2. [18 points] For each of the following series, write whether the series "Converges" or "Diverges" on the space provided next to the series. Support your answer by stating the test(s) you used to prove convergence or divergence, and show complete work and justification.

a. [6 points] $\sum_{n=2}^{\infty} \frac{\sqrt{n^2+1}}{n^2-1}$ +1 diverges

+1 $0 \le \frac{\sqrt{n^2+1}}{n^2-1}$ when $n \ge 2$, so use +1 limit comparison test with $\frac{1}{n}$.

$\lim_{n\to\infty} \frac{n\sqrt{n^2+1}}{n^2-1} \overset{+1}{=} 1 \ne 0, \infty$. Then $\sum_{n=2}^{\infty} \frac{1}{n}$ +1 diverges by p-test.

Hence, the series +1 diverges by limit comparison test.

b. [6 points] $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(2n)!}$ +1 converges.

+1 $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty}$ +1 $\frac{(n+1)!\,(n+2)!}{(2n+2)!} \frac{(2n)!}{(n+1)!\,n!} = \lim_{n\to\infty} \frac{(n+1)(n+2)}{(2n+2)(2n+1)}$ +1 $= \frac{1}{4}$ +1 < 1.

Therefore converges by ratio test.
+1

c. [6 points] $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2-3}$ +1 converges.

More than 6 points, but you would probably get subtracted points for missing steps.

$0 \le |\sin(n)| \le 1$, so +1 $0 \le \frac{|\sin(n)|}{n^2-3} \le \frac{1}{n^2-3}$,

$\lim_{n\to\infty} \frac{n^2}{n^2-3} \overset{+1}{=} 1 \ne 0, \infty$, and $\sum_{n=2}^{\infty} \frac{1}{n^2}$ +1 converges by p-test, so $\sum_{n=2}^{\infty} \frac{1}{n^2-3}$

+1 converges by limit comparison test, so $\sum_{n=2}^{\infty} \frac{|\sin(n)|}{n^2-3}$ converges by comparison,
+1

so $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2-3}$ converges by absolute convergence test.

+1