Math 116 — Practice for Exam 1

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Name: SOLUTIONS

Instructor: ______________________ Section Number: ________

1. This exam has 1 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

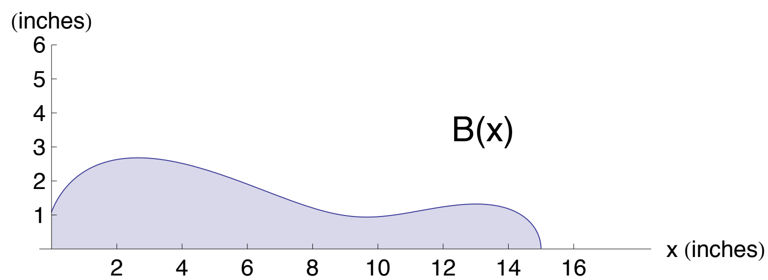
Semester	Exam	Problem	Name	Points	Score
Fall 2011	1	5	bowling pin	11	
Total				11	

Recommended time (based on points): 10 minutes

5. [11 points] During a friendly game of ten-pin bowling, your friends Walter and Smokey begin to quarrel over whether Smokey's toe slipped over the foul line. Meanwhile, you decide to pass the time by finding a mathematical model for the shape of a bowling pin. After some careful thought, you find that a fallen pin is a solid of revolution given by rotating the region under the curve

$$B(x) = \sqrt{1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4}$$

over the interval $[0, 15]$ about the x-axis. The region is pictured below. All measurements are in inches. A helpful stranger in the bowling alley informs you that the wood used to make the pin has density $\delta = 17$ grams per cubic inch.



a. [3 points] Write a definite integral that gives the mass of the bowling pin. You do not need to evaluate this integral.

Solution: Since the bowling pin is a solid of revolution, the volume of a cylindrical slice located x inches from the base of the pin can be approximated by $\pi B(x)^2 \Delta x$. Thus, the mass of the slice is approximately

$$\delta \pi B(x)^2 \Delta x = 17\pi \left(\sqrt{1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4}\right)^2 \Delta x,$$

so that the mass of the entire pin is

$$\int_0^{15} 17\pi B(x)^2 dx = \int_0^{15} 17\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx \text{ grams.}$$

b. [6 points] What are the coordinates $(\bar{x}, \bar{y})$ of the bowling pin's center of mass? You may use your calculator to answer this question.

Solution: Since the bowling pin has uniform density, we know immediately from rotational symmetry that $\bar{y} = 0$. Using the formula for the x-coordinate of the center of mass, we have

$$\bar{x} = \frac{\int_0^{15} \delta x \pi B(x)^2 dx}{\int_0^{15} \delta \pi B(x)^2 dx} = \frac{\int_0^{15} 17x\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx}{\int_0^{15} 17\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx} = 5.$$

Thus, the coordinates of the center of mass of the bowling pin are $(5, 0)$, where each coordinate is measured in inches.

c. [2 points] Suppose the wood used to make the pin had density $\delta = 16$ grams per cubic inch. How does this affect the position $(\bar{x}, \bar{y})$ of the center of mass?

Solution: The center of mass is not affected since the integral

$$\bar{x} = \frac{\int_0^{15} \delta x \pi B(x)^2 dx}{\int_0^{15} \delta \pi B(x)^2 dx} = \frac{\int_0^{15} x \pi B(x)^2 dx}{\int_0^{15} \pi B(x)^2 dx}$$

is independent of δ. The same is true for $\bar{y}$.