Math 116 — Practice for Exam 3

Generated December 14, 2015

Name: ______________________________

Instructor: ____________________ Section Number: __________

1. This exam has 8 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2014	3	11		9	
Fall 2014	3	6		8	
Winter 2014	3	5		12	
Fall 2013	3	3		8	
Winter 2013	3	1		10	
Fall 2012	3	1		12	
Winter 2012	3	1		12	
Fall 2011	3	1		10	
Total				81	

Recommended time (based on points): 97 minutes

11. [9 points] Circle all true statements.

a. [3 points] The integral $\displaystyle\int_0^1 \frac{1}{\sin(x)}dx$

I. converges by the comparison test because $\dfrac{1}{\sin(x)} \leq C$ for some constant C for $0 < x \leq 1$ and $\displaystyle\int_0^1 C dx$ converges.

II. diverges by the comparison test because $\dfrac{1}{\sin(x)} \geq \dfrac{1}{x}$ for $0 < x \leq 1$ and $\displaystyle\int_0^1 \frac{1}{x}dx$ diverges.

III. diverges because $\displaystyle\lim_{x\to 0} \frac{1}{\sin(x)} \neq 0$.

IV. converges by the alternating series test because the values of $\sin(x)$ oscillate between -1 and 1.

b. [3 points] The series $\displaystyle\sum_{n=0}^{\infty} \frac{e^{n^2}}{n!}$

I. converges because $\displaystyle\lim_{n\to\infty} \frac{e^{n^2}}{n!} = 0$.

II. converges because factorials grow faster than exponential functions.

III. diverges by the ratio test.

IV. diverges by the comparison test because $\dfrac{e^{n^2}}{n!} \geq e^n$ for $n = 0, 1, 2, 3, ...$ and $\displaystyle\sum_{n=0}^{\infty} e^n$ diverges.

c. [3 points] The differential equation $\dfrac{dy}{dt} = t(y-2)(\ln(y))$ defined for $t > 0$ and $y > 0$ has

I. an unstable equilibrium solution at $t = 0$.

II. a stable equilibrium solution at $y = 2$.

III. a stable equilibrium solution at $y = 1$.

IV. an unstable equilibrium solution at $y = 2$.

6. [8 points] Suppose that $f(x)$, $g(x)$, $h(x)$ and $k(x)$ are all positive, differentiable functions. Suppose that

$$0 < f(x) < \frac{1}{x} < g(x) < \frac{1}{x^2}$$

for all $0 < x < 1$, and that

$$0 < h(x) < \frac{1}{x^2} < k(x) < \frac{1}{x}$$

for $x > 1$. Determine whether the following statements are always, sometimes or never true by circling the appropriate answer. No justification is necessary.

a. [2 points] $\displaystyle\int_0^1 g(x)dx$ converges.

Always **Sometimes** **Never**

b. [2 points] $\displaystyle\int_0^1 f(x)dx$ diverges.

Always **Sometimes** **Never**

c. [2 points] $\displaystyle\sum_{n=1}^{\infty} h(n)$ diverges.

Always **Sometimes** **Never**

d. [2 points] $\displaystyle\sum_{n=1}^{\infty} k(n)$ converges.

Always **Sometimes** **Never**

5. [12 points] For each of the following statements, circle True if the statement is always true and circle False otherwise. No justification is necessary.

a. [2 points] Suppose that an object has constant density δ and center of mass $(\bar{x}, \bar{y}, \bar{z})$. If the density of the object is doubled to 2δ then the center of mass changes to $(2\bar{x}, 2\bar{y}, 2\bar{z})$.

True False

b. [2 points] Every solution of the differential equation $y' = y$ is increasing.

True False

c. [2 points] If $f(x)$ is a continuous function and $F(x)$ is an antiderivative of $f(x)$, then $F(x) = \int_3^x f(t)dt + K$ for some constant K.

True False

d. [2 points] If $g(x) = \displaystyle\int_{-e^x}^{e^x} t^2\,dt$ and $h(x) = \int_0^{2x} e^{t^2}\,dt$ then $g'(x) \leq h'(x)$ for all $x > 1$.

True False

e. [2 points] If $w(x)$ is a positive continuous function and the series $\sum_{n=1}^{\infty} w(n)$ converges then the integral $\int_1^{\infty} w(x)\,dx$ must also converge.

True False

f. [2 points] Suppose that a_n is a decreasing sequence and $0 \leq a_n \leq 1$ then $b_n = \cos(a_n)$ is a convergent sequence.

