DAY

Can you write down a quadratic poly $f(x)$ so that

$$f(0)=2 \qquad f'(0)=-3 \qquad f''(0)=5 \quad ?$$

$$f(x) = a_0 + a_1 x + a_2 x^2$$

$$f(0) = a_0$$

$$f'(0) = a_1$$

$f''(0) = 2a_2$ so $f(x) = 2 - 3x + \frac{5}{2}x^2$ Nice.

How about $f(0)=3$ $f'(0)=6$ $f''(0)=1$?

↳ easy! $f(x) = 3 + 6x + \frac{1}{2}x^2$

Okay, but now suppose I want a quadratic so that

$$f(7)=2 \qquad f'(7)=3 \qquad f''(7)=-5$$

$$f(x) = a_0 + a_1 x + a_2 x^2$$

$$2 = f(7) = a_0 + 7a_1 + 49a_2$$

$$3 = a_1 + 2a_2 \cdot 7 = f'(7)$$

$$-5 = f''(7) = 2a_2$$

yuck.

There's a better way!

$$f(x) = b_0 + b_1(x-7) + b_2(x-7)^2$$

$$f(7) = b_0$$

$f'(7) = b_1$ so $f(x) = 2 + 3(x-7) - \frac{5}{2}(x-7)^2$

$$f''(7) = 2b_2$$

Let's use the ratio test.

$$b_n = a_n (x-c)^n$$

$$\lim_{n\to\infty} \left| \frac{b_{n+1}}{b_n} \right| = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \frac{|x-c|^{n+1}}{|x-c|^n}$$

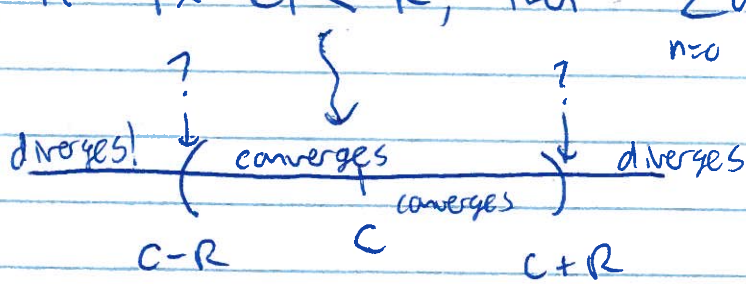
$$= \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| |x-c| < 1$$

In order for series to converge, need this limit to be < 1

$$\therefore \ |x-c| < \frac{1}{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|} = \frac{1}{L} = R$$

Define R to be $\frac{1}{L}$ where $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$ is the limit from the ratio test. R is called the <u>radius of convergence</u>

So, if $|x-c| < R$, then $\sum_{n=0}^{\infty} a_n (x-c)^n$ converges.



If $|x-c| > R$, the series diverges.

If $|x-c| = R$, we don't know.

The domain of $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ is called the <u>Interval of convergence</u>.

ex. $\sum_{n=0}^{\infty} \frac{2^n}{n 3^n} x^n$

What is the center?

What is the radius?

What is the interval of convergence?

ex. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n 3^n} x^n$ Same Q's.

↳ same radius & center!

↳ different interval.

ex. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n 3^n} (x-7)^n$ Same Q's.

↳ only thing that changed is the center.

ex. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ Same Q's.

$\therefore$ I.O.C of $f(x)$ is $(-\sqrt{3}, \sqrt{3})$

hmm...

ex. $\sum_{n=0}^{\infty} \frac{x^{2n}}{3^n} = f(x)$. ⟵ $f(x) = g(x^2)$ where $g(x) = \sum_{n=0}^{\infty} \frac{x^n}{3^n}$ ⟵ ~~which~~ converges when $\left|\frac{x}{3}\right| < 1$

$|x| < 3$

So, $f(x)$ converges when $|x^2| < 3 \Rightarrow |x| < \sqrt{3}$