An Introduction to the Theory of Mechanism Design

Tilman Börgers

With a chapter by

Daniel Krähmer and Roland Strausz
The authors will donate all payments that they receive from the publisher for this book to *Amnesty International.*
CONTENTS

Preface xi
Acknowledgments xv

1. Introduction 1

2. Screening 6
  2.1. Introduction 6
  2.2. Pricing a Single Indivisible Good 6
  2.3. Nonlinear Pricing 18
  2.4. Bundling 27
  2.5. Remarks on the Literature 29
  2.6. Problems 29

3. Bayesian Mechanism Design: Examples 31
  3.1. Introduction 31
  3.2. Single Unit Auctions 31
    3.2.1. Setup 31
    3.2.2. Mechanisms, Direct Mechanisms, and the Revelation Principle 33
    3.2.3. Characterizing Incentive Compatibility and Individual Rationality 36
    3.2.4. Expected Revenue Maximization 39
    3.2.5. Welfare Maximization 42
    3.2.6. Numerical Examples 43
  3.3. Public Goods 45
    3.3.1. Setup 45
    3.3.2. Incentive- and Individually Rational Direct Mechanisms 46
    3.3.3. Ex Ante and Ex Post Budget Balance 47
    3.3.4. Welfare Maximization 50
    3.3.5. Profit Maximization 57
    3.3.6. A Numerical Example 58
  3.4. Bilateral Trade 63
    3.4.1. Setup 63
    3.4.2. Direct Mechanisms 63
    3.4.3. Welfare Maximization 65
3.4.4. Profit Maximization 72
3.4.5. A Numerical Example 73
3.5. Remarks on the Literature 74
3.6. Problems 75

4. Dominant Strategy Mechanisms: Examples 76
4.1. Introduction 76
4.2. Single Unit Auctions 78
4.2.1. Setup 78
4.2.3. Characterizing Dominant Strategy Incentive Compatibility and Ex Post Individual Rationality 80
4.2.4. Canonical Auctions 81
4.3. Public Goods 84
4.3.1. Setup 84
4.3.2. Direct Mechanisms 85
4.3.3. Characterizing Dominant Strategy Incentive Compatibility and Ex Post Individual Rationality 85
4.3.4. Canonical Mechanisms 87
4.3.5. Ex Post Exact Budget Balance 88
4.4. Bilateral Trade 90
4.4.1. Setup 90
4.4.2. Dominant Strategy Incentive-Compatible and Ex Post Individually Rational Direct Mechanisms 90
4.4.3. Canonical Mechanisms 92
4.4.4. Ex Post Exact Budget Balance 92
4.5. Remarks on the Literature 93
4.6. Problems 93

5. Incentive Compatibility 95
5.1. Introduction 95
5.2. Setup 95
5.3. Weak Monotonicity 96
5.4. Cyclical Monotonicity 99
5.5. Cyclical Monotonicity When Outcomes Are Lotteries 102
5.6. One-Dimensional Type Spaces 103
5.7. Rich Type Spaces 108
5.8. Revenue Equivalence 109
5.9. Individual Rationality 110
5.10. Remarks on the Literature 111
5.11. Problems 111
6. Bayesian Mechanism Design

