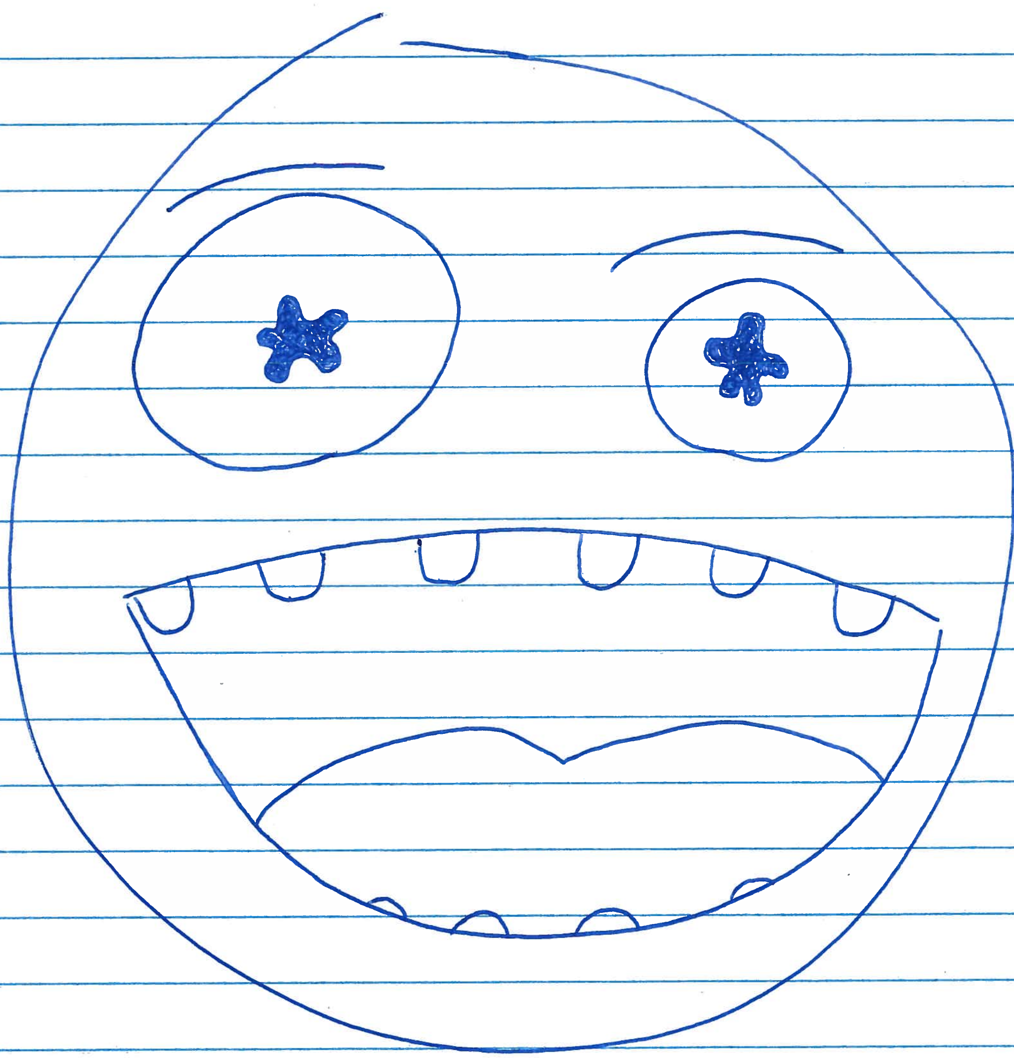
TEAM CALCULUS WHISPERERS



Starring:

Tammy Toppel............manager
Russ Weterson............clarifier
Jackman Raul.........scribe
Debra McQueath.......reporter

1. a) Error 1: it's called "u-sub" not "w-sub," duh.

(joking, of course)

For real now, I see ~~two~~ three errors.

