

Optimal Contract Design for Energy Procurement

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ABSTRACT

We consider a mechanism design problem for strategic agents with multi-dimensional private information and uncertainty in their utility/cost functions. We show that the optimal mechanism is a menu of contracts that can be implemented as a nonlinear pricing scheme. We illustrate the result by considering an optimal energy procurement mechanism from a strategic seller with conventional (deterministic) and renewable (random) plants. We address the problem of risk-sharing and ex-post voluntary participation (commitment) under uncertainty.

Keywords: contract under uncertainty, multi-dimensional private information, renewable energy, mechanism design.

I. INTRODUCTION

With the regulation of electricity industry [23], contract has become one of the main schemes used in electricity markets. In liberalized electricity markets, generation and utility companies are privately owned and seek to maximize their own profit. These companies have private information about their generation cost and demand and behave strategically. Contract theory provides an appropriate framework to study electricity markets with strategic players that possess private information.

Bilateral trade, as a long-term contract between a generator and a utility company, has been utilized by players in electricity markets to help them hedge themselves against the risk of day-ahead and real-time markets. It is actually suggested that bilateral trades between players in electricity markets are necessary along with the existing pooling markets to guarantee the stability and reliability of electricity markets [2].

One of the main goals of the developing smart grids is to involve actively the demand side in electricity markets. This gives the opportunity to utilize the existing consumption behavior flexibility and increase the operational efficiency of the grid. To do so, appropriate schemes are required to be designed that incentivise the demand side to actively participate in electricity markets [21]. Contract with incentive payment is considered as one of

the potential options; a group of loads signs contracts with an aggregator and give the direct control of their loads to it; the aggregator utilizes the aggregated resource in electricity markets (*e.g.* ancillary service market). Compared to real-time pricing or load direct market participation, contracts with incentive payments provide a reliability guarantee, are simpler to implement, and more appealing to smaller loads (*e.g.* households).

In this paper we study a general contract design problem for electricity markets. We assume general utility/cost functions for both the buyer and the seller, where both have multi-dimensional private information. The generality of the formulated problem enable us to capture many instances in electricity markets including bilateral trades and contracts with incentive payments. Furthermore, we explicitly consider uncertainty in our problem formulation and address issues of ex-post voluntary participation and risk sharing. With the increase in the share of renewable generation and the development of demand responses programs, uncertainty becomes an important consideration in electricity markets.

To illustrate our results, we consider a forward bilateral trade between a buyer and a seller with both conventional and renewable generation capability. Currently, renewable energy generation is guaranteed to be procured in real-time, and is not required to follow any schedule or commitment [17]. However, as the share of renewable generation increases the current scheme cannot be sustained [15], and it becomes necessary for renewable generators to commit to an agreed schedule [7], [8].

A. Related Literature

There is a rich literature on mechanism design for electricity with information asymmetry and strategic behavior. A contract design problem for demand management with one-dimensional private information is studied in [9]. The work in [1] studies the problem of contract design for deferrable demands with constant marginal utility for demand. The work in [4] considers a mechanism design problem for the forward reserved market assuming that the participants have constant marginal cost and no market power. Although the private information assumed in [1],

and [4] is multi-dimensional, the simplifying assumption of constant marginal cost/utility enables the authors in [1] and [4] to rank different types, and is critical to the solution approaches they provide. A contract design problem for demand response with quadratic cost function is studied in [11], and the solution is found numerically. The scenarios proposed in [18] and [6] investigate the idea of selling uncertain (random) power to consumers and explicitly consider uncertainty in their model, but assume one-dimensional heterogeneous types for consumers. The work in [19] considers a mechanism design problem for energy procurement for a general utility/cost function and uncertainty and applies the Vickery-Clacks-Gloves (VCG) mechanism. However, the VCG mechanism is suboptimal for the problem formulated in [19] when the generation cost function cannot be parameterized by only a one-dimensional variable for different types. (see [12], Ch. 14).

From the economics point of view, the problem we formulate in this paper belongs to the class of screening problems. In economics, the one-dimensional screening problem has been well-studied with both linear and nonlinear utility functions [3]. However, the extension to the multi-dimensional screening problem is not straightforward and no general solution is currently available for this problem. The authors in [13] study a general framework for a static multi-dimensional screening problem with linear utilities. They discuss two general approaches, the parametric-utility approach and the demand-profile approach. The methodology we use to solve the problem formulated in this paper is similar to the demand-profile approach. We consider a multi-dimensional screening problem with nonlinear utilities. The presence of nonlinearities results in additional complications that are not present in [13] where the utilities are linear¹.

