Solutions to Homