8. [7 points] Consider the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n n}$.

a. [2 points] At which x-value is the interval of convergence of this power series centered?

$$x = -2.$$

b. [5 points] The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n n}$ is 3. Find the interval of convergence for this power series. Thoroughly justify your answer.

$x = 1$ $\quad \sum_{n=1}^{\infty} \frac{3^n}{3^n n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-test

$x = -5$ $\quad \sum_{n=1}^{\infty} \frac{(-3)^n}{3^n n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$a_n = \frac{1}{n}$

1. $\frac{1}{n} > \frac{1}{n+1}$

2. $\lim_{n\to\infty} \frac{1}{n} = 0$

So, this converges by alternating series test.

$$\boxed{IOC = [-5, 1)}$$

9. [5 points] Find the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$

$$1 > \lim_{n\to\infty} \left| \frac{\frac{(2(n+1))!\, x^{2(n+1)}}{(n+1)!^2}}{\frac{(2n)!\, x^{2n}}{(n!)^2}} \right| = \lim_{n\to\infty} \frac{(2n+2)!}{(2n)!} \left(\frac{n!}{(n+1)!}\right)^2 |x|^2$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)^2} |x|^2$$

$$= 4|x|^2$$

So, $4|x|^2 < 1$

$|x|^2 < \frac{1}{4}$

$|x| < \frac{1}{2}.$

$$\boxed{\text{Therefore, } R = \frac{1}{2}}$$