SOLUTIONS BY TREVOR HYDE.

7. [12 points] Determine whether the following series converge or diverge (circle your answer). Be sure to mention which tests you used to justify your answer. If you use the comparison test or limit comparison test, write an appropriate comparison function.

a. [3 points] $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{1+2\sqrt{n}}$

$\displaystyle\lim_{n\to\infty}(-1)^n\frac{\sqrt{n}}{1+2\sqrt{n}}$ +1 does not exist, hence the series diverges +1 (b/c $(-1)^n$) by the n^{th} term test. +1

b. [4 points] $\displaystyle\sum_{n=1}^{\infty}ne^{-n^2}$

$0 \le n e^{-n^2}$ +1, so one can use integral comparison test.

$$\int_1^{\infty} x e^{-x^2}\,dx = \frac{1}{2}\int_1^{\infty} e^{-u}\,du = \frac{1}{2}\lim_{b\to\infty}\int_1^{b} e^{-u}\,du \quad +1$$

$u = x^2 \quad du = 2x\,dx$

$$= \frac{1}{2}\lim_{b\to\infty}\frac{1}{e}-\frac{1}{e^b}=\frac{1}{e},$$ hence converges.

+1 Therefore, the series converges by integral comparison test. +1

c. [5 points] $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n^2)}{n^2}$

$$0 \le \overbrace{\frac{|\cos(n^2)|}{n^2} \le \frac{1}{n^2}}^{+1}$$

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by p-test. +1

$\displaystyle\sum_{n=1}^{\infty}\frac{|\cos(n)|}{n^2}$ converges by comparison. +1 Therefore the series converges by absolute convergence test. +1 +1