Math 116 — Practice for Exam 3

Generated December 10, 2015

Name: ____________________

Instructor: ____________________ Section Number: ____________

1. This exam has 7 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2010	3	6		10	
Winter 2011	3	9	Fourier	9	
Fall 2014	3	12	chicken coop	8	
Fall 2011	3	5	circuit	10	
Fall 2012	3	3		12	
Fall 2013	3	4		9	
Fall 2013	3	8		12	
Total				70	

Recommended time (based on points): 84 minutes

6. [10 points] Consider the function $f(x) = \ln(1+x)$ and its Taylor series about $x = 0$.

a. [4 points] Determine the first four non-zero terms of the Taylor series for $f(x) = \ln(1+x)$ about $x = 0$. Be sure to show enough work to support your answer.

b. [4 points] Find the first three non-zero terms of the Taylor series for $g(x) = \ln\left(\frac{1+x}{1-x}\right)$ about $x = 0$. Be sure to show enough work to support your answer. *(Hint: You may find it helpful to utilize properties of logarithms.)*

c. [2 points] Find the exact value of the sum of the series

$$2\left(\frac{3}{4}\right) + \frac{2}{3}\left(\frac{3}{4}\right)^3 + \frac{2}{5}\left(\frac{3}{4}\right)^5 + \ldots\ .$$

9. [9 points]

a. [2 points] Find the Taylor series about $x = 0$ of $\sin(x^2)$. Your answer should include a formula for the general term in the series.

b. [2 points] Let m be a positive integer, find the Taylor series about $x = 0$ of $\cos(m\pi x)$. Your answer should include a formula for the general term in the series.

c. [5 points] Use the second degree Taylor polynomials of $\sin(x^2)$ and $\cos(m\pi x)$ to approximate the value of b_m, where

$$b_m = \int_{-1}^{1} \sin(x^2) \cos(m\pi x) dx.$$

(The number b_m is called a *Fourier coefficient of the function* $\sin x^2$. These numbers play a key role in *Fourier analysis*, a subject with widespread applications in engineering and the sciences.)

12. [8 points] Franklin, your friendly new neighbor, is building a large chicken sanctuary. You decide to help Franklin build a special chicken coop with volume (in cubic km) given by the integral

$$\int_0^1 x\sqrt{1-\cos(x^2)}\,dx.$$

This integral is difficult to evaluate precisely, so you decide to use the methods you've learned this semester to help out Franklin. Your friend and president-elect, Kazilla, stops by to give you a hand. She suggests finding the 4th degree Taylor polynomial, $P_4(x)$, for the function $1-\cos(x^2)$ near $x=0$.

a. [4 points] Find $P_4(x)$.

b. [4 points] Substitute $P_4(x)$ for $1-\cos(x^2)$ in the integral and compute the resulting integral by hand, showing all of your work.

5. [10 points] When a voltage V in volts is applied to a series circuit consisting of a resistor with resistance R in ohms and an inductor with inductance L, the current $I(t)$ at time t is given by

$$I(t) = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \qquad \text{where } V, R, \text{ and } L \text{ are constants.}$$

a. [2 points] Show that $I(t)$ satisfies

$$\frac{dI}{dt} = \frac{V}{L}\left(1 - \frac{R}{V}I\right).$$

b. [6 points] Find a Taylor series for $I(t)$ about $t = 0$. Write the first three nonzero terms and a general term of the Taylor series.

c. [2 points] Use the Taylor series to compute

$$\lim_{t \to 0} \frac{I(t)}{t}.$$

3. [12 points] Let
$$I = \int_0^1 \left(1 + \frac{t^2}{2}\right)^{\frac{5}{2}} dt$$

a. [5 points] Approximate the value of I using Right(2) and Mid(2). Write each term in your sums.

b. [2 points] Are your estimates of the value of I obtained using Right(2) and Mid(2) guaranteed to be overestimates, underestimates or neither?

c. [3 points] Find the first three nonzero terms of the Taylor series for $g(t) = \left(1 + \frac{t^2}{2}\right)^{\frac{5}{2}}$ about $t = 0$.

d. [2 points] Use your answer from part (c) to estimate I.

4. [9 points]
Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. If the sequence converges, identify the limit. Circle all your answers. No justification is required.

a. [3 points] $a_n = \displaystyle\int_1^{n^3} \frac{1}{(x^2+1)^{\frac{1}{5}}} dx.$

Converges to ________________ Diverges.

Increasing Decreasing Neither.

b. [3 points] $b_n = \displaystyle\sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!}.$

Converges to ________________ Diverges.

Increasing Decreasing Neither.

c. [3 points] $c_n = \cos\left(a^n\right)$, where $0 < a < 1$.

Converges to ________________ Diverges.

Increasing Decreasing Neither.

8. [12 points]

a. [4 points] Let a be a positive constant. Determine the first three nonzero terms of the Taylor series for

$$f(x) = \frac{1}{(1+ax^2)^4}$$

centered at $x = 0$. Show all your work. Your answer may contain a.

b. [2 points] What is the radius of convergence of the Taylor series for $f(x)$? Your answer may contain a.

c. [3 points] Determine the first three nonzero terms of the Taylor series for

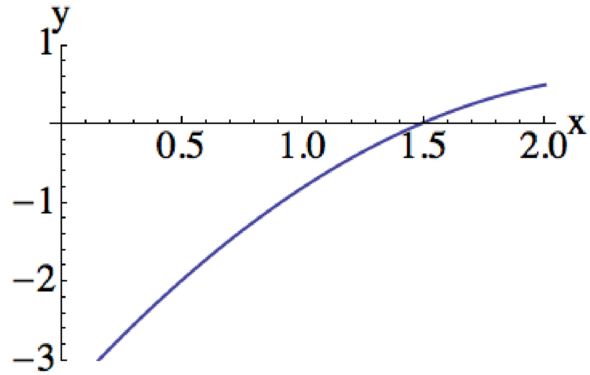
$$g(t) = \int_0^t \frac{1}{(1+ax^2)^4}dx,$$

centered at $t = 0$. Show all your work. Your answer may contain a.

d. [3 points] The degree-2 Taylor polynomial of the function $h(x)$, centered at $x = 1$, is

$$P_2(x) = a + b(x-1) + c(x-1)^2.$$

The following is a graph of $h(x)$:



What can you say about the values of a, b, c? You may assume a, b, c are nonzero. Circle your answers. No justification is needed.

a is: Positive Negative Not enough information

b is: Positive Negative Not enough information

c is: Positive Negative Not enough information