Math 116 — Practice for Exam 1

Generated October 4, 2015

Name: SOLUTIONS

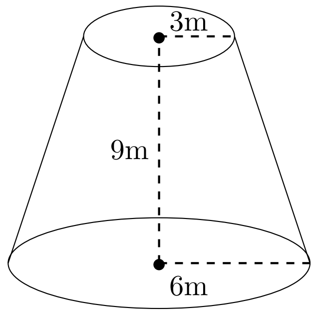
Instructor: ______________________ Section Number: __________

1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2014	1	9	hot chocolate	12	
Winter 2013	1	6	swimming pool	11	
Fall 2010	1	7	bio willie	12	
Total				35	

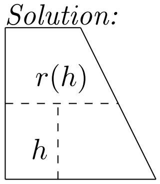
Recommended time (based on points): 32 minutes

9. [12 points] The Nub's Nob Ski Area keeps a massive supply of hot chocolate. The hot chocolate is stored in a container shaped like a cone with the point end removed as shown below. The height of the container is 9 meters, and it has lower radius 6 meters and upper radius 3 meters. The hot chocolate has a density of 3000 kg/m^3. Recall the gravitational constant is $g = 9.8\text{m/s}^2$.



a. [3 points] Write a formula for $r(h)$, the radius of a circular cross section of the container h meters above the base.

Solution:



Looking at a vertical cross section of the cone we see that $r(h)$ is the width of a trapezoid at height h. The width of the trapezoid is decreasingly linearly thus $r(h)$ must be a linear function with $r(0) = 6$ and $r(9) = 3$. Therefore $r(h) = 3 + \frac{3(9-h)}{9} = 6 - h/3$.

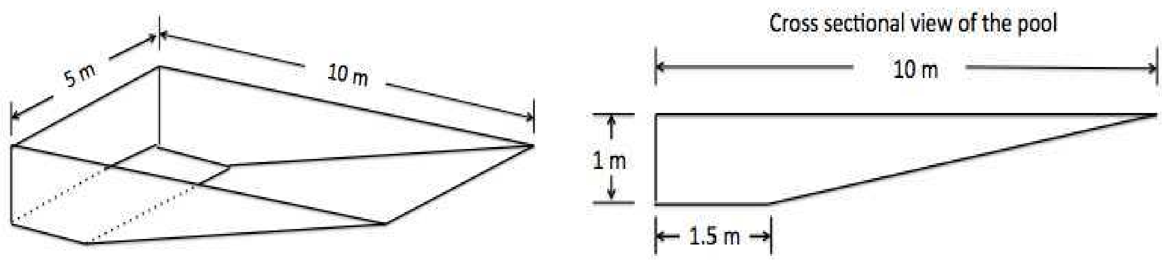
b. [6 points] Write a formula in terms of $r(h)$ for the work required to lift a slice of hot chocolate of thickness Δh from height h to the top of the container.

Solution: The mass of the slice is $3000\pi r(h)^2\Delta h$. The slice must be lifted $9 - h$ meters. Therefore the work to lift the slice is $3000g\pi r(h)^2(9 - h)\Delta h$.

c. [3 points] Write an integral that gives the work required to lift all of the hot chocolate to the top of the container. Do not evaluate this integral.

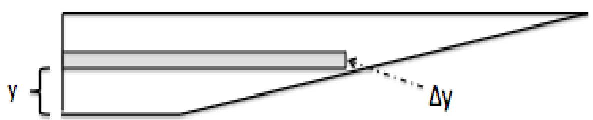
Solution: Integrating the above function from 0 to 9 the work is $\int_0^9 3000g\pi r(h)^2(9-h)dh$

6. [11 points] A swimming pool 10 m long and 5 m wide has varying depth. Its maximum depth is 1 m as shown in the picture below



The swimming pool has water up to a level of maximum depth of 0.6 m. The density of water is 1000 kg per m^3. Use $g = 9.8$ m/s^2 for the acceleration due to gravity.