True False

3. [8 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] The equation $y^3 - x^3 = xy$ in Cartesian coordinates can be written in polar coordinates as

$$r = \frac{\sin\theta\cos\theta}{\sin^3\theta - \cos^3\theta}.$$

True False

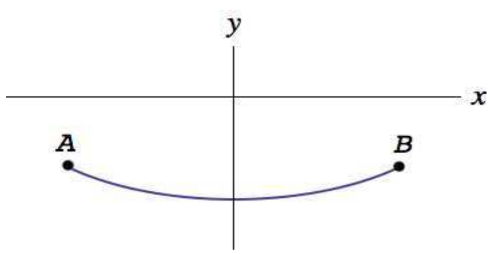
b. [2 points] If $g(x) = \int_1^x f(t)dt$, then $g(4) - g(2) = \int_2^4 f(t)dt$.

True False

c. [2 points] The function $h(x) = \int_0^{\sin x} e^{-t^2} dt$ has a local maximum at $x = \frac{\pi}{2}$.

True False

d. [2 points] The graph of the parametric equations $x = f(t)$ and $y = f'(t)$ for some function $f(t)$ is shown below:



As t increases, the curve is traced from A to B.

True False

1. [10 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let $-1 < q < 1$, then

$$\sum_{n=1}^{\infty} q^n = q + q^2 + q^3 + \cdots + q^n + \cdots = \frac{q}{1-q}.$$

True False

b. [2 points] Let $F(t)$ be an antiderivative of a continuous function $f(t)$. If the units of $f(t)$ are meters and t is in seconds, then the units of $F(t)$ are meters per second.

True False

c. [2 points] If the motion of a particle is given by the parametric equations

$$x = \frac{at}{1+t^3}, \quad y = \frac{at^2}{1+t^3} \qquad \text{for} \quad a > 0,$$

then the particle approaches the origin as t goes to infinity.

True False

d. [2 points] Let a_n be a sequence of positive numbers satisfying $\lim_{n\to\infty} a_n = \infty$. Then the series $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

True False

e. [2 points] Let $f(x)$ be a continuous function. Then

$$\int_0^1 f(2x)dx = \frac{1}{2}\int_0^1 f(x)dx.$$

True False

1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] In polar coordinates, $(r_1, \theta_1) = (2, \frac{\pi}{5})$ and $(r_2, \theta_2) = (-2, -\frac{4\pi}{5})$ represent the same point.

True False

b. [2 points] If a particle moves according to the parametric equations $x(t) = t^3 + t^2$ and $y(t) = t^4$, then the particle has speed zero at the origin.

True False

c. [2 points] The Taylor series for $f(x) = \sqrt{1+2x}$ centered about $x = 0$ converges for $-1 < x < 1$.

True False

d. [2 points] If $P(t)$ is a cumulative distribution function, then the sequence $x_n = P(n)$ converges.

True False

e. [2 points] The sequence $a_n = \displaystyle\int_{\frac{1}{n}}^{1} \frac{1}{x^3}\, dx$ converges.

True False

f. [2 points] The function $F(x) = \displaystyle\int_{1}^{x^2} \sin(t^2) dt$ is an even function.

True False

1. [12 points] Indicate whether each of the following statements are true or false by circling the correct answer. **You do not need to justify your answers.**

a. [2 points] The curve defined by the parametric equations $x = 1 - \cos t$ and $y = t - \sin t$ has a vertical tangent line when $t = \pi$.

True False

b. [2 points] If the sequence a_n converges to 0 and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty}(a_n + b_n)$ converges.

True False

c. [2 points] The graph of a polar function $r = f(\theta)$ in the (x, y)-plane has a horizontal tangent line at $\theta = a$ if $f'(a) = 0$.

True False

d. [2 points] The integral $\int_0^1 \pi x^4 dx$ computes the volume of the solid obtained by rotating the graph of $y = x^2$ around the x axis for $0 \le x \le 1$.

True False

e. [2 points] Let $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1} x^n$ be the Taylor series of $f(x)$ about 0. Then $f(x)$ is concave up at $x = 0$.

True False

f. [2 points] The integral test says that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \int_1^{\infty} \frac{1}{x^2} dx$.

True False

1. [10 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let a_n be a sequence of positive numbers. If $a_n \leq \frac{7^n}{2^{3n}-1}$ for all values of $n \geq 1$, then a_n must converge.

True False

b. [2 points] The trapezoid rule is guaranteed to give an underestimate of $\int_{-\pi}^{\pi} \cos t dt$.

True False

c. [2 points] If the area A under the graph of a positive continuous function $f(x)$ is infinite, then the volume of the solid generated by rotating A around the x-axis could be either infinite or finite depending on the function $f(x)$.

True False

d. [2 points] If $H(x) = \int_0^x f(t)g(t)dt$, then $H'(x) = f'(x)g(x) + f(x)g'(x)$.

True False

e. [2 points] If $(x(t), y(t))$ gives a parametrization of the unit circle centered at the origin, then $\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 2\pi$.

True False