5. [10 points]

a. [5 points] Determine the **radius** of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^{4n}}{n^5 (16)^n}$$

notice the 4!

$$1 > \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(x-5)^{4(n+1)}}{(n+1)^5 (16)^{n+1}}}{\frac{(-1)^n (x-5)^{4n}}{n^5 (16)^n}} \right| = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^5 \frac{(16)^n}{(16)^{n+1}} \frac{|x-5|^{4n+4}}{|x-5|^{4n}}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^5 \left(\frac{1}{16}\right) |x-5|^4 = \frac{1}{16}|x-5|^4$$

$$\Longrightarrow \frac{|x-5|^4}{16} < 1 \Longrightarrow |x-5|^4 < \underset{2^4}{16} \Longrightarrow |x-5| < 2$$

The radius of convergence is 2.

b. [5 points] The power series $\sum_{n=0}^{\infty} \frac{(n+2)x^n}{n^4+1}$ has radius of convergence 1. Determine the **interval** of convergence for this power series.

Just need to check the endpoints. Center is 0, so check $x = \pm 1$.

$x = 1$: $\sum_{n=0}^{\infty} \frac{(n+2)}{n^4+1}$

let's use LCT with $\frac{1}{n^3}$, note $\frac{n+2}{n^4+1} \geq 0$.

1. $\lim_{n\to\infty} \frac{n^3 (n+2)}{n^4+1} = 1 \neq 0, \infty$.
2. $\sum_{n=0}^{\infty} \frac{1}{n^3}$ converges by p-test.
3. $\sum_{n=0}^{\infty} \frac{n+2}{n^4+1}$ converges by LCT.

$x = -1$: $\sum_{n=0}^{\infty} \frac{(n+2)(-1)^n}{n^4+1}$

Since $\left|\frac{(n+2)(-1)^n}{n^4+1}\right| = \frac{(n+2)}{n^4+1}$ and $\sum_{n=0}^{\infty} \frac{(n+2)}{n^4+1}$ converges (see)

Then $\sum_{n=0}^{\infty} \frac{(n+2)(-1)^n}{n^4+1}$ converges by ~~alter~~ absolute convergence test.

The interval of convergence is $[-1, 1]$.