6.1. Introduction

6.2. Setup

6.3. Independent Types

6.4. Correlated Types

6.4.1. Framework

6.4.2. Failure of Revenue Equivalence

6.4.3. Characterizing Bayesian Incentive Compatibility

6.4.4. A Numerical Example

6.4.5. Individual Rationality and Budget Balance

6.4.6. Discussion

6.5. Remarks on the Literature

6.6. Problems

7. Dominant Strategy Mechanisms

7.1. Introduction

7.2. Dominant Strategy Incentive Compatibility

7.3. Implementing Efficient Decision Rules

7.4. Positive Association of Differences

7.5. Individual Rationality and Budget Balance

7.6. Remarks on the Literature

7.7. Problems

8. Nontransferrable Utility

8.1. Introduction

8.2. The Gibbard–Satterthwaite Theorem

8.2.1. Setup

8.2.2. Statement of the Result and Outline of the Proof

8.2.3. Every Monotone Direct Mechanism Is Dictatorial

8.3. Dominant Strategy Incentive Compatibility on Restricted Domains

8.4. Bayesian Incentive Compatibility

8.5. Remarks on the Literature

8.6. Problems

9. Informational Interdependence

9.1. Introduction

9.2. An Example

9.3. Impossibility of Implementing Welfare-Maximizing Decision Rules

9.4. Characterizing All Incentive-Compatible Mechanisms

9.5. Remarks on the Literature

9.6. Problems
The first objective of this text is to give rigorous but accessible explanations of classic results in the theory of mechanism design. The second objective is to take the reader in selected areas to the frontiers of research. The description of recent areas of research is, by necessity, a little more subjective than the description of classic results. The reader may turn to recent literature in the field to learn about perspectives that are different from the authors’.

This book is meant for advanced undergraduate and graduate students of economics who have a good understanding of game theory. Fudenberg and Tirole (1993) contains more than the reader needs for this book. I shall also assume a basic knowledge of real analysis that can, for example, be acquired from Rudin (1976).

This book started out as lecture notes for a class that I taught at the University of Michigan in the Winter Semester 2006. At the time, as far as I was aware, no books on mechanism design existed. As I was working excruciatingly slowly on my text, several excellent books appeared that cover topics similar to mine. Whenever I discovered such a book, to maintain my own motivation, I needed to persuade myself that there are important differences between other authors’ books and my own. These excellent books, along with the justifications for the existence of my own book, are listed below in chronological order. I include them here, even though they are in some sense competitors of my own text, to serve the primary purpose of a book such as mine, which is to help others to obtain a comprehensive picture of the field. The following books, hopefully together with my own book, will be extremely useful as the reader embarks on this endeavor.


The theory of mechanism design was created by Leonid Hurwicz, who won the 2007 Nobel Prize for it, together with Eric Maskin and Roger Myerson. The focus of this text is on informational efficiency and privacy preservation in mechanisms. Incentive aspects play a much smaller role than they do in this book.
This book covers material similar to that of Hurwicz and Reiter. The emphasis that both books place on the size of the message space in a mechanism differentiates them from more modern treatments of mechanism design. However, as we shall discuss in this book, in particular in Chapter 10, the complexity or simplicity of mechanisms, one aspect of which is the size of the message space, seems to be of continuing importance and may be central for future research.

This book is closest to mine among those listed here, but it covers more than I do, such as the theory of Nash implementation, the theory of matching markets, and empirical evidence on mechanisms. Sometimes I wish I had written this book. My own book is more narrowly focused, perhaps goes somewhat into greater depth, and places a greater emphasis on the relation between game theoretic foundations and mechanism design.

This is a superb book, demonstrating how large parts of the theory of mechanism design can be developed as an application of results from linear programming. Vohra puts less emphasis than I do on the game theoretic aspects of mechanism design.

My comments on the books listed above already indicate some limitations of this text. Two more require emphasis. First, I have not covered the theory of implementation. I explain in the remainder of this paragraph what specifically I mean by this, but I note that this paragraph may be comprehensible to readers only after they have read this book. I understand the literature on implementation to be that part of the literature on mechanisms that requires the mechanism designer to consider all equilibria of the mechanism that she proposes, as opposed the literature on mechanism design, which allows the mechanism designer to select one among the equilibria of the mechanism that she proposes. This terminology is not universally used, but it will be useful in this book. Note that whenever we invoke the revelation principle, we are in the
realm of the theory of mechanism design rather than the theory of implementation. I mentioned already that the book by Diamantaras et al. cited above includes chapters on implementation. Extremely useful is also:


I have left out the subject of implementation, not because it would not be important, but because it has been explained so well by others and because it would require many techniques and arguments very different from the ones invoked in this book. Having said this, I continue to feel guilty for leaving out this subject.¹

