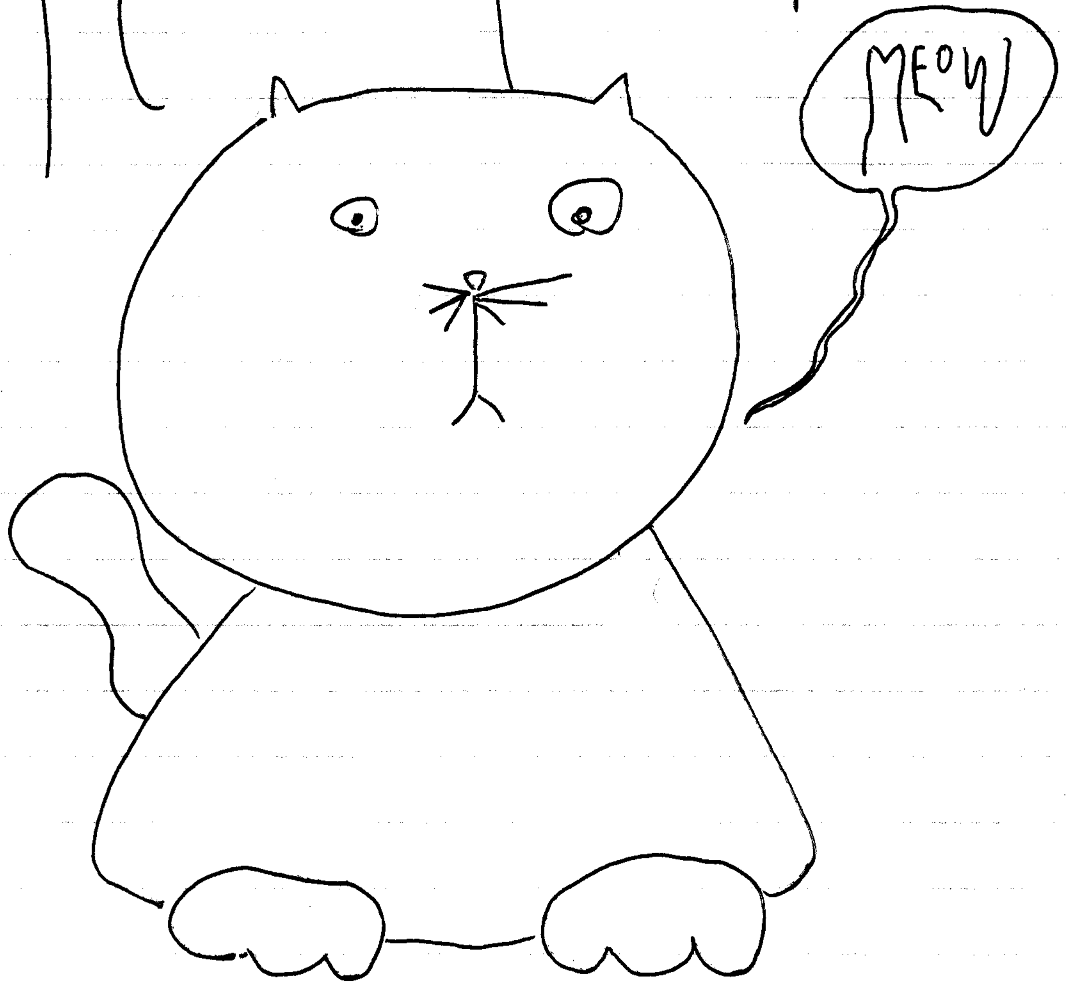
TEAM HOMEWORK 4



(1) We solve the intial value problems.

V: $y' = y\cos x$ $\quad V(0) = 100$ $\quad$ These are both separable.

G: $xy' = 4y$ $\quad G(1) = 17.$

Let's do V first.

$$y' = y\cos x \implies \frac{dy}{y} = \cos x \, dx. \qquad \int \frac{dy}{y} = \int \cos x \, dx$$

$$\implies \ln|y| = \sin x + C$$

$$\left.\begin{aligned} |y| &= e^{\sin x + C} = C e^{\sin x} \\ y &= C e^{\sin x} \end{aligned}\right\} \text{C changes 2 times.}$$

↳ Plug in initial value: $100 = C e^{\sin(0)} = C$

$\therefore \quad V(x) = 100 e^{\sin(x)}$

Now G. $\quad xy' = 4y \implies \frac{dy}{y} = \frac{4}{x} dx \implies \int \frac{dy}{y} = 4\int \frac{dx}{x}$

$$\implies \ln|y| = 4\ln|x| + C = \ln|x^4| + C$$

$|y| = e^{\ln|x^4| + C} = C|x^4|$ $\qquad$ Now initial value:

$y = Cx^4$ $\qquad$ $17 = C\, 1^4 = C$ so

$G(x) = 17x^4.$

Now we need to compute maximum values of each function on $[0, 2]$.