B. Contribution

We consider an optimal contract design problem that assumes general utility/cost function for the buyer and the seller with multi-dimensional private information. The generality of the model enables us to capture many applications in electricity markets as well as other disciplines. We explicitly incorporate uncertainty in the cost/utility of the buyer and the seller. This consideration becomes crucial for the emerging electricity markets, as (1) the share of renewable generation increases, (2) the existing demand becomes more responsive and less shielded from the market outcome, and (3) new resources/loads (e.g. storage, plug-in electric vehicle) enter the market. Due to uncertainty, interim voluntary participation of the seller

¹When a problem is linear, expectation of any random variable can be replaced by its expected value and reduce the problem to a deterministic one.

does not necessarily imply ex-post voluntary participation. We determine an optimal multi-dimensional contract that satisfies both interim and ex-post voluntary participation. We show that in general the optimal mechanism is a menu of contracts that can be implemented as a nonlinear pricing scheme. We show that by allowing the payment to be dependent on the uncertainty, one can achieve ex-post voluntary participation, and a desired risk-sharing (associated with the uncertainty) among the buyer and the seller. To the best of our knowledge, our results present the first optimal mechanism for buyer/seller with multi-dimensional private information that guarantees the seller's ex-post voluntary participation. We illustrate our result by considering the problem of optimal energy procurement from a strategic seller with both conventional and renewable generation, and determine the optimal contract which is a nonlinear pricing².

C. Organization

The paper is organized as follows. In Section 2, we introduce the model and formulate the contract design problem. For ease of exposition, we formulate the problem as an energy procurement from a seller. However, the model and results presented in this paper are general and not limited to this example. In Section 3, we present an outline of our approach and key ideas toward the solution of the problem formulated in Section 2, and state the main result on the optimal mechanism. We illustrate the result by an example in Section 4. We discuss the nature of our results in Section 5. We extend our results to address the problem of ex-post voluntary participation, and arbitrary risk allocation between the buyer and the seller in Section 6. We conclude in Section 7.

II. MODEL SPECIFICATION AND PROBLEM FORMULATION

A. Model Specification

A buyer wishes to make an agreement to buy energy from an energy seller³. The seller has the ability to produce energy from conventional (deterministic) generators or from renewable (random) generators that she owns.

Let q be the amount of energy the buyer buys, and t be his payment to the seller. We proceed to formulate the energy procurement problem by making the following assumptions.

²One can interpret the problem as an interaction between one buyer and one seller, where the seller has multi-dimensional private information and the buyer has prior belief (probability distribution) about the seller's private information, or equivalently, an interaction between a buyer and a heterogeneous population of sellers with multi-dimensional private information, where the buyer knows the frequency distribution of the seller's private information.

³From now on, we refer to the buyer as "he" and to the seller as "she".

(A1) The buyer is risk-neutral and his total utility is given by $\mathcal{V}(q) - t$, where $\mathcal{V}(q)$ is the utility that he gets from receiving q amount of energy, and $\mathcal{V}(0) = 0$. $\mathcal{V}(\cdot)$ is the buyer's private information⁴.

(A2) The seller's production cost is given by $C(q, x, w)$, where $x \in \chi \subseteq \mathbb{R}^n$ is the seller's type (technology and cost) and w denotes the realization of a random variable W , e.g. weather. $C(q, x, w)$ is convex and increasing in q . The start-up cost $C(0, x, w)$ does not depend on the weather w and is given by x_1 , i.e. $C(0, x, w) = C(0, x) = x_1$.

(A3) The probability distribution function of W , i.e. weather forecast, is common knowledge between the buyer and the seller and is given by $F_W(w)$.

(A4) The seller is risk-neutral and her utility is given by her total expected revenue

$$\mathbb{E}_W \{t - C(q, x, W)\}. \quad (1)$$

(A5) Define $c(q, x) = \frac{\partial \mathbb{E}_W \{C(q, x, W)\}}{\partial q}$ as the expected marginal cost for the seller's type x . Without loss of generality, there exists m , $1 < m \leq n$, such that $c(q, x)$ is increasing in x_i for $1 \leq i \leq m$, and decreasing in x_i for $m < i \leq n$. Moreover, there is an $\underline{x} \in \chi$ (the seller's worst type) such that $\underline{x}_i \leq x_i$ and $\underline{x}_j \geq x_j$ for all $x \in \chi$, $1 \leq i \leq m$ and $m < j \leq n$.