I have also not covered the contributions to the theory of mechanism design made by computer scientists. A survey of these contributions can be found in Chapters 9–16 of:


I have omitted this work from my book because I am not sufficiently familiar with it. For example, I am currently reading at my usual snail’s pace through the survey chapters mentioned above. But while I am doing this, the number of papers on mechanism design written by computer scientists seems to grow at breathtaking speed. Finding out how to stay abreast of these developments and how to integrate my knowledge of economists’ research with whatever I can learn about computer scientists’ research is a task so large that I cannot realistically tackle it in this book.

A very large subject that readers might find fascinating is the subject of mechanism design in practice. This subject requires a separate book, or multiple books. Examples of surveys which the reader might find useful are:


In contrast to these works, the emphasis of this book is on the methodology of the theory of mechanism design. I may now already have lost readers with a more practical bent. But, to keep those readers who have stuck with me up to now, it is time to begin.²

Ann Arbor, August 2014

Tilman Börgers
ACKNOWLEDGMENTS

This text explains the parts of the theory of mechanism design that I believe I have understood. Whatever I have understood, I have mostly learned from others’ books and articles, from students who have politely listened to me and told me where I was going wrong, and from my co-authors during our joint research.

One area in which my lack of knowledge was particularly comprehensive is the theory of dynamic mechanism design. Daniel Krähmer and Roland Strausz generously agreed to take over the task of writing a chapter on this subject. Their work is included here as Chapter 11.

Christoph Kuzmics allowed me to see some problems that he gave to his students when teaching a class based on this book. I have included some problems here that were inspired by Christoph’s problems.

I would also like to thank the anonymous referees of Oxford University Press as well as Stefan Behringer, George Chen, Shaowei Ke, Christoph Kuzmics, Xian Li, Vitor Farinha Luz, Stephen Morris, Colin von Negenborn, Martin Polrich, Arunava Sen, Xianwen Shi, Jan-Henrik Steg, Roland Strausz, and Tobias Widmer for very helpful corrections, comments, and suggestions.

Trevor Burnham, Yan-Min Choo, and Nikoleta Scekic have proofread various parts of this book. They have caught many embarrassing errors. I am immensely grateful to them.

I have benefited a lot from Vijay Krishna’s (2002) and Paul Milgrom’s (2004) books on auction theory. The exposition in Chapters 2 and 3 owes a lot to these books. The idea to present the screening problem in Chapter 2 before turning to mechanism design proper has its origin in a conversation with Mark Armstrong.

I am grateful to Daniel and Roland for agreeing to donate all authors’ income from this book to Amnesty International. Even if you feel after reading the book that you have wasted your time, rest assured that you have not completely wasted your money. Some of it benefits a good cause.

Ann Arbor, August 2014

Tilman Börgers
Suppose you want to sell your house, and your realtor has identified several potential buyers who are willing to pay your ask price. You might then wish to conduct an auction among these buyers to obtain a higher price. There are many different auction formats that you could use: For example, each buyer could be asked to send in one binding and final bid. Alternatively, buyers could bid in several rounds, and in each round they are all informed about the highest bid of the previous round and are then asked to revise their bids. You could also use some combination of these formats. How should you choose among different auction formats? This is one of the questions that the theory of mechanism design aims to answer.

Now imagine that you and your colleagues are considering whether to buy a new refrigerator to be kept at work, in which you can store food that you bring from home. While everyone is in favor, it is not so clear how much the refrigerator is worth to different people. How can you find out whether the sum of the amounts that everyone would at most be willing to contribute covers the cost of the refrigerator? You could ask everyone to submit pledges simultaneously and then see whether the sum of the pledges covers the expense. Alternatively, you could go around and tell each colleague how much everyone else has pledged so far. Or you could divide the cost by the
number of colleagues involved and commit to buying the refrigerator only if everyone is willing to pay their share. Which of these procedures is best? Again, this is one of the questions that the theory of mechanism design addresses.

