Math 116 — Practice for Exam 2

Generated October 27, 2015

Name: ____________________

Instructor: ____________________ Section Number: ________

1. This exam has 6 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2011	2	4	ventilation	11	
Winter 2015	2	3		4	
Winter 2012	2	6	parachute	8	
Fall 2012	2	4	bees	11	
Fall 2013	2	7	termites	8	
Winter 2014	2	1		10	
Total				52	

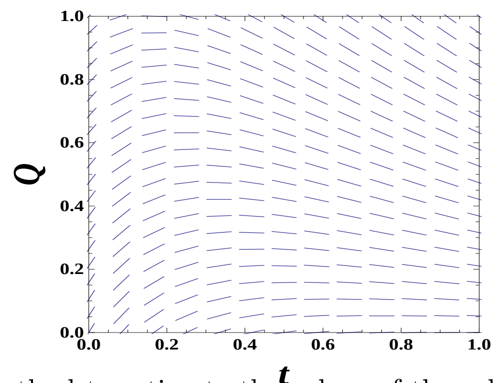
Recommended time (based on points): 47 minutes

4. [11 points] A restaurant installs a kitchen ventilation system to control the amount of grease in the air due to cooking. The ventilation system reduces the amount of grease in the air by 90 percent every hour. Let $Q(t)$ be the amount in grams of grease in the air t hours after the ventilation is activated. Then Q satisfies the differential equation

$$\frac{dQ}{dt} = 2e^{-5t} - \frac{9}{10}Q,$$

where $2e^{-5t}$ is the rate at which the kitchen produces grease in grams per hour at time t.

a. [2 points] The slope field of the differential equation is shown below. Suppose that the air in the kitchen initially has 0.4 grams of grease. Sketch the solution curve in the slope field.

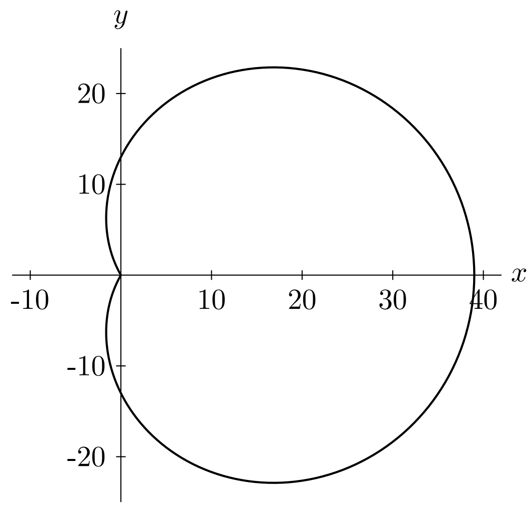


b. [7 points] Use Euler's method to estimate the values of the solution curve $Q(t)$ through $(0, 0.4)$ for all values of t given in the table below. Show all your work.

t	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$Q(t)$				

c. [2 points] Does your approximation for $Q(1)$ using Euler's method give an overestimate or an underestimate? Justify.

2. [6 points] Your friend the goliath frog is going to decorate the boundary of his lily pad with a string of tiny flowers. The boundary of the lily pad is given by a portion of the curve $r = 13 + 26\cos(\theta)$ where r is measured in inches and θ is measured in radians. The part of the curve that traces out the lily pad is shown below in the xy-plane.



If the goliath frog is going to decorate only the part of the boundary of the lily pad for which $x \leq 0$, write an expression involving integrals for the length of the string of flowers required. Do not evaluate your integral.

3. [4 points] We can approximate the value of $\ln(1.5)$ by using the fact that $y = \ln(x)$ solves the differential equation

$$\frac{dy}{dx} = \frac{1}{x}$$

Approximate $\ln(1.5)$ by using Euler's method for the differential equation above with initial condition $y(1) = 0$ and with $\Delta x = 0.25$. Fill in the table with the y-values obtained at each step.

x	y
1.00	
1.25	
1.50	

Thus, $\ln(1.5) \approx$ __________

6. [8 points] A box is dropped from an airplane. The downward velocity $v(t)$ of the box , once its parachute opens, satisfies the differential equation

$$\frac{dv}{dt} = 10 - \frac{1}{10}(1 + e^{-t})v^2.$$

Suppose the parachute opens when the velocity of the box is 11 m/s. Use Euler's method with three steps to approximate the velocity of the box one second after the parachute opens. Fill in the table below with the approximations at each step. Be sure to include all your work to receive full credit.

t	0			
$v(t)$				

4. [11 points] The function $P(t)$ models the number of bees (in thousands) in a colony at time t (in years). Suppose the function $P(t)$ satisfies the differential equation