1. In this step: $2e^w\Big|_2^4$ the bounds 2, 4 are for the variable x, not w. We should change back to x first.

2. The last step: $2e^4 - 2e^2 + C$ what the hell is C doing here? No no no.

3. $\int_2^4 2e^w dw$ should be $\int_2^4 \frac{1}{2}e^w dw$

Correct sol'n:

$$\int xe^{x^2}dx = \int \frac{1}{2}e^u du = \frac{1}{2}e^{x^2}, \text{ so}$$

$u = x^2$

$du = 2x dx$

$\frac{1}{2}du = x dx$

$$\int_2^4 xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big]_2^4 = \boxed{\frac{1}{2}e^{16} - \frac{1}{2}e^4}.$$

b) This is total nonsense. Every step is wrong. Use integration by parts.

$$\int_5^3 x\ln x\,dx = \frac{1}{2}x^2\ln x\Big]_5^3 - \int_5^3 \frac{1}{2}x\,dx = \frac{1}{2}9\ln 3 - \frac{1}{2}25\ln 5 - \frac{1}{4}x^2\Big]_5^3$$

$u = \ln x$ $\quad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx$ $\quad dv = x dx$

$$= \boxed{\frac{1}{2}9\ln 3 - \frac{1}{2}25\ln 5 - \frac{1}{4}9 + \frac{25}{4}}.$$

c) Ugh. This is u-sub gone wrong. Just do it like this:

$$\int_0^2 \left(\frac{x-1}{2}\right)^3 dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2u^3 du = 0$$ because $2u^3$ is odd and we're integrating from $-\frac{1}{2}$ to $\frac{1}{2}$.

$u = \frac{x-1}{2}$

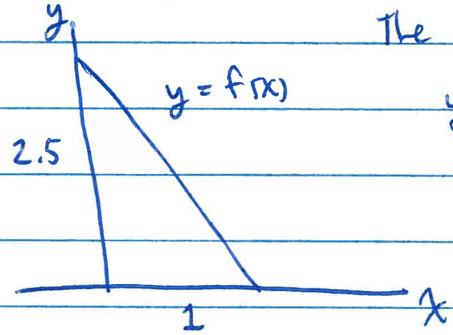
$du = \frac{1}{2}dx$ $\quad 2du = dx$ $\quad$ I changed the bounds using $u = \frac{x-1}{2}$

d) This function has the wrong value at 1 and we don't add +C when we are looking for a specific antiderivative.

$$Q(x) = Q(1) + \int_1^x t^2\,dt = 3 + \left.\tfrac{1}{3}t^3\right]_1^x = \boxed{3 + \tfrac{1}{3}x^3 - \tfrac{1}{3}}$$

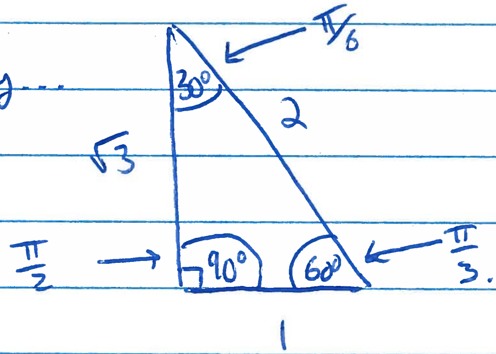
use 1 here to get the right value at $x=1$

2. a) The triangle on the floor looks like this:



The line $y=f(x)$ has formula $y = 2.5 - 2.5x$ with $0 \le x \le 1$.

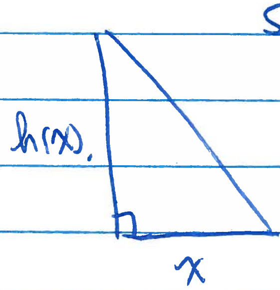
b) i. Okay...



$$\text{Area} = \tfrac{1}{2}(1)(\sqrt{3}) = \boxed{\tfrac{\sqrt{3}}{2}\,m^2}$$

ii. Let $h(x)$ be the height as a function of the base.

