Math 116 — Practice for Exam 1

Generated October 1, 2017

Name: SOLUTIONS

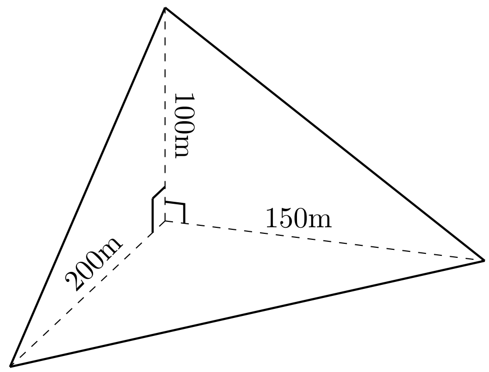
Instructor: ______________________ Section Number: __________

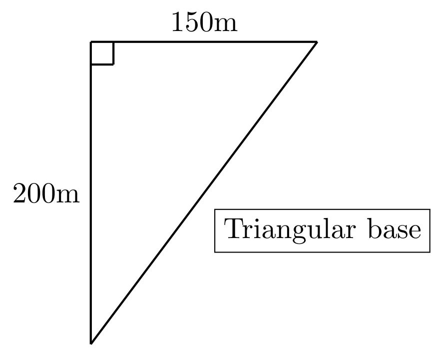
1. This exam has 1 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2017	1	9	nano pyramid	11	
Total				11	

Recommended time (based on points): 10 minutes

9. [11 points] Advanced beings from another planet recently realized they left a stockpile of nanotechnology here on Earth. These tiny devices are stacked in the shape of a pyramid with a triangular base that is flat on the ground. Its base is a right triangle with perpendicular sides of length 150m and 200m. Two of the other three sides are also right triangles, and all three right angles meet at one corner at the base of the pile. The fourth side is a triangle whose sides are the hypotenuses of the other three triangles. (See diagrams below.)





The density of the contents of the pile at a height of h meters above the ground is given by

$$\delta(h) = \frac{2}{\sqrt{1+h^2}} \text{ kg/m}^3.$$

For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2.

a. [4 points] Write an expression in terms of h that approximates the volume (in cubic meters) of a horizontal slice of thickness Δh of the contents of the pile at a distance h meters above the ground.

Solution: The cross section of the pile h meters above the ground is a right triangle with legs of length $200 - 2h$ and $150 - 1.5h$ meters. (One can use e.g. linearity or similar triangles to find these lengths.) Therefore, the volume (in cubic meters) of the horizontal slice is approximately $\frac{1}{2}(200-2h)(150-1.5h)\Delta h$.

b. [3 points] Write, but do **not** to evaluate, an integral that gives the total mass of the pile of nanotechnology. Include units.

Solution: We mutiply the expression in (a) by the density to get an approximation for the mass of the slice and integrate that expression from 0 to 100 while replacing Δh by dh to find a total mass of

$$\int_0^{100} \frac{1}{2}(200-2h)(150-1.5h)\frac{2}{\sqrt{1+h^2}}\, dh \quad \text{kilograms.}$$

c. [4 points] The beings must return to Earth and collect the nanotech that they left behind. Suppose that the spaceship hovers 150 meters above the ground (directly above the pile) while recovering the nanotechnology. Write, but do **not** evaluate, an integral which gives the total work that must be done in order to lift all of the nanotech from the pile into the ship. Include units.

Solution: We multiply the expression in (a) by the density, acceleration due to gravity, and the distance required to lift it to a height of 150 m to estimate the work done to lift that slice up to the spaceship. Then we integrate this expression from 0 to 100 and replace Δh by dh to find that the total work done is

$$\int_0^{100} g \cdot (150-h) \cdot \frac{1}{2}(200-2h)(150-1.5h)\frac{2}{\sqrt{1+h^2}}\, dh \quad \text{Joules.}$$