Practice final
Phil 303
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The exam will be "open book", open notes, open mind, etc. (I've made question 3) a bit longer than the analogous question on the actual exam will be, to give you extra practice questions)

1. a) Prove (with a derivation) that $\{P \vee Q,\ P \to R,\ R \to W,\ Q \to (R \wedge R)\} \vdash W$

   b) What metatheorem allows you to conclude, on the basis of the derivation in a), that $\{P \vee Q,\ P \to R,\ R \to W,\ Q \to (R \wedge \neg R)\} \Vdash W$?

2. i) Give a categorical derivation of $\neg(\neg A \vee B) \to (A \wedge \neg B)$

   ii) Give a derivation of $W \vee R$ from $\{(P \to P) \to (A \wedge \neg A),\ Q \to W,\ Q \to R\}$

3. For each of the following statements, prove it if it is true and give a counter-example if it is false:

   a) If $\Gamma \vdash P$ and $\Delta \vdash Q$ then $\Gamma \cup \Delta \ \vdash P \wedge Q$

   b) If $\Gamma \Vdash P$ and $\Delta \Vdash Q$ then $\Gamma \cup \Delta \ \Vdash P \wedge Q$

   c) If Γ is satisfiable, then $\Gamma \cup \{P \vee \neg P\}$ is satisfiable.

   d) If Γ is satisfiable, and Δ is satisfiable, then $\Gamma \cup \Delta$ is satisfiable.

   e) If $\Gamma \Vdash P$ then $\Gamma \cup \{\neg P\}$ is satisfiable.

   f) If Γ is satisfiable, then $\{\neg S \,|\, S \ \in \Gamma\}$ is satisfiable.

4. Say that we have a system of inference with just the rule of Modus Ponens. (You are allowed to make hypotheses only at the beginning of the proof.) Prove by induction that this system is sound.

5. Say that Γ is a V-saturated set and $(\neg P \vee \neg Q) \in \Gamma$. Explain how we know that either $P \notin \Gamma$ or $Q \notin \Gamma$.