(A6) The seller's type x is her own private information, the set χ is common knowledge, and there is a prior probability distribution f_X over χ which is common knowledge between the buyer and the seller.

(A7) Both the buyer and the seller are strategic and perfectly rational, and this is common knowledge.

(A8) The buyer has all the bargaining power; thus, he can design the mechanism/set of rules that determines the agreement for energy procurement quantity q , and payment t .⁵

(A9) After the buyer announces the mechanism for energy procurement and the seller accepts it, both the buyer and the seller are fully committed to following the rules of the mechanism.

⁴We assume that the buyer either has an elastic demand, or needs to meet some fixed demand and has an outside option to buy energy if he cannot buy it from the seller (which is the result of his interaction with other players in the market).

⁵Usually there is no competition on the demand side, but different sellers compete to win a contract with the demand side. Therefore, it is reasonable to assume that in a non-cooperative setting, the demand side has all the bargaining power. If one assumes that the seller has all the bargaining power, the resulting problem can be solved in the same way. However, modeling and studying of the bargaining game between the buyer and the seller is an interesting problem, but bit is beyond the scope of this paper.

The above model is appropriate for other problems arising in electricity markets. For example, consider a mechanism design problem for a demand response (DR) program [20]. Then, in the proposed model q denotes the amount of the seller's load that participate in the DR program. The seller's cost is given by $C(q, x, W)$, where x denotes the seller's type, and W denotes a random variable that captures the uncertainty in the number of times the participating load in the DR program will be actually required to shift/change its consumption.

As a consequence of Assumption (A8) on the buyer's bargaining power and the fact that the seller's utility does not directly depend on the buyer's private information, the solution of the problem formulated in this paper does not depend on whether the buyer's utility $\mathcal{V}(\cdot)$ is private information or common knowledge⁶.

Note that in the one-dimensional screening problem, the cost of production induces a complete order among the seller's types, which is crucial to the solution to the optimal mechanism design problem. However, in multi-dimensional screening problems, the expected cost of production induces, in general, only a partial order among the seller's types.

Definition 1. We say the seller's type x is better (resp. worse) than the seller's type \hat{x} if $\mathbb{E}_W \{C(q, x, W)\} \leq \mathbb{E}_W \{C(q, \hat{x}, W)\}$ for all $q \geq 0$ (resp. $\mathbb{E}_W \{C(q, x, W)\} \geq \mathbb{E}_W \{C(q, \hat{x}, W)\}$) with strict inequality for some q .

From (A5), the seller's type x is better than the seller's type \hat{x} if and only if $x_i \leq \hat{x}_i$ for $1 \leq i \leq m$, and $x_i \geq \hat{x}_i$ for $m < i \leq n$ with strict inequality for some i . The following example illustrates (A2)-(A5) and Definition 1.

A simple case. Consider a seller with a wind turbine and a gas generator. The generation from the wind turbine is free and given by γw^3 , where γ is the turbine's technology and w is the realized weather. The gas generator has a fixed marginal cost θ_c . We assume that there is a fixed cost c_0 which includes the start-up cost for both plants and the capital cost for the seller. Therefore, the seller's type has $n = 3$ dimensions. The generation cost for the seller is given by

$$C(q, x, w) = c_0 + \theta_c \max \{q - \gamma w^3, 0\}. \quad (2)$$

The seller's type $x = (c_0, \theta_c, \gamma)$ is better than the seller's type $\hat{x} = (\hat{c}_0, \hat{\theta}_c, \hat{\gamma})$ if and only if $c_0 \leq \hat{c}_0$, $\theta_c \leq \hat{\theta}_c$, and $\gamma \geq \hat{\gamma}$, with one of the above inequalities being strict.

B. Problem Formulation

Let (\mathcal{M}, h) be the mechanism/game form (see [14], Ch. 23) for energy procurement designed by the buyer. In this

⁶This becomes more clear by looking at the main result given by Theorem 1.

game form, \mathcal{M} describes the message/strategy space and h determines the outcome function; $h : \mathcal{M} \rightarrow \mathbf{R}_+ \times \mathbf{R}$ is such that for every message/action $m \in \mathcal{M}$ it specifies the amount q of procured energy and the payment t made to the seller, i.e. $h(m) = (q(m), t(m)) = (q, t)$.