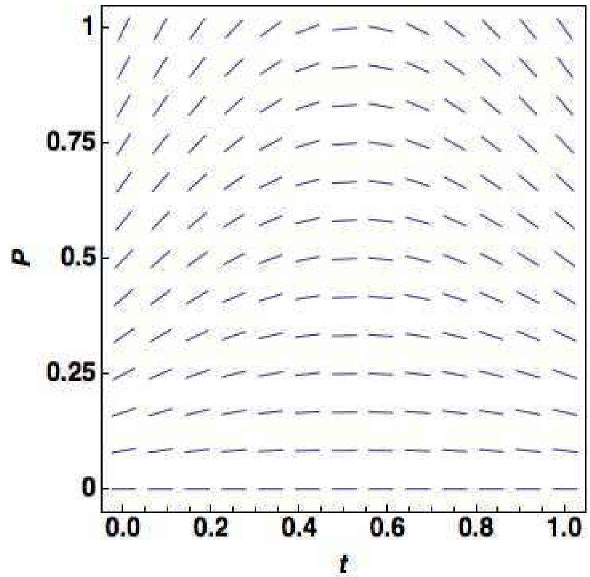
$$\frac{dP}{dt} = 2(1 - 2\sin t)P.$$

The colony initially has 500 bees.

a. [6 points] Use Euler's method, with three steps, to find the approximate number of bees (in thousands) in the farm after one year. Fill in the table with the appropriate values of t and your approximations.

t (in years)	0			1
$P(t)$ (in thousands)				

b. [1 point] The slope field of the differential equation $\frac{dP}{dt} = 2(1 - 2\sin t)P$ is shown below. Use it to sketch the graph of $P(t)$, the number of bees (in thousands) in the colony after t years.



c. [2 points] Use the differential equation $\frac{dP}{dt} = 2(1 - 2\sin t)P$ to find the exact value of t during the first year at which the number of bees in the colony has a maximum.

d. [2 points] Does the approximation of $P(1)$ obtained with Euler's method in (a) guarantee an underestimate, an overestimate or neither? Justify without solving the differential equation.

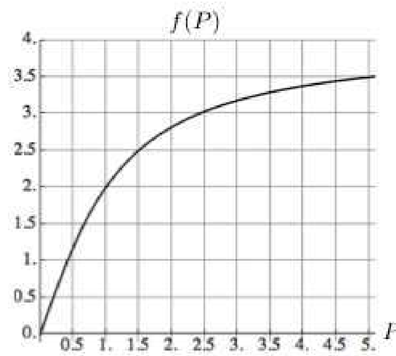
7. [8 points] The population of termites $P(t)$ (in thousands) in a tree grows at a rate $f(P)$, in thousands of termites per day. A pesticide is applied to the tree to eliminate the termites. As a result, the population of termites $P(t)$ satisfies

$$\frac{dP}{dt} = f(P) - 3e^{-\frac{1}{3}t},$$

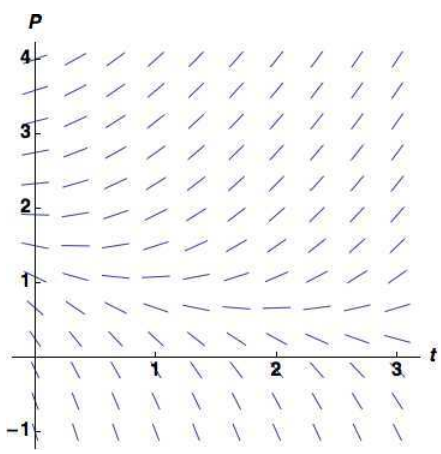
where t is measured in days since the pesticide is applied.

a. [4 points] Use Euler's method with steps of $\Delta t = 0.5$ to estimate the amount of termites in the tree one day after the pesticide is applied. It is estimated that there are 2500 termites in the tree at the time the pesticide is applied ($t = 0$). The graph of $f(P)$ is given below. Show all your computations.

t	0	0.5	1
$P(t)$			



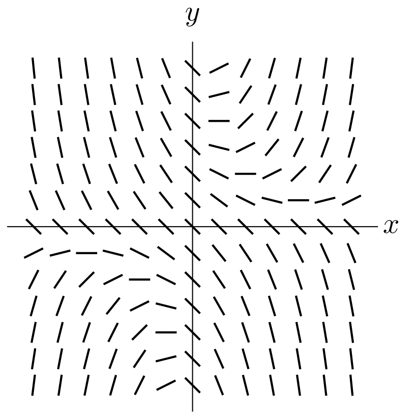
b. [2 points] The slope field of the differential equation satisfied by $P(t)$ is shown below. Sketch the graph of $P(t)$.



c. [2 points] Is the estimate obtained in part (**a**) guaranteed to be an overestimate or an underestimate? Justify.

1. [10 points] Consider the differential equation $y' = xy - 1$.

a. [2 points] The slope field of $y' = xy - 1$ is shown below. On the graph, sketch a solution curve passing through the point $(0, 0)$.



b. [5 points] Starting with the initial condition $y(0) = 0$, use Euler's method with 3 steps to estimate $y(3/2)$. Show your work for each step.

c. [3 points] Can you determine if your estimate of $y(3/2)$ is an underestimate or overestimate? Circle your answer and **explain** your reasoning in one sentence.

Underestimate **Overestimate** **Not enough information**