Math 116 — Practice for Exam 3

Generated October 24, 2017

Name: _____

Instructor: _____ Section Number: _____

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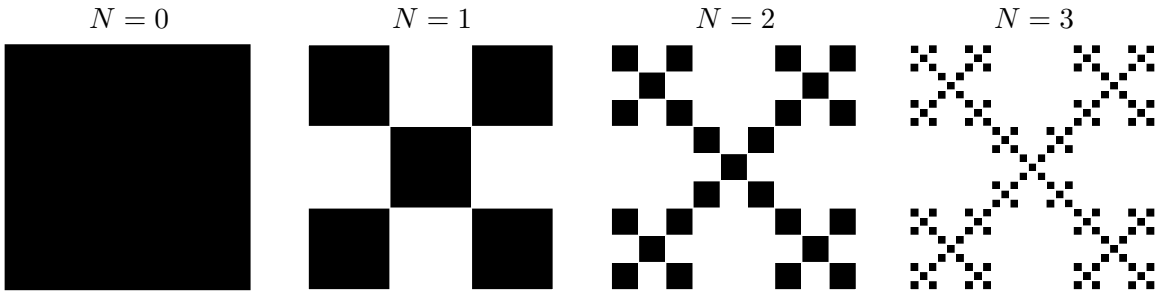
1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2015	3	11	checkers	12	
Fall 2014	3	4	robot army	8	
Winter 2014	3	2	pool	10	
Total				30	

Recommended time (based on points): 36 minutes

11. [12 points] You construct a snowflake by starting with a square piece of paper of side length 3 inches. You divide the square into a three by three grid of squares of side length one and remove the four squares in the grid that share a side with the center square in the grid. For each remaining square in the grid, subdivide each of them into 9 equally sized squares and remove the four squares in each of these new grids that share a side with the center square in the grid. You continue in this manner for a long time.



a. [3 points] Write a formula that gives the perimeter, P_N, of the black squares that make up the snowflake after N steps.

b. [2 points] Find $\lim_{N\to\infty} P_N$.

c. [3 points] Suppose $N \geq 1$. Write a sum that gives the area, A_N of all the squares you have **removed** after N steps.

d. [2 points] Write a closed form expression for A_N.

e. [2 points] Find the limit as $N \to \infty$ of your expression in (d).

4. [8 points] Franklin's robots start building more robots to replace their deactivated comrades. The initial number of robots in Franklin's army is 800. Each minute, the number of robots increases by 15%. At the end of each minute, you fire an EMP which immediately deactivates 50 robots.

a. [3 points] Let R_n denote the number of active robots in Franklin's army immediately after the EMP is fired for the n-th time. Find R_1 and R_2.

b. [4 points] Find a closed form expression for R_n (i.e. evaluate any sums and solve any recursion).

c. [1 point] Find $\lim_{n\to\infty} R_n$. No justification is necessary.

2. [10 points] Consider an outdoor pool initially filled with 20,000 gallons of water. Each day 4% of the water in the pool evaporates. Each morning at 10:00am, W gallons of water are added back to the pool where W is a constant.

a. [3 points] Let A_n be the number of gallons of water in the pool immediately after water is added back to the pool for the nth time. Given that $A_1 = 19200 + W$, find A_2 and A_3. Put your final answers in the answer blanks.

$A_2 =$ _____

$A_3 =$ _____

b. [4 points] Find a closed form expression for A_n (i.e. evaluate any sums and solve any recursion). Note your answer may contain the constant W.

c. [3 points] If the pool has a maximum capacity of 25,000 gallons, find the largest value of W so that the pool does not overflow eventually.