The objective is to determine a mechanism (\mathcal{M}, h) so as to

$$\underset{(\mathcal{M}, (q, t))}{\text{maximize}} \quad \mathbb{E}_{\mathcal{X}, W} \{ \mathcal{V}(q) - t \}, \quad (3)$$

under Assumptions (A1)-(A9) and the constraint that the seller is willing to voluntarily participate in the energy procurement process. The willingness of the seller to voluntarily participate in the mechanism for energy procurement is called voluntary participation (VP) (or individual rationality) and is written as

$$\text{VP: } t(m^*) - \mathbb{E}_W \{ C(q(m^*), x, W) \} \geq 0, \quad \forall x \in \chi \quad (4)$$

where $m^* \in \mathcal{M}$ is a Bayesian Nash equilibrium (BNE) of the game induced by the mechanism (\mathcal{M}, h) . That is, at equilibrium the seller has a non-negative payoff.

We call the above problem **(P1)**.

III. OUTLINE OF THE APPROACH & RESULTS

We prove that the optimal energy procurement mechanism is a pricing scheme that the buyer offers to the seller and the seller chooses a production quantity based on her type. We characterize the optimal energy procurement mechanism by the following theorem, which reduces the original functional maximization problem (P1) to a set of equivalent pointwise maximization problems.

Theorem 1. *The optimal mechanism (q, t) for the buyer is a menu of contracts (nonlinear pricing) given by*

$$p(q) = \arg \max_{\hat{p}} \{ P[x \in \chi | \hat{p} \geq c(q, x)] (\mathcal{V}'(q) - \hat{p}) \}, \quad (5)$$

$$t(q) = \int_0^q p(l) dl + C(0, \underline{x}), \quad (6)$$

$$q(x) = \arg \max_{l \in \mathbf{R}_+} \mathbb{E} \{ t(l) - \mathbb{E}_W \{ C(l, x, w) \} \} \quad (7)$$

where $\mathcal{V}'(q) := \frac{d\mathcal{V}(q)}{dq}$.

The proof of Theorem 1 proceeds in several steps. Below we present these steps and the key ideas behind each step. The detailed proof of all results (theorems and lemmas) appearing below can be found in [10].

Step 1. By invoking the revelation principle [5], we restrict attention, without loss of optimality, to direct revelation mechanisms that are incentive compatible (defined below) and individually rational.

Definition 2. *A direct revelation mechanism is defined by functions $q : \chi \rightarrow \mathbf{R}_+$ and $t : \chi \rightarrow \mathbf{R}_+$, and works*

as follows: First, the buyer announces functions q and t . Second, the seller declares some $x' \in \chi$ as her report for her technology and cost. Third, the seller is paid $t(x')$ to deliver $q(x')$ amount of energy.

The seller is strategic and may lie and misreport her private information x , i.e. we do not necessarily have $x' = x$ unless it is to the seller's interest to report truthfully. We call incentive compatibility (IC) the requirement for truthful reporting, and define the following problem that is equivalent to (P1).

Problem P2: Determine functions $q : \chi \rightarrow \mathbf{R}_+$ and $t : \chi \rightarrow \mathbf{R}_+$ so as to

$$\underset{(q, t)}{\text{maximize}} \quad \mathbb{E}_{\mathcal{X}, W} \{ \mathcal{V}(q(x)) - t(x) \} \quad (8)$$

subject to

$$\text{IC: } x = \arg \max_{x'} \mathbb{E}_W [t(x') - C(q(x'), x, w)], \quad \forall x \in \chi \quad (9)$$

$$\text{VP: } \mathbb{E}_W [t(x) - C(q(x), x, W)] \geq 0, \quad \forall x \in \chi. \quad (10)$$

Step 2. We utilize the partial order among the seller's different types to rank the seller's utility for her different types, and reduce the VP constraint for all the seller's types to the VP constraint only for the seller's worst type.

Lemma 1. *For a given mechanism (q, t) , a better type of the seller gets a higher utility. That is, let $U(x) := \mathbb{E}_W \{ t(q(x)) - C(q(x), x, W) \}$ denote the expected profit of the seller with type x . Then,*

- 1) $\frac{\partial U}{\partial x_i} \leq 0, 1 \leq i \leq m,$
- 2) $\frac{\partial U}{\partial x_i} \geq 0, m < i \leq n.$

A direct consequence of Lemma 1, is that the seller's worst type \underline{x} receives the minimum utility among all the seller's types.

Corollary 1. *The voluntary participation constraint is only binding for the worst type \underline{x} , that is the general VP constraint (7) can be reduced to*

$$U(\underline{x}) := \mathbb{E} \{ t(q(\underline{x})) - C(q(\underline{x}), \underline{x}, w) \} = 0. \quad (11)$$

Step 3. We show, via Lemma 2 below, that without loss of optimality, we can restrict attention to a set of functions $t(\cdot)$ that depend only on the amount of energy q . That is, the optimal mechanism is a pricing scheme.

