TEAM

HOMEWORK II: the sequel.

U WANT SOLS?

If you printed, go that way

If you no printed, go that way

gl, hf.

—T

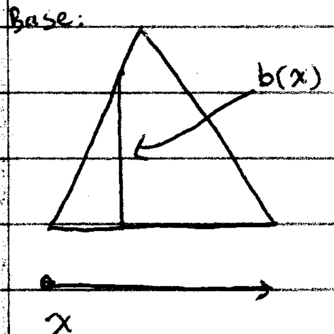
(1) We need to know if Oguk's object was heavier than 9000 kg, so let's compute the mass of his weirdo object

Run-down:

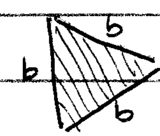
- $\delta = 1000\ kg/m^3$ (constant density! Yay!)
- base is eq. triangle of side length 4 m
- cross-sections perpendicular to base are eq. triangles

What does it look like? I dunno, it's kinda pointy. Seems dangerous. But, doesn't matter, because volume = $\int$(area of cross section)dx.

Base:



Recall the area of an eq. triangle with base b is $\boxed{\frac{\sqrt{3}}{4} b^2}$



$$\text{Total mass} = \int_0^4 \delta \frac{\sqrt{3}}{4} b(x)^2\, dx = 2 \int_0^2 \delta \frac{\sqrt{3}}{4} b(x)^2\, dx$$

since density is constant, we make life easier by computing volume of half the object and then multiplying by 2.

Finally we find a formula for $b(x)$. Guess what? It's a linear function of x on $[0,2]$.

$b(0) = 0 \qquad b(2) = 4\sin(\frac{\pi}{3}) = 2\sqrt{3}$

do you see what I'm doing?

So, $b(x) = \sqrt{3}\, x$

Alright, $\text{total mass} = 2 \cdot 1000 \cdot \frac{\sqrt{3}}{4} \int_0^2 (\sqrt{3}\, x)^2\, dx = 1500\sqrt{3}\left(\frac{1}{3}x^3\Big]_0^2\right)$

$= 1500 \frac{\sqrt{3}}{3} \cdot 8 = 4000\sqrt{3} \approx 6928.2\ kg. < 9000\ kg.$ Loser.

② a) Notice that $C = \int_0^1 f'(x)\,dx = f(1) - f(0) = f(1)$ by the fundamental theorem of calculus.

This smells like integration by parts.

$$D = \int_0^1 x f'(x)\,dx = xf(x)\Big]_0^1 - \int_0^1 f(x)\,dx = f(1) - \int_0^1 f(x)\,dx$$

$u = x \qquad du = dx$

$dv = f'(x)\,dx \qquad v = f(x)$

$$= C - A$$

Now we play a matching game!

So, $\boxed{D = C - A.}$ Now we know.

$$E = \int_0^1 x^2 f'(x)\,dx = x^2 f(x)\Big]_0^1 - 2\int_0^1 x f(x)\,dx = f(1) - 2\int_0^1 x f(x)\,dx$$

$u = x^2 \qquad du = 2x\,dx$

$dv = f'(x)\,dx \qquad v = f(x)$

$$= C - 2B.$$

$$\boxed{E = C - 2B}$$

b) Follow your nose...

$$\int_0^2 \frac{g(2x+1)}{2x+1}\,dx = \frac{1}{2}\int_1^5 \frac{g(u)}{u}\,du = \frac{1}{2}\int_1^5 \frac{g(y)}{y}\,dy = \frac{1}{2}\left(g(y)\ln(y)\Big]_1^5 - \int_1^5 g'(y)\ln(y)\,dy \right)$$

← notice that I changed the bounds.

$u = 2x+1$

$du = 2\,dx$ ← Pretend this never happened

$\frac{1}{2}du = dx$

Let me change the variable name to avoid confusion in the next step.

$u = g(y) \qquad du = g'(y)\,dy$

$dv = \frac{1}{y}\,dy \qquad v = \ln(y)$

$$= \frac{1}{2}\left(g(5)\ln(5) - \int_1^5 g'(y)\ln(y)\,dy\right)$$

$$= \frac{1}{2}\left(a\ln(5) - b\right).$$

$$\boxed{\text{So, } \int_0^2 \frac{g(2x+1)}{2x+1}\,dx = \frac{1}{2}\left(a\ln(5) - b\right)}$$

③ These are SO MUCH FUN!!!

a) $$\int e^{x+e^x}\,dx = \int e^{e^x}\cdot e^x\,dx = \int e^u\,du = e^u + C$$

$u = e^x$ $\qquad\qquad$ $= e^{e^x} + C$

$du = e^x\,dx$

So $$\int e^{e^x+x}\,dx = e^{e^x} + C.$$

b) $$\int e^x\left(\ln x + \frac{1}{x^2}\right)dx = \int e^x \ln x\,dx + \int x^{-2} e^x\,dx$$

Let's try to do these one at a time.