Each of the procedures that you might consider in the two examples above creates a strategic game in the sense of noncooperative game theory among the participants. Participants in these procedures will understand that the outcome will depend not only on their own choices but also on others’ choices and that therefore their own optimal strategy may depend on others’ strategies. In other words, the participants in these procedures will understand that they are playing a noncooperative game. The theory of mechanism design therefore builds on the theory of games (Fudenberg and Tirole, 1993). Game theory takes the rules of the game as given, and it makes predictions about the behavior of strategic players. The theory of mechanism design is about the optimal choice of the rules of the game.

We are more frequently involved in the design of rules for games than might be obvious at first sight. How should shareholders’ votes be conducted? How should promotion procedures in companies be organized? What are optimal prenuptial agreements? All these questions are about the optimal rules of games. The theory of mechanism design seeks to study the general structure underlying all these applications, but it also considers a number of particularly prominent applications in detail.

There are at least two reasons why we study mechanism design. First, the theory of mechanism design aids in practice the designers of real-world mechanisms. The theory of optimal auctions, for example, is frequently invoked in discussions about the design of government and industry auctions. One could call this first aspect of the theory of mechanism design the “normative” side of mechanism design. Second, we can explain why real-world institutions are as they are by interpreting them as rational choices of those who designed them. For example, we might seek to explain the use of auctions in some house sales, as well as the use of posted prices in other house sales by appealing to the theory of mechanism design which indicates that posted prices are optimal in some circumstances and auctions are optimal in other circumstances. One could call this second aspect of the theory of mechanism design the “positive” side of mechanism design.

The incentives created by the choice of rules of games are central to the theory of mechanism design. Incentives are also at the center of contract theory (Bolton and Dewatripont, 2005). At first sight the distinction between the theory of mechanism design and contract theory is simple: In contract theory, we study the optimal design of incentives for a single agent. In mechanism design, we study the optimal design of incentives for a group of agents, such as the buyers in our first example and the colleagues in the second example. Contract theory therefore, unlike the theory of mechanism design, does not have to deal with strategic interaction.
The relation between contract theory and the theory of mechanism design is, however, more subtle. One part of the theory of mechanism design is, in fact, a straightforward extension of insights from contract theory. This is surprising because one might have expected the element of strategic interaction, which is present in mechanism design but absent in contract theory, to create substantial new problems. It is interesting and conceptually important to understand why this is not the case, and we shall address this issue in detail below. The close parallel between contract theory and mechanism design applies only to some parts of mechanism design. Other parts of mechanism design are unrelated to contract theory.

Contract theory has traditionally been divided into two parts: the theory of hidden information (also referred to as the theory of “adverse selection”) and the theory of hidden action (also referred to as the theory of “moral hazard”). The distinction is easily explained within the context of contracts for health insurance. Whether you have experienced severe chest pain in the past is something that you know, but that the company from which you are trying to buy health insurance does not know. It is “hidden information.” Whether you exercise regularly, or take it a little more easy once you have bought complete insurance coverage for heart surgery, is a choice that you make that your insurance company does not observe unless it puts into place a surveillance operation. It is a “hidden action.” Both hiddens, information and actions, matter for contract design. For example, by offering you a menu of insurance contracts and observing your choice from this menu, an insurance company might be able to infer information about your health risks that you might wish to conceal from the company. By introducing deductibles, an insurance company might seek to maintain your incentives to look after your own health and thus alleviate moral hazard problems.

Mechanism design, as traditionally understood, is about hidden information, not hidden actions, with multiple agents. In our first example, the hidden information that the seller of a house seeks to find out is the buyers’ true willingness to pay for the house. In our second example in this Introduction, the hidden information that we seek to find out is the colleagues’ true willingness to pay for an office refrigerator. In voting, the hidden information that we seek to find out is individuals’ true ranking of different alternatives or candidates. Of course, hidden action with many agents involved is a subject of great interest, and the theory that deals with it is, like the theory of mechanism design, concerned with the optimal choice of rules for a game. For example, promotion schemes within a company set work incentives for a group of employees, and the optimal choice of such schemes is an important subject of economic theory. However, it is not the subject of mechanism design as the term has traditionally been interpreted.