Lemma 2. *For any pair of functions (q, t) that satisfies the IC constraint, we can rewrite $t(x')$ as $t(q(x'))$.*

Step 4. As a consequence of Lemma 2, we determine an optimal mechanism sequentially. First, we determine the optimal payment function $t(\cdot)$, then the optimal energy procurement function $q(\cdot)$. For any function $t(\cdot)$, we determine, for each type x of the seller, the optimal quantity $q^*(x)$ that she wishes to produce as

$$q^*(x) = \arg \max_l \mathbb{E}_W \{t(l) - C(l, x, w)\}. \quad (12)$$

Incentive compatibility then requires that the seller must tell the truth to achieve this optimal value, and cannot do better by lying, *i.e.* $q(x) = q^*(x)$ for all $x \in \chi$. For any function $t(\cdot)$, this last equality can be taken as the definition for the associated function $q(\cdot)$. Thus, we eliminate the IC constraint by defining $q(\cdot) := q^*(\cdot)$ and reduce the problem of designing the optimal direct revelation mechanism (q, t) to an equivalent problem where we determine only the optimal payment function $t(\cdot)$ subject to the voluntary participation constraint for the worst type.

Step 5. To solve this new equivalent problem, we write the buyer's expected utility as the integration of his marginal expected utility, and express the marginal expected utility in terms of the marginal price $p(q) := \frac{\partial t(q)}{\partial q}$ and the minimum payment $t(0)$ (which along with $p(\cdot)$ uniquely determines the payment function $t(\cdot)$). Specifically, in [10] we show that

$$\begin{aligned} \mathbb{E}_X[\mathcal{V}(q^*(X))] - \mathbb{E}_X[t(q^*(X))] = \\ \int_0^\infty P(x \in \chi | q^*(x) \geq l) \mathcal{V}'(l) dl \\ - t(0) - \int_0^\infty P(x \in \chi | q^*(x) \geq l) p(l) dl, \end{aligned} \quad (13)$$

where $\mathcal{V}'(q) := \frac{d\mathcal{V}(q)}{dq}$. That is, the buyer's total expected utility is obtained by integrating his marginal utility at quantity l , times the probability that the seller's production exceeds l , and subtracting the minimum payment $t(0)$. We show in [10] that the seller's optimal decision $q^*(x)$ depends only on the marginal price $p(q)$. Thus, we can write the probability associated with the seller's decision as

$$P(x \in \chi | q^*(x) \geq l) = P[x \in \chi | p(l) \geq c(l, x)]. \quad (14)$$

That is, the seller is willing to produce the marginal quantity at l if the resulting expected marginal profit is positive, *i.e.* marginal price $p(l)$ exceeds marginal expected cost of generation $c(l, x)$ ⁷. Using (13) and (14), we define the following problem that is equivalent to (P2) and is in terms of the marginal price $p(q)$ and the minimum payment $t(0)$.

⁷This relies on quasi-concavity of the seller's optimal decision program. This is a standard assumption in the literature, *e.g.* see [13] and [22]. Basically, it gives the seller the freedom to decide for each marginal unit of production independently. Therefore, the continuity of the resulting generation quantity must be checked posterior to the design of the optimal contract for each type of the seller.

Problem P3:

$$\begin{aligned} \max_{p(\cdot), t(0)} \int_0^\infty P[x \in \chi | p(l) \geq c(l, x)] (\mathcal{V}'(l) - p(l)) dl - t(0) \quad (15) \\ \text{subject to} \end{aligned}$$

$$\text{VP: } \mathbb{E}_W \left\{ t(0) + \int_0^{q(\underline{x})} p(r) dr - C(q^*(\underline{x}), \underline{x}, w) \right\} \geq 0. \quad (16)$$

Step 6. We provide a ranking for the seller's optimal decision $q^*(x)$ based on the partial order among the seller's types.

Lemma 3. *For a given mechanism specified by $(t(\cdot), q(\cdot))$, a better type of the seller produces more. That is, the optimal quantity $q^*(x)$ that the seller with true type x wishes to produce satisfies the following properties:*

- a) $\frac{\partial q^*(x)}{\partial x_i} \leq 0, 1 \leq i \leq m,$
- b) $\frac{\partial q^*(x)}{\partial x_i} \geq 0, m < x \leq n.$

In [10] we show that a consequence of Corollary 1 and Lemma 3 is the following result.

