Practice Midterm Solutions
Posted Feb. 16 2005

1. a) xxax

   b) abxx

2. a) Base case: $0 \cdot 25 = 0$ Recursion clause: $(x+1) \cdot 25 = x \cdot 25 + 25$

   This one line ($\Uparrow$) is a *complete* answer to the question.

   b) The induction goes most smoothly if you do it on the number of concatenations of elements of $\{aa\}$ that form the given member of $\{aa\}^*$.

   Base case: The number of concatenations is 0, then x is the empty string. On the empty string, F_1 remains in q_0, which is not a final state of F_1.

   Induction step:
   Induction assumption:Say that every string x in $\{aa\}^*$ that is formed by n or fewer concatenations of elements of aa is not accepted by F_1. Say that x' is formed by the concatenation of n+1 instances of aa.

   Since $0 < n+1$, there must be strings y $\in \{aa\}^*$ and $y' = aa$, such that $x' = y * y'$. Since y is formed with n concatenations of aa, it is not accepted by F_1, by the induction assumption. If y is the empty string, then x' = aa. F_1 enters q_1 upon reading a, and then enters q_2 upon reading the final a. q_2 is not a final state of F_1, so the computation is unsuccessful.

   If y is not the empty string, then F_1 leaves q_0 when it reads the first a and there is no transition that can bring it back to q_0. q_1 is a final state, and since y is not accepted by F_1, F_1 must finish reading y in q_2. Now F_1 has only y' = aa to read. Since F_1 begins reading aa in q_2, it will begin by reading a and moving into q_1, and then reading a again and moving into q_2, terminating in q_2, which is not a final state.

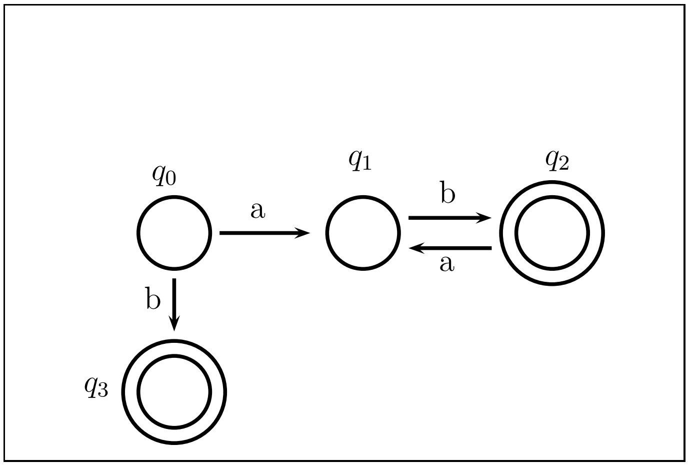
   So x' is not accepted by F_1. This completes the inductive step, and hence the proof.

3. $\{\langle q_0, a, q_1\rangle, \langle q_1, a, q_2\rangle, \langle q_2, a, q_1\rangle\}$ are the transitions, q_0, q_1 and q_2 are the states, with q_0 and q_1 the final states.

   To keep the machine from accepting the empty string, don't make q_0 a final state.

   [For this question, it would be acceptable to draw the labelled graph rather than to spell out the tuples.]

b)



c) This machine recognizes strings that are either “b” (just the single letter) or consist of a string $abab \cdots ab$ of some number of “ab”’s concatenated together.