3$

- **T4**: Filled & Temp. low  
  $\varphi(T_4) = i_2 \land \neg i_3$

- **t1**: $\varphi(t_1) = i_4$

- **t2**: $\varphi(t_2) = i_2$

- **t3**: $\varphi(t_3) = i_3$

- **t4**: $\varphi(t_4) = \neg i_1 \land \neg i_4$

- **t5**: $\varphi(t_5) = i_4 \lor i_5 \lor i_6$

- **t6**: $\varphi(t_6) = (0,1,1,0,1)$
RG\_SIPN

\[ m_0 = (1,0,0,0,0) \]
\[ \Omega = (0,0,0,0) \]
\[ t_1: \neg i_1 \wedge \neg i_2 \wedge i_4 \]
\[ t_2: i_2 \wedge \neg i_3 \]
\[ t_3: i_3 \]
\[ t_4: \neg i_1 \wedge \neg i_3 \wedge \neg i_4 \]

\[ m_1 = (0,1,0,1,0) \]
\[ \Omega = (1,0,1,0) \]

\[ m_2 = (0,0,1,0,1) \]
\[ \Omega = (0,1,1,1) \]
\[ t_5: i_2 \wedge i_3 \]

\[ m_3 = (0,1,0,0,1,1) \]
\[ \Omega = (r_0,1,r_0,0,0,0) \]
\[ (t_1 \rightarrow t_5): \neg i_2 \wedge i_4 \]

\[ m_4 = (0,0,2,0,0,0,1) \]
\[ \Omega = (0,0,1,1) \]
\[ t_2: i_2 \wedge \neg i_3 \]
\[ t_3: i_3 \wedge (i_1 \wedge i_4) \]

\[ m_5 = (1,0,1,0,0,0,1) \]
\[ \Omega = (r_0,0,0,0,0,0) \]
\[ (t_1 \rightarrow t_5) \]
\[ (t_2 \rightarrow t_3 \rightarrow t_4): i_2 \wedge i_3 \wedge \neg i_4 \]

\[ m_0 = (1,0,1,0,0,0,1) \]
\[ \Omega = (r_0,0,0,0,0,0) \]
\[ (t_1 \rightarrow t_5) \]
\[ (t_2 \rightarrow t_3 \rightarrow t_4): i_2 \wedge i_3 \wedge \neg i_4 \]
Requirements in Code-Generation

Transparency

- Structure of SIPN has to be visible somehow in PLC code

Correctness: Especially correct Representation of Dynamics

- Concurrency
- Iteration

Efficiency

- Method of code generation
- Generated code
TRANSPARENCY OF THE IL-CODE

Implementation of SIPN token play

- One Boolean variable for each place

Implementation of net elements

- One-to-one correspondence between SIPN elements and code segments. Example:

T1:  
LD  P1 (* T1: if pre-place P1 is marked *)
ANDN P2 (* and post-place P2 is not marked *)
AND (i3 (* and firing condition is fulfilled *)
OR  i4
)
R  P1 (* then unmark pre-place P1 *)
S  P2 (* and mark post-place P2 *)
P2:  
LD  P2 (* P2: if place P2 is marked *)
S  O2 (* set O2 *)
R  O1 (* and reset O1 *)
**Correctness of the IL-Code**

**Concurrency**

- No problem if all transition codes are evaluated during one PLC cycle

**Iteration**

- Direct implementation (very inefficient code)
- Special ordering of transition codes
  - Analytic [Frey: ACC 2000]
**Conclusions and Outlook**

**Conclusions**

- SIPN allows the formal specification of logic controllers
- SIPN can be implemented on PLC automatically
- No process model needed for analysis
- Essential correctness criteria defined
- Transparency metrics defined
- All algorithms prototyped in *Mathematica*

**Outlook**

- Complexity metric [*Frey, Litz and Klöckner, SMC 2000 Nashville]*
- Extension to timed SIPN and to analog I/O-Signals [*Frey ADPM2000*]

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**Papers / Further Info:** http://www.eit.uni-kl.de/litz/ENGLISH/frey.htm