Corollary 2. *The VP constraint is satisfied if $t(0) = C(0, \underline{x})$ and the lowest seller's type payment is equal to her expected production cost, *i.e.* $t(q(\underline{x})) = \mathbb{E}_W \{C(q(\underline{x}), \underline{x}, W)\}$.*

Based on Corollary 2, we define a problem (P4) that is equivalent to (P3) and is only in terms of the marginal price $p(l)$ and the constraint that the payment the seller's lowest type receives is equal to her cost of production.

Problem (P4)

$$\begin{aligned} \max_{p(\cdot)} \int_0^\infty P[x \in \chi | p(l) \geq c(l, x)] (\mathcal{V}'(l) - p(l)) dl \quad (17) \\ \text{subject to} \end{aligned}$$

$$\text{VP: } C(0, \underline{x}) + \int_0^{q(\underline{x})} p(l) dl = \mathbb{E}_W [C(q(\underline{x}), \underline{x}, W)]. \quad (18)$$

Step 7. We consider a relaxed version of (P4) without the VP constraint (18). The unconstrained problem can be solved pointwise at each quantity l to determine the optimal $p(l)$ as

$$p(l) = \arg \max_{\hat{p}} \{P[x \in \chi | \hat{p} \geq c(q, x)] (\mathcal{V}'(q) - \hat{p})\} \quad (19)$$

which is the same as (5). We show in [10] that the solution to the unconstrained problem automatically satisfies the VP constraint (18), therefore, the solution to the unconstrained problem determines the optimal marginal price $p(\cdot)$ for the original problem. Using $p(\cdot)$ and the fact that $t(0) = C(0, \underline{x})$, we determine the optimal function $t(\cdot)$. Using $t(\cdot)$ we find the seller's best response function $q^*(\cdot)$. By incentive compatibility $q^*(\cdot) = q(\cdot)$, and this completely determines the optimal direct revelation mechanism $(q(\cdot), t(\cdot))$ described by Theorem 1.

In essence, Theorem 1 states that at each quantity l , the optimal marginal price $p(l)$ is chosen so as to maximize the expected total marginal utility at l , which is given by the total marginal utility $(\mathcal{V}'(l) - p(l))$ times the probability that the seller generates at least l .

Remark 1. *In a setup with startup cost for the seller, it might not be optimal for the buyer to require all the seller's types to voluntarily participate in the energy procurement process, since the minimum payment $t(0)$ depends on the production cost of the seller's worst type. In such cases, it might be optimal for the buyer to exclude some "less efficient" types of the seller from the contract, select an admissible set of seller's types, and then design the optimal contract for this admissible set of the seller's types⁸. Note that, this is not the case for setups without startup cost. In such setups, if it is not optimal for some type x to be included in the optimal contract, it is equivalent to have $q(x) = 0$ for the optimal contract that considers all types of the seller.*

Remark 2. *In problem (P1), we assume that there exists a seller's worst type which has the highest cost at any quantity among all the seller's types, and we reduce the VP constraint for all the seller's type to only the VP constraint for this worst type. As a result, we pin down the optimal payment function by setting $t(0) = C(0, \underline{x})$ to ensure the voluntary participation of the worst type, which consequently implies the voluntary participation for all the seller's types. In absence of the assumption on the existence of the seller's worst type, the argument used to reduce the VP constraint is not valid anymore and we cannot pin down the payment function and specify $t(0)$ a priori. Assuming that all types of the seller participate in the contract, their decision on the optimal quantity q^* only depends on the marginal price $p(q)$, and therefore, the optimal marginal price $p(q)$ given by (19) is still valid without the assumption on the existence of the worst type. To pin down the payment function $t(\cdot)$, we find the minimum payment $t(0)$ a posteriori so that all types of the seller voluntarily participate. That is,*

$$t(0) = \max_{x \in \mathcal{X}} \left[\mathbb{E}_W \{C(q(x), x, w)\} - \int_0^{q(x)} p(q) dq \right], \quad (20)$$

where the optimal decision of type x is given by

$$q(x) = \arg \max_q \left[\int_0^q p(q) - \mathbb{E}_W \{C(q, x, w)\} \right]. \quad (21)$$

⁸To find the optimal admissible set, the optimal contract can be computed for different potential admissible sets. Then, the resulting utilities can be compared to find the best admissible set.