When choosing the rules for the strategic interaction among agents, we might restrict ourselves to a small subset of all conceivable rules; or we might try to cast our net
wide, and consider as large a set of rules as possible. For example, when considering how to auction your house, you might restrict attention to the choice of the minimum bid and take for granted that the auction will proceed with all potential buyers submitting their bids simultaneously. You would then focus on the choice of only one parameter in a much larger set of possible choices. Alternatively, you might consider all conceivable ways of proceeding, not just auctions, but, for example, also simultaneous negotiations with all buyers that follow some predetermined format. It has been one of the accomplishments of the theory of mechanism design to develop a framework in which one can find the optimal rules of the game among all conceivable rules. Indeed, mechanism design has traditionally been understood as the field in which this grand optimization among all conceivable procedures is considered. In this book, we shall stick to this interpretation.

Suppose you have considered all possible rules for proceeding with your house sale, and you have come to the conclusion that an auction with just one round of bidding is optimal. After the highest bid has been revealed, one of the losing bidders approaches you with a new and improved bid that is higher than the winning bid in the auction. Will you accept? This is an obvious temptation, but if you accept later bids, are you still conducting an auction with just a single round of bidding? In this book we shall assume that the mechanism designer has full commitment power. The rules, once announced, are set in stone. The mechanism designer will not deviate from them. In our example, the mechanism designer will absolutely refuse to renegotiate after the auction results have been revealed. This is obviously a strong assumption. In contract theory, much attention has been given to the optimal design of contracts if full commitment cannot always be achieved, and this line of research has been very productive. It is also likely to be an interesting line of research in mechanism design. We do not consider this line of argument in this book because we want to maintain a focus on the central arguments of the traditional theory of mechanism design, and these arguments have assumed full commitment by the mechanism designer.

I have chosen a somewhat unusual beginning for the book with Chapter 2, where I explain the basic theory of screening. The theory of screening is sometimes not regarded as part of the theory of mechanism design because it constructs an incentive scheme for only one agent rather than multiple interacting agents. However, the theories that are covered in Chapter 2 are intimately linked to the theories of optimal mechanisms that are explained in later chapters, particularly in Chapter 3. My hope is that by juxtaposing the theory of screening and the theory of mechanism design, I can help the reader understand which features of optimal mechanism design are due to strategic interaction and which features of optimal mechanism design are identical to the corresponding features of optimal screening.
Chapter 3 then reviews the classic “Bayesian” theory of mechanism design in the context of some prominent examples. Chapter 5 develops a generalization of the single agent theory of Chapter 2, covering a very large class of models. Chapter 6 then develops in an analogous way the “Bayesian” theory of mechanism design for a very general framework.

The game theoretic models that are used in Chapters 3 and 6 are built on several restrictive assumptions, which we shall discuss in detail. One of these assumptions is that for given rules of the mechanism, agents play a Bayesian equilibrium of the mechanism for a particular specification of agents’ beliefs about each other’s private information. This assumption might attribute more information to the mechanism designer than is realistic, and therefore the literature has sought to develop mechanisms that require the mechanism designer to know less about agents’ beliefs. The classic approach to this problem is to seek dominant strategy mechanisms. We present this approach in Chapter 4, in the context of examples, and in Chapter 7, in general.

In Chapters 8 and 9 we relax other assumptions of the classic model. In Chapter 10 we return to the issue of what the mechanism designer knows about agents’ beliefs about each other and investigate more modern approaches to this problem which do not necessarily require the construction of a dominant strategy mechanism. Chapter 11 presents models of mechanism design that apply to dynamic contexts. Like robust mechanism design, this is an area of current research interest.