Practice final
Phil 303
Posted April 23, 2009
The exam will be "open book" and open notes. No electronic aids.

Answer all parts of a question, unless the question explicitly says you need only answer one.

1. Answer **one** of these two questions:

   (a) Say that $CS =_{definition} \{X | X \notin S\}$.
   Prove that $C(S_1 \cup S_2) = CS_1 \cap CS_2$

   (b) Using the Pumping Lemma, explain why there cannot be an FSA over the alphabet $\{a, b\}$ that accepts only strings that consist of just an unbroken sequence of a's, and then an unbroken sequence of exactly the same number of b's.

2. a) Prove (with a derivation) that $\{P \vee Q,\ P \rightarrow R,\ R \rightarrow W,\ Q \rightarrow (R \wedge R)\} \vdash W$

   b) What metatheorem allows you to conclude, on the basis of the derivation in a), that $\{P \vee Q,\ P \rightarrow R,\ R \rightarrow W,\ Q \rightarrow (R \wedge \neg R)\} \Vdash W$?

3. i) Give a categorical derivation of $\neg(\neg A \vee B) \rightarrow (A \wedge \neg B)$

   ii) Give a derivation of $W \vee R$ from $\{(P \rightarrow P) \rightarrow (A \wedge \neg A),\ Q \rightarrow W,\ Q \rightarrow R\}$

4. For each of the following statements, prove it if it is true and give a counter-example if it is false:

   a) If $\Gamma \vdash P$ and $\Delta \vdash Q$ then $\Gamma \cup \Delta \vdash P \wedge Q$

   b) If $\Gamma \Vdash P$ and $\Delta \Vdash Q$ then $\Gamma \cup \Delta \Vdash P \wedge Q$

   c) If Γ is satisfiable, then $\Gamma \cup \{P \vee \neg P\}$ is satisfiable.

   d) If Γ is satisfiable, and Δ is satisfiable, then $\Gamma \cup \Delta$ is satisfiable.

   e) If $\Gamma \Vdash P$ then $\Gamma \cup \{\neg P\}$ is satisfiable.

   f) If Γ is satisfiable, then $\{\neg S \,|\, S \in \Gamma\}$ is satisfiable.

5. Say that we have a system of inference with just the rule of Modus Ponens. (You are allowed to make hypotheses only at the beginning of the proof.) Prove by induction that this system is sound.

6. (a) Say that Γ is a V-saturated set and $(\neg P \vee \neg Q) \in \Gamma$. Explain how we know that either $P \notin \Gamma$ or $Q \notin \Gamma$.

   (b) Explain why i) follows from ii):
   i) If $\Gamma \Vdash S$ then $\Gamma \vdash S$
   ii) If no contradictions can be derived from the sentences in Γ, then there is a truth-assignment making every sentence in Γ true.

7. Answer **one** of these two questions:

   (a) Derive $\forall x\,(P(x) \to Q(x))$ using just the hypothesis $\forall x\, P(x) \to \forall x\, Q(x)$.

   (b) Is it possible for a relation to be symmetric and transitive, but irreflexive? If yes, give an example. If no, explain why not.