IV. EXAMPLE

Consider a seller with a conventional plant and a wind turbine. The wind turbine's output power curve $g(w)$ is as in Figure 1. We assume that for wind speeds between v_{ci} and v_r the energy generation is given by γw^3 , where γ captures the technology and size of the turbine. Energy generation remains constant for wind speeds between v_r and v_{co} , and is zero otherwise.

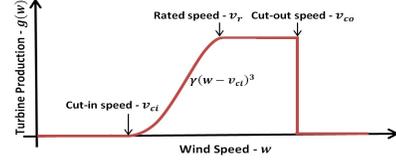


Fig. 1: Example - the wind turbine generation curve

We assume that there is a fixed marginal operational cost θ_w for the wind turbine and a fixed marginal operational cost θ_c for the conventional plant. There is a no-production cost c_0 that captures the start-up cost for both plants and the capital cost for the seller. Therefore, the seller's type $x = (c_0, \theta_w, \theta_c, v_{ci}, v_r, v_{co}, \gamma)$ is 7-dimensional and her total cost is given by

$$C(q, x, w) = c_0 + \theta_w \min\{q, g(w)\} + \theta_c \max\{q - g(w), 0\}, \quad (22)$$

where $g(w)$ is as in Figure 1. The wind profile is a class $k=3$ Weibull distribution with average speed $5m/s$.

We only consider 6 types for the seller here:

$$\begin{aligned} a &= (c_0=4, \theta_w=0.2, \theta_c=1.2, v_{ci}=3, v_r=13, v_{co}=20, \gamma=1), \\ b &= (c_0=4, \theta_w=0.2, \theta_c=1.2, v_{ci}=3, v_r=13, v_{co}=20, \gamma=2), \\ c &= (c_0=5, \theta_w=0.1, \theta_c=1.2, v_{ci}=3, v_r=13, v_{co}=20, \gamma=1), \\ d &= (c_0=5, \theta_w=0.2, \theta_c=1.0, v_{ci}=1, v_r=17, v_{co}=28, \gamma=2), \\ e &= (c_0=6, \theta_w=0.1, \theta_c=1.0, v_{ci}=1, v_r=17, v_{co}=28, \gamma=1), \\ f &= (c_0=6, \theta_w=0.1, \theta_c=1.0, v_{ci}=1, v_r=13, v_{co}=28, \gamma=2), \end{aligned}$$

where the cost unit is \$ and the energy unit is MWh , and there is no worst type. The optimal contract from Theorem 1 is depicted in Figure 2. It is a nonlinear pricing scheme. The marginal price varies between 2.2 and 3.1 ¢/kWh . The variation in the marginal price is of the same order as the variation in the expected marginal production cost across the seller's different types. Since there is no complete order among the different seller's types, we can not compare the seller's types based on the expected revenue or amount of production prior to the design of the mechanism. However, wherever we have a partial order and can rank a subset of types, we can utilize Lemmas 1 and 3 to predict a ranking a priori. For instance, the seller with type (b) is better than the one with type (a), and we can say a priori that the former has a higher

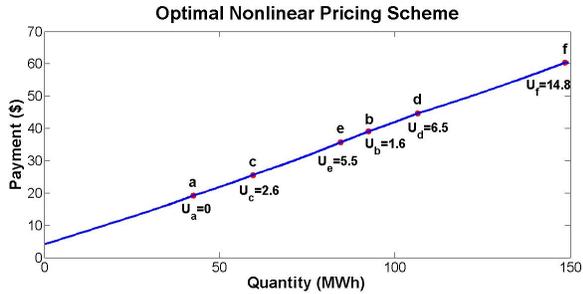


Fig. 2: Example - the optimal pricing scheme

production and revenue. However, we cannot order types (b) and (c). For the setup of our example, according to the optimal contract, type (c) gets a higher expected revenue than type (b) but produces a lower amount of energy than type (b).

V. DISCUSSION

The optimal mechanism/contract for the energy procurement problem formulated in this paper is a nonlinear pricing scheme. The nonlinearity is due to three factors. First, the buyer's utility function $\mathcal{V}(q)$ is not linear in the quantity q . Second, for each type of the seller, the cost of production is a nonlinear function of the amount of produced energy. Third, the seller has private information about her technology and cost (seller's type).

The uncertainty about the production from the non-conventional plant makes the total expected production cost function nonlinear even with a fixed marginal cost of production for conventional and renewable plants. The buyer has to pay information rent (monetary incentive) to the seller to incentivize her to reveal her true type. Therefore, the payment the buyer makes to the seller includes the cost of production the seller incurs plus the information rent, which varies with the seller's type; the better the seller's type, the higher is the information rent.

