DAY 22

- do a "drag problem"
- practice. geom series (quick review)

Infinite series.

Say a_n is a sequence.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

is called an <u>infinite series</u>.

We are now going to focus on figuring out whether infinite series converge.

$$\cancel{S_m = \sum_{n=1}^{m} a_n \Big\} \text{partial sums.}}$$

$$\sum_{n=1}^{\infty} a_n \overset{\text{def}}{=} \lim_{m\to\infty} \sum_{n=1}^{m} a_n$$

just like improper integrals.

ex. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n}$ converge?

$$\frac{n-1}{n} = 0, \frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \ldots$$

we're adding at least $\frac{1}{2}$ every time... nah!

Test 1: n^{th} term test.

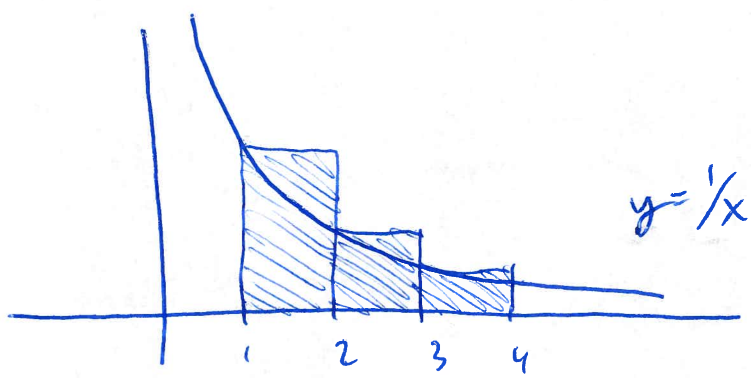
If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Note: can only be used to show <u>divergence</u>.

ex. $\lim_{n\to\infty} \frac{n-1}{n} = 1 \neq 0$, so $\sum_{n=1}^{\infty} \frac{n-1}{n}$ diverges by the n^{th} term test.

ex. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

terms go to 0... so?



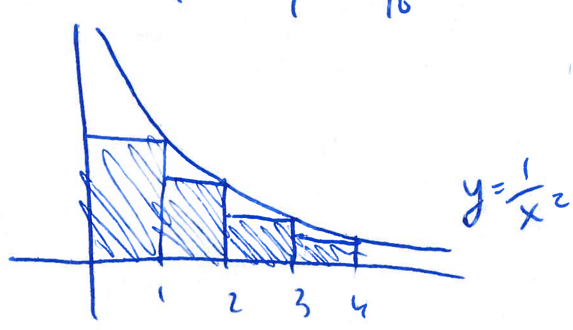
So

$$\int_1^{\infty} \frac{1}{x}\,dx \leq \sum_{n=1}^{\infty} \frac{1}{n}$$

diverges

so this must too!

ex. $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$



$$\sum_{n=1}^{\infty} \frac{1}{n^2} \leq 1 + \int_1^{\infty} \frac{1}{x^2}\,dx$$

converges by p-test.

So this must converge too!

Test 2: Integral Test!

Say $a_n = f(n)$ and

- $f(x)$ is decreasing
- $f(x)$ is positive

$$\int_1^\infty f(x)\,dx \text{ converges}$$

$$\Updownarrow$$

$$\sum_{n=1}^\infty f(n) \text{ converges.}$$

both converge or both diverge.

ex. $\displaystyle\sum_{n=1}^\infty \frac{2n}{e^{n^2}}$ $\qquad f(x) = \dfrac{2x}{e^{x^2}}$ $\qquad$ decreasing ✓ positive ✓

$$\int_1^\infty \frac{2x}{e^{x^2}}\,dx = \int_1^\infty \frac{1}{e^u}\,du = \lim_{b\to\infty} -e^{-u}\Big]_1^b$$

$$= \lim_{b\to\infty} e^{-1} - e^{-b} = \frac{1}{e}.$$

so this converges.

$\therefore \displaystyle\sum_{n=1}^\infty \frac{2n}{e^{n^2}}$ converges by integral test.

Note: the sum does not equal $\frac{1}{e}$.

Test 3: The p-test.

$\displaystyle\sum_{n=1}^\infty \frac{1}{n^p}$ $\quad$ converges $p>1$
diverges $p \le 1$

$\displaystyle\sum_{n=1}^\infty \frac{1}{n^p}$ $\qquad f(x) = \dfrac{1}{x^p}$

Test 4: Comparison.

If $0 \le a_n \le b_n$, then

$$\sum_{n=1}^{\infty} a_n \text{ diverges} \Longrightarrow \sum_{n=1}^{\infty} b_n \text{ diverges.}$$

$$\sum_{n=1}^{\infty} b_n \text{ converges} \Longrightarrow \sum_{n=1}^{\infty} a_n \text{ converges.}$$

ex. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+n^3+n^2}$

ex $\displaystyle\sum_{n=1}^{\infty} \frac{6n^2+1}{2n^3-1}$

Test 5: Limit Comparison Test.

Say $a_n, b_n > 0$. If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c \neq 0, \infty$ (samesies)

then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges $\Longleftrightarrow$ $\displaystyle\sum_{n=1}^{\infty} b_n$ converges.

both conv. or both diverge.