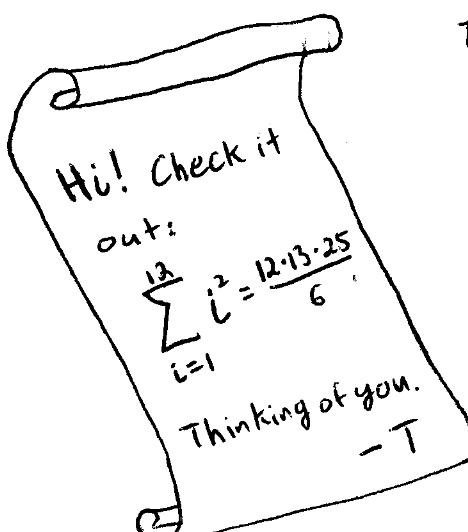
# THE WORD ON Σ NOTATION

—a handout by Trevor Hyde

You could say mathematics is a language of patterns. Any time you come across a pattern, there's a good chance there is a mathematical way to say it concisely. Consider the following example: Let's say you want to correspond with your penpal about this sum which recently entered your life:

$$S = 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 + 121 + 144.$$

Stationary sells for a premium so it is desirable to express this sum in fewer strokes without being vague. Sigma notation is the tool for this. Notice that our sum $S$ is all of the squares from 1 to 12.




Then we write

$$S = \sum_{i=1}^{12} i^2.$$

So easy! Let's break it down.

$$\sum_{i=1}^{12} i^2$$

- 12 ← This is the last value of $i$.
- This is the sigma (think greek 's') we use it for a sum. → $\sum$
- $i^2$ ← For each value of $i$, we compute the expression here and add it to our sum.
- The letter $i$ is being used as an index. we let it take on all the whole number values from 1 to 12. → $i = 1$ ← This is the starting value for $i$.

Remember that the only letter that's changing is the index $i$. The letter we choose for the index is immaterial — can you guess why I chose $i$?

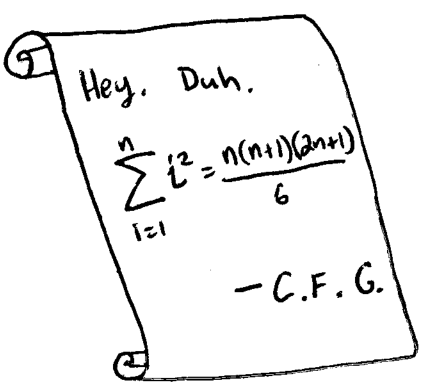
I often want to define a function using the sum of a bunch of numbers. Sigma notation saves my life in these circumstances.

Example: Define $S(n) = \sum_{i=1}^{n} i^2$. Then we have

- $S(1) = 1^2$
- $S(5) = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$
- and so on...

The variable $n$ controls how many terms are being summed. Without sigma notation, this is awkward to describe.

This notation will be used at various times throughout the course.




Practice:

1. Write $1 + 3 + 5 + 7 + 9 + 11 + 13$ in sigma notation.

2. Evaluate $\sum_{i=3}^{8} i(i+1)$.