G is easier: since it is increasing, the global max occurs at the right endpoint.

$G(2) = 17 \cdot 16 = (16+1)16 = 2^8 + 2^4 = 256 + 16 = \boxed{272}.$

↳ no calculator handy...

For $V(x) = 100e^{\sin(x)}$, I note that e^x is increasing, hence maxes out at larger values. So $V(x)$ is at a maximum when $\sin(x)$ is, and the max of $\sin(x)$ is 1. So, $V(x) = 100e^{\sin(x)} \le 100e^1$

$$\le 100(2.7183)$$
$$= \boxed{271.83}$$

upper bound on value of e.

Since $272 > 271.83$,

O-guk wins.

② The book gives us a general differential equation describing how things heat up and cool down. It's called Newton's Law of Heating and Cooling. (Page 613)

The law says objects heat or cool exponentially till they reach the ambient temperature of their surroundings. Here is the differential equation:

$$\frac{dT}{dt} = k(T - A)$$

k: constant of proportionality

A: ambient temperature.

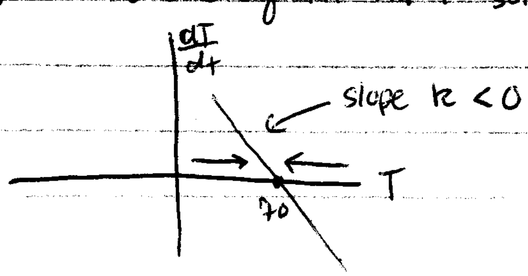
$\boxed{\text{Note:}}$ the equilibrium solutions of the heating equation are always <u>stable</u> because k is <u>negative</u>.

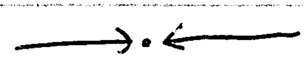
a) We apply this to our situation.

$$\frac{dT}{dt} = k(T - 70) \qquad T(0) = 30$$

be sure to state the initial condition.

b) There is one equilibrium solution at $T = 70$. It is stable.





stable.

c) We need to solve the diff'eq

$$\frac{dT}{dt} = k(T-70) \Rightarrow \frac{dT}{T-70} = k\,dt$$

$$\int \frac{dT}{T-70} = k \int dt$$

$\ln|T-70| = kt + C$

$|T-70| = Ce^{kt}$

$T-70 = Ce^{kt}$

evaluate at initial condition: $T(0) = 30$

$-40 = 30 - 70 = Ce^0 = C$

So, $T(t) = 70 - 40e^{kt}$.

Now we use $T(60) = 50$ to determine k.

$50 = 70 - 40e^{60k}$

$-20 = -40e^{60k}$

$\frac{1}{2} = e^{60k}$

$\ln(\frac{1}{2}) = 60k$

$k = \frac{1}{60}\ln(\frac{1}{2})$

So, $T(120) = 70 - 40e^{\frac{1}{60}\ln(\frac{1}{2})120}$

$= 70 - 40(\frac{1}{2})^2 = 70 - 10 = 60.$

So the orange juice is 60° after 2 hours. Great.

(3) a) $\frac{dP}{dt} = (\text{rate increase}) - (\text{rate decrease})$

$$\underbrace{kP^2}_{\text{rate proportional to the square of the population}} \qquad \underbrace{.1}_{\text{note the units are in thousands.}}$$

So, $\frac{dP}{dt} = kP^2 - .1$

b) With $k = 7$ we have $\frac{dP}{dt} = 7P^2 - .1$, and $\Delta t = 1$.

n	x_n	y_n	m_n
0	0	0	$-.1$
1	1	$-.1$	$-.03$
2	2	$-.13$	$.0183$
3	3	$-.1117$	

$m_1 = 7(-.1)^2 - .1 = 7(.01) - .1$
$= -.03$

$m_2 = 7(-.13)^2 - .1 = .0183$

here are my 3 other points.

④ a) This is cryptic, but they just want you to take the derivative of the differential equation.

$$y' = xe^y + x^3 \implies y'' = xe^y y' + e^y + 3x^2$$

now use the original diff'eq to get rid of y'.

$$\boxed{y'' = xe^y(xe^y + x^3) + e^y + 3x^2}$$

b) Bad wording. They mean the original (O.G.) diff'eq. we do this by showing y'' is always positive.

$$y'' = \underbrace{x^2e^{2y} + x^4e^y + e^y + 3x^2}_{\text{all parts are always } \geq 0}$$

so $y'' \geq 0$

so all solutions are concave up.

THE END.