The optimal contract/mechanism discovered in this paper can be implemented indirectly (without reporting the seller's private information) as follows: the buyer offers the seller a menu of contracts (nonlinear pricing scheme); the seller chooses one of these contracts based on her type, and there is no need for information exchange stage between the seller and the buyer.

The optimal contract specified by Theorem 1 induces incentives for investment in infrastructure and technology development. From Lemma 1, the seller with the higher type has a higher utility. Therefore, there is an incentive for the seller to improve her technology and decrease her cost of generation.

It is well-known that in the presence of private information and strategic behavior, in general, there exists no mechanism/contract that is (1) individually rational,

(2) incentive compatible, and (3) efficient (Pareto-optimal generally) [16]. In the optimal contract/mechanism given by Theorem 1, the allocation for the seller's different types is not ex-post efficient (Pareto-optimal) except for the seller's worst type who gets zero utility.

VI. CONTRACT UNDER UNCERTAINTY

A. Commitment and Ex-post Voluntary Participation

The voluntary participation constraint imposed in problem (P1) is interim. That is, the expected profit with respect to the weather for each type of the seller must be non-negative. By assumption (A9), once the seller agrees on the contract (this agreement takes place before the realization of the weather) she is fully committed to following the agreement, even if the realized profit is negative (because of the realization of the weather)⁹. To ensure that the seller's realized profit is non-negative for all weather realizations, we impose an ex-post voluntary participation constraint. We replace the interim VP constraint (7) by

$$\text{Ex-post VP: } t(m^*) - C(q, x, w) \geq 0, \forall w, \forall x \in \mathcal{X}. \quad (23)$$

To obtain an ex-post voluntary participation constraint, we modify the payment function of the mechanism given by Theorem 1 as follows:

$$\tilde{t}(q, w) = t(q) - \mathbb{E}_W \{C(q(x), x, W)\} + C(q(x), x, w). \quad (24)$$

We have $\mathbb{E}_W \{\tilde{t}(q, w)\} = t(q)$, and therefore, the seller always chooses the same quantity q under the modified payment function $\tilde{t}(\cdot)$ as under the original payment function $t(q)$ given by (6). Note that we have $\tilde{t}(q(x), w) - C(q(x), x, w) = U(x)$, and therefore, the seller's realized utility is the same as the expected utility $U(x) \geq 0$ for all realizations w of the random variable W .

B. Risk Allocation

In the optimal mechanism/contract menu presented by Theorem 1, the buyer faces no uncertainty, and he is guaranteed to receive quantity $q(x)$, and all the risk associated with the realization of the weather is taken by the seller. We wish to modify the mechanism to reallocate the above-mentioned risk between the buyer and the seller. To do so, we modify the payment function so that the risk is reallocated between the buyer and the seller. Consider the following modified payment function with $\alpha \in [0, 1]$,

$$\hat{t}(x, w) = t(q(x)) + \alpha [C(q(x), x, w) - \mathbb{E}_W \{C(q(x), x, w)\}]. \quad (25)$$

From (25) it follows that $\mathbb{E}_W \{\hat{t}(x, w)\} = t(q(x))$. Therefore, the strategic behavior of the seller does not change

⁹Since the seller's reserved utility is zero by not participating (outside option), we can always think of the seller walking away from the agreement for these negative profit realizations and not following the mechanism rules.

and the seller chooses the same quantity under the modified payment function $\hat{t}(\cdot)$ as under the original payment function $t(q)$ given by (6). Note that for $\alpha = 0$ we have the same payment as $t(q)$. For $\alpha = 1$, the seller is completely insured against any risk and all the risk is taken by the buyer. The parameter α determines the allocation of the risk between the buyer and the seller; the buyer undertakes α and the seller undertakes $(1 - \alpha)$ share of the risk.

VII. CONCLUSION

We investigated a contract design problem where each strategic player possesses multi-dimensional private information. The consideration of multi-dimensional private information along with the general structure of the utility/cost functions enables us to capture many applications in electricity markets as well as other disciplines. We assumed that the buyer and/or the seller have uncertainty in their utility/cost function which is realized after the time the agreement takes place. We addressed the problem of ex-post voluntary participation, and risk sharing in the presence of uncertainty. Our results were illustrated by considering a problem of energy procurement from a strategic seller with both conventional and renewable (random) generation. We showed that the optimal procurement method is a nonlinear pricing scheme that the buyer offers and the seller chooses one quantity and the corresponding payment based on her private technology (private information).

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