- $$\int e^x \ln x\,dx = e^x \ln x - \int x^{-1} e^x\,dx$$

$u = \ln x$ $\qquad$ $du = x^{-1}dx$

$dv = e^x\,dx$ $\qquad$ $v = e^x$

now let's try to do this one...

- $$\int x^{-1} e^x\,dx = e^x x^{-1} + \underbrace{\int x^{-2} e^x\,dx}$$

$u = x^{-1}$ $\qquad$ $du = -x^{-2}dx$ $\qquad$ ↳ this looks familiar...

$dv = e^x\,dx$ $\qquad$ $v = e^x$

Putting this all together we have

$$\int e^x \ln x\,dx = e^x \ln x - e^x x^{-1} - \int x^{-2} e^x\,dx$$ (add $\int x^{-2} e^x\,dx$ to both sides)

$$\boxed{\int e^x \ln x\,dx + \int x^{-2} e^x\,dx = e^x \ln x - e^x x^{-1}}$$ Amazing!

c) $\int_0^1 x^3 \sin^2 x + (1-x)^3 \cos^2(1-x)dx = \int_0^1 x^3 \sin^2 x\, dx + \underbrace{\int_0^1 (1-x)^3 \cos^2(1-x)dx}_{\text{Let's focus on this.}}$

don't forget her!

$$\int_0^1 (1-x)^3 \cos^2(1-x)dx = -\int_1^0 u^3 \cos^2 u\, du = \int_0^1 u^3 \cos^2(u)du$$

$u = 1-x$

$du = -dx$

notice the change in limits

flip limits of integration, eat a sign.

Let's replace the letter u with the letter x.

$$\int_0^1 x^3 \cos^2 x\, dx = \int_0^1 x^3(1-\sin^2 x)\, dx = \int_0^1 x^3 dx - \int_0^1 x^3 \sin^2 x\, dx$$

$$= \frac{1}{4} - \int_0^1 x^3 \sin^2 x\, dx.$$

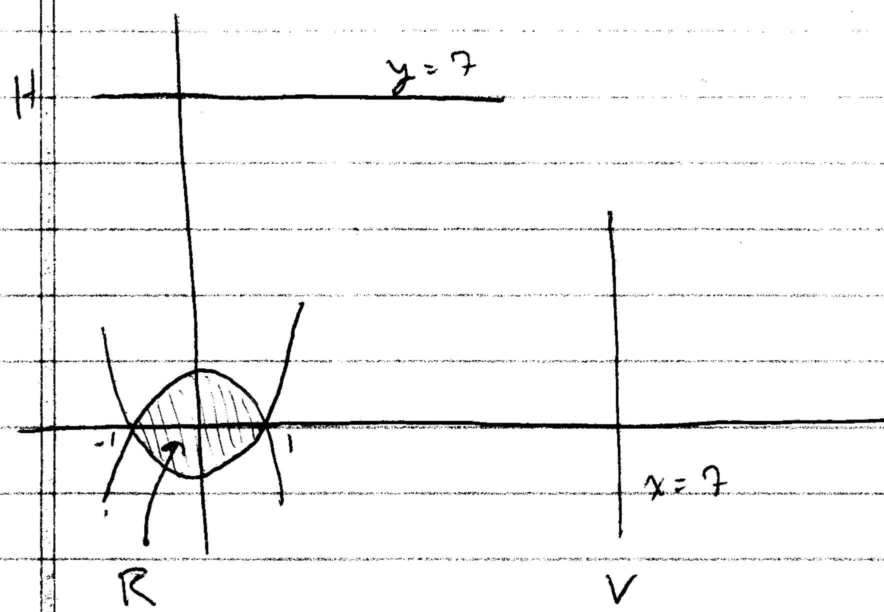
So...

$$\int_0^1 (1-x)^3 \cos^2(1-x)dx = \frac{1}{4} - \underbrace{\int_0^1 x^3 \sin^2 x\, dx}_{\text{add this to both sides}}$$

$$\boxed{\int_0^1 x^3 \sin^2 x\, dx + \int_0^1 (1-x)^3 \cos^2(1-x)dx = \frac{1}{4}}$$

Beautiful.

(4) Here is the picture:



Let's compute the volume of V. We will use SHELL METHOD.

$$\text{volume of } V = \int_{-1}^{1} 2\pi(x+7)\,2(1-x^2)\,dx = \frac{112\pi}{3}\ \text{mm}^3$$

distance from slice at x to line $x = 7$
= radius of shell.

For H, I'll use "washer method"

$$\text{volume of } H = \cancel{\int_{-1}^{1} \pi(x^2-1)^2 - \pi(1-x^2)^2\,dx} =$$

NO JK!

Let's try again.

$$\text{volume of } H = \int_{-1}^{1} \pi\left(7 - x^2+1\right)^2 - \pi\left(7-1+x^2\right)^2 dx = \frac{112\pi}{3}\ \text{mm}^3$$

that's a minus.

So... they're the same volume.

But H looks less painful to wear, Make H.