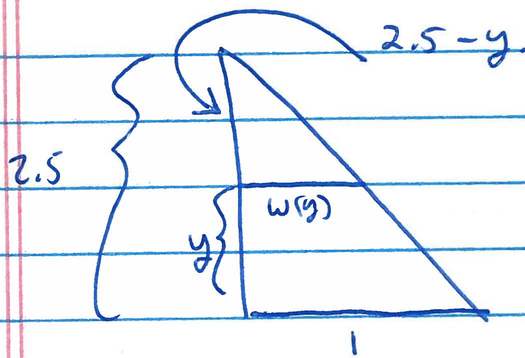
Similar triangles $\Rightarrow \dfrac{h(x)}{x} = \dfrac{\sqrt{3}}{1}$



So $h(x) = \sqrt{3}\,x$.

$$\text{Area} = \tfrac{1}{2}x\,h(x) = \boxed{\tfrac{\sqrt{3}}{2}x^2\,m^2}$$

iii. Let's look at the floor again.



Similar triangles

$$\frac{w(y)}{2.5-y} = \frac{1}{2.5}$$

$$w(y) = \frac{2.5-y}{2.5} = 1 - \frac{y}{2.5}.$$

By the last problem, the area is $\frac{\sqrt{3}}{2} w(y)^2 = \boxed{\frac{\sqrt{3}}{2}\left(1-\frac{y}{2.5}\right)^2 m^2}$

c) Given the previous parts, a slice at y has "approximate volume" $\underbrace{\frac{\sqrt{3}}{2}\left(1-\frac{y}{2.5}\right)^2}_{\text{area}} \underbrace{\Delta y}_{\text{thickness}}\ m^3$

So my integral is

$$\int_0^{2.5} \frac{\sqrt{3}}{2}\left(1-\frac{y}{2.5}\right)^2 dy = -2.5\frac{\sqrt{3}}{2}\int_1^0 u^2\,du = 2.5\frac{\sqrt{3}}{2}\int_0^1 u^2\,du$$

$u = \left(1-\frac{y}{2.5}\right)$

$du = -\frac{1}{2.5}\,dy$

I changed the bounds

$$\boxed{= 2.5\,\frac{\sqrt{3}}{2}\cdot\frac{1}{3}\ m^3}$$

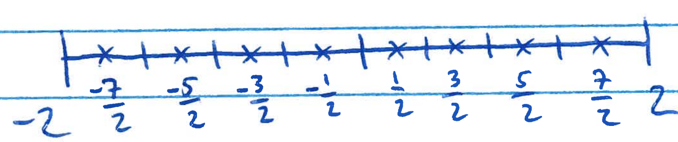
3 a) $f(x)$ is concave down so TRAP is an underestimate and MID is an overestimate.

key word.

$$\text{TRAP}(8) \le \int_{-2}^{2} f(x)\,dx \le \text{MID}(8).$$

b) Ugh... this sucks. First figure out the midpoints.

$$\Delta x = \frac{2-(-2)}{8} = \frac{1}{2}.$$



Then MID(8) is

$$\left(f\left(-\tfrac{7}{2}\right)\Delta x + f\left(-\tfrac{5}{2}\right)\Delta x + f\left(-\tfrac{3}{2}\right)\Delta x + f\left(-\tfrac{1}{2}\right)\Delta x\right.$$

$$\left. + f\left(\tfrac{1}{2}\right)\Delta x + f\left(\tfrac{3}{2}\right)\Delta x + f\left(\tfrac{5}{2}\right)\Delta x + f\left(\tfrac{7}{2}\right)\Delta x\right)$$

notice that $f(x)$ is even, so $f(-x) = f(x)$, and thus

$$MID(8) = 2\left(f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{2}\right) + f\left(\tfrac{5}{2}\right) + f\left(\tfrac{7}{2}\right)\right)\Delta x$$

(Note $2\Delta x = 1$)

$$= \frac{8 - e^{1/2} - e^{-1/2}}{2} + \frac{8 - e^{3/2} - e^{-3/2}}{2} + \frac{8 - e^{5/2} - e^{-5/2}}{2} + \frac{8 - e^{7/2} - e^{-7/2}}{2}.$$

oy...

c) They're saying $f(x) = \frac{8 - (e^x + e^{-x})}{2} = 4 - g(x)$.