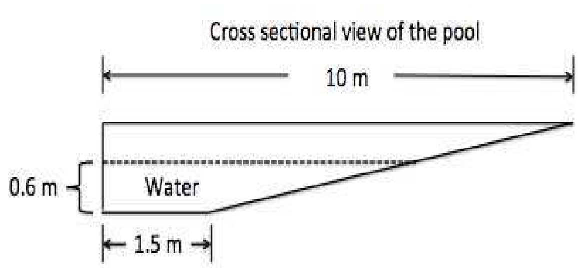
a. [9 points] Write an expression that approximates the work done in lifting a horizontal slice of water with thickness Δy meters, that is at a distance of y meters above the bottom, to the top of the swimming pool.



Solution: First we must find a formula for the length of the swimming pool at depth for a given height above the bottom. Let's call this function $l(y)$. We know that $l(0) = 1.5$ and $l(1) = 10$. Since $l(y)$ is a linear function, this tells us that $l(y) = 8.5y + 1.5$.
The volume of such a slice is $\Delta y(8.5y+1.5)\cdot 5$. Multiplying by 1000 kg/m^3 and 9.8 m/s^2 gives us the weight of the water in Newtowns. The amount the water needs to be lifted is $(1 - y)$. We therefore get:

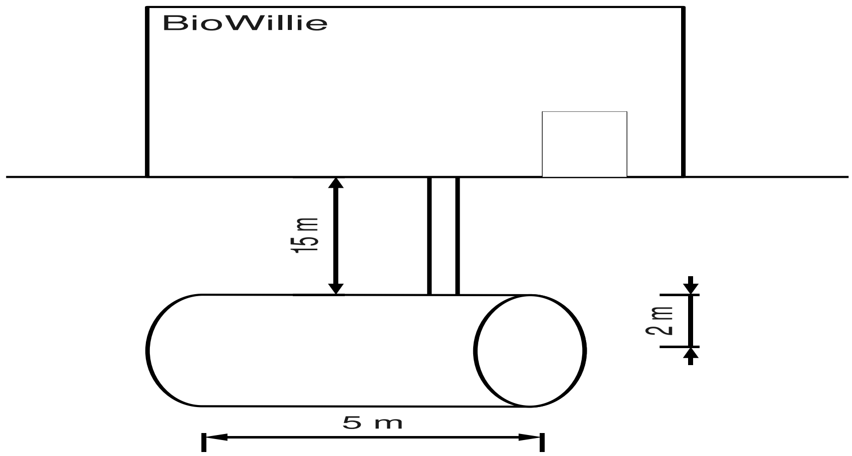
$$W_{slice} \approx 1000 \cdot 9.8 \cdot (8.5y + 1.5) \cdot 5 \cdot (1 - y)\Delta y.$$

b. [2 points] Write a definite integral that computes the work required to pump all the water to the top of the pool.



Solution: Work $= \displaystyle\int_0^{0.6} 1000 \cdot 9.8 \cdot 5(8.5y + 1.5)(1 - y)dy$ Joules.

7. [12 points] Country music legend Willie Nelson is concerned about our dependence of fossil fuels. In 2005, he started a company which sells a bio-diesel fuel called BioWillie. He recently added a new cylindrical underground storage tank at his factory, and he needs to know how much work is required to pump all the fuel in a full tank to the surface. The tank is pictured below. It is 5 meters long and has a radius of 2 meters. Its center line is 17 meters underground. BioWillie fuel has a density of 900 kg per cubic meter. Make sure to include appropriate units and justification to support your answers.



a. [7 points] Write an expression that approximates the work done in lifting a horizontal slice of fuel that is h_i meters below the ground's surface, given that the thickness of the slice is Δh meters.

Solution: Using the ground's surface as our horizontal axis and h be the variable on the vertical axis, then the cross section of the tank can be described by $x^2+(h+17)^2 = 4$.

Volume$_{slice} = 2\sqrt{4-(h+17)^2}(5)\Delta h\ m^3$.

Force$_{slice}$ = (density)(Volume$_{slice}$)g Newtons

Distance= $h_i = -h$ m.

Work$_{slice}$ =(Force)(Distance)= $(900)(2\sqrt{4-(h+17)^2}(5)\Delta h)g(-h)$ Joules.

b. [5 points] Help Willie Nelson by computing the work required to pump all the fuel in a full tank to the ground's surface. You can use your calculator to compute your final answer.

Solution: $Work = \int_{-19}^{-15} -9000\sqrt{4-(h+17)^2}ghdh = 9,421,008.05$ Joules.