Philosophy 303
Symbolic Logic
Winter, 2009

Solutions to Homework Assignment 9

1. Say that $\Gamma \Vdash A$ and $\Gamma \Vdash \neg B$. Show that $\Gamma \Vdash (\neg A \vee B) \rightarrow Q$

   Say we have an interpretation I that makes every sentence in Γ true. Then I makes A true, so it makes $\neg A$ false, and I makes $\neg B$ false, so it makes B true. By the truth-table for $\vee$, if $\neg A$ is false and B is false, then $\neg A \vee B$ is false. Hence, by the truth table for $\rightarrow$, $(\neg A \vee B) \rightarrow Q$ is true whatever the truth-value of Q.

2. Metatheorem 27: Say that $\Gamma \cup \{A\} \Vdash B$. Show $\Gamma \Vdash A \rightarrow B$.

   Say that $\Gamma \cup \{A\} \Vdash B$.
   Say we have an interpretation Î that makes every sentence in Γ true. If Î makes $\neg A$ true, it also makes $A \rightarrow B$ true, since Î would have to correspond to one of the lines in boldface:

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	**T**	**T**
F	**F**	**T**

   On the other hand, if Î makes A true, then Î makes $\Gamma \cup \{A\}$ true, so it makes B true (since $\Gamma \cup \{A\} \Vdash B$). Here too, Î makes $A \rightarrow B$ true, since Î would have to correspond to one of the lines in boldface:

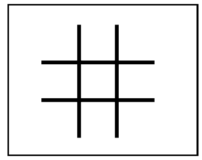
A	B	$A \rightarrow B$
T	**T**	**T**
T	F	F
F	**T**	**T**
F	F	T

   Either way, Î makes $A \rightarrow B$ true. Since Î was arbitrary, this shows that $\Gamma \Vdash A \rightarrow B$.

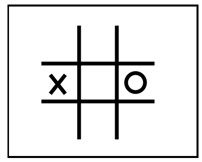
   Metatheorem 29: Say that $\Gamma \cup \{A\} \Vdash B$ and $\Gamma \cup \{A\} \Vdash \neg B$ Show $\Gamma \Vdash \neg A$.

   The point here is that any interpretation that simultaneously satisfies $\Gamma \cup \{A\}$ would satisfy both B and $\neg B$, which is impossible. So no interpretation satisfies $\Gamma \cup \{A\}$. Say we have an interpretation Î which satisfies Γ. If Î also satisfied A, then it would satisfy $\Gamma \cup \{A\}$, which we've just seen is impossible. So Î satisfies $\neg A$.
   This shows that $\Gamma \Vdash \neg A$.

3. (a) $(Empty(1) \wedge O(6)) \rightarrow \neg X(4))$

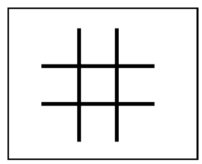


Satisfies

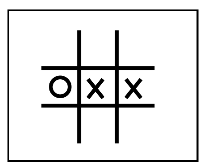


Doesn't Satisfy

(b) $O(4) \rightarrow (X(5) \rightarrow Empty(6))$

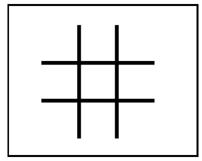


Satisfies

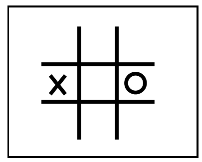


Doesn't Satisfy

(c) $Empty(1) \wedge Empty(4) \wedge Empty(7)$

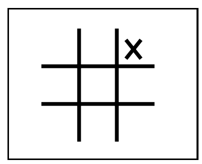


Satisfies

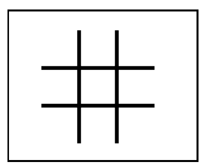


Doesn't Satisfy

(d) $(Empty(1) \vee Empty(4) \vee Empty(7)) \rightarrow (X(3) \vee X(6) \vee X(9))$



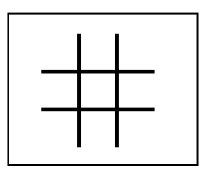
Satisfies



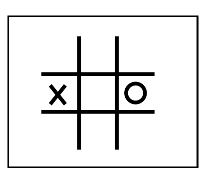
Doesn't Satisfy

f)

$Empty(1) \wedge Empty(4) \wedge Empty(7)$

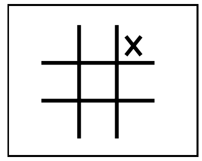


Satisfies

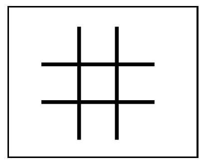


Doesn't Satisfy

j)
$(Empty(1) \lor Empty(4) \lor Empty(7)) \rightarrow (X(3) \lor X(6) \lor X(9))$



Satisfies



Doesn't Satisfy

4. You need to account for the moves each player must do to be playing the game (play exactly one of: rock, paper, scissors), what situations result in a win for which player and which situations are draws.
   Say the players are Andrea and Bill. For the moves and outcomes, we'll need six letters for moves, and three for outcomes: $R_a, R_b, P_a, P_b, S_a, S_b, W_a, W_b, D$.

   Axioms:

   (a) Each player plays one of the moves, and *only* one of the moves:
   $(R_a \land \neg S_a \land \neg P_a) \lor (S_a \land \neg R_a \land \neg P_a) \lor (P_a \land \neg R_a \land \neg S_a)$
   and: $(R_b \land \neg S_b \land \neg P_b) \lor (S_b \land \neg R_b \land \neg P_b) \lor (P_b \land \neg R_b \land \neg S_b)$

   (b) Rock beats Scissors:
   $(R_b \land S_a) \rightarrow W_b$ and $(R_a \land S_b) \rightarrow W_a$

   (c) Scissors beats Paper:
   $(P_b \land S_a) \rightarrow W_a$ and $(P_a \land S_b) \rightarrow W_b$

   (d) Paper beats Rock:
   $(P_b \land S_a) \rightarrow W_a$ and $(P_a \land S_b) \rightarrow W_b$

   (e) Identical moves draw:
   $(P_b \land P_a) \rightarrow D$ and $(S_a \land S_b) \rightarrow D$ and $(R_b \land R_a) \rightarrow D$

   Those were the bits of information the question specifically asked for, so that will suffice for an answer. However, there are other bits of information about the game that you could choose to build in if you wanted to model that. For example, that two people cannot win, or that someone cannot both win and draw. To this end, another good axiom would be:

5. $(W_a \land \neg W_b \land \neg D_a \land D_b) \lor (W_b \land \neg W_a \land \neg D_a \land D_b) \lor (\neg W_a \land \neg W_b \land D_a \land D_b)$