The left hand side is...

$$1 + g'(x)^2 = 1 + \left(\frac{e^x - e^{-x}}{2}\right)^2 = 1 + \frac{e^{2x} - 2 + e^{-2x}}{4}$$

$$\left(g'(x) = \frac{e^x - e^{-x}}{2}\right)$$

$$= 1 - \frac{1}{2} + \frac{e^{2x} + e^{-2x}}{4}$$

$$= \frac{e^{2x} + 2 + e^{-2x}}{4}$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 = g(x)^2$$

...which is the right hand side.

d) use the arclength formula.

$$\int_{-2}^{2} \sqrt{1 + g'(x)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{g(x)^2}\, dx = \int_{-2}^{2} g(x)\, dx$$

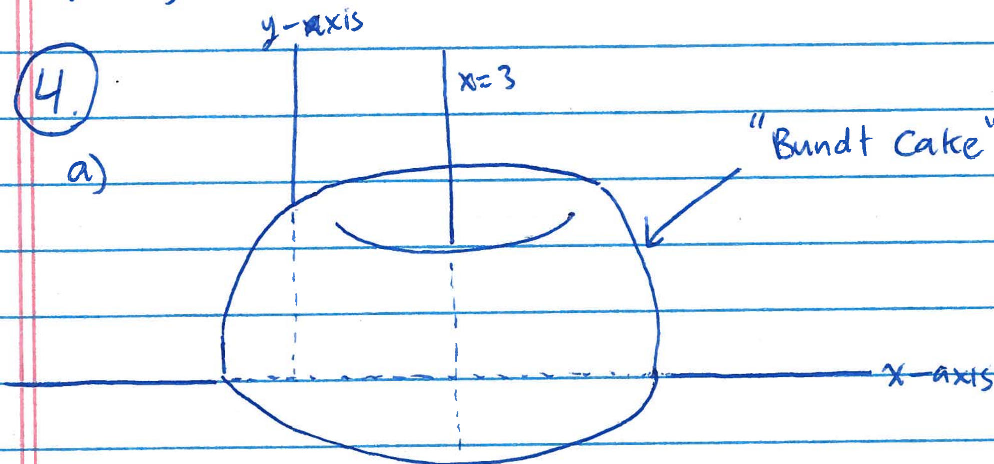
b/c $g(x)$ is even $\longrightarrow$ $= 2\int_0^2 \frac{e^x + e^{-x}}{2}\, dx = \int_0^2 e^x + e^{-x}\, dx = e^x - e^{-x}\Big]_0^2$

$$\boxed{= e^2 - e^{-2}}$$

e) Yes, translation and reflection do not affect length.

4.

a)



b) Alas, I will do both.

i. This is the (oops) "washer method"

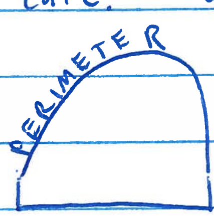
A. Wait a second, this is an awful way to do this problem. It's a trap. ABORT!

ii. This is the "shell method".

A. approx. vol. of one shell: $\boxed{2\pi\underbrace{(3-x)}_{\text{radius of shell}}\underbrace{\left(\frac{8-e^{x}-e^{-x}}{2}\right)}_{\text{height of shell}}\Delta x}$

B. $$\boxed{\int_{-2}^{2} 2\pi(3-x)\left(\frac{8-e^{x}-e^{-x}}{2}\right)dx.}$$

c) This is a weird way of asking for the perimeter of a slice of cake. We use our arclength computation for the curve on top and then add in other sides.



$$\underbrace{e^{2}-e^{-2}}_{\text{top}} + \underbrace{4}_{\text{bottom}} + \underbrace{2\left(\frac{8-e^{2}-e^{-2}}{2}\right)}_{\text{short sides}} = \boxed{8-2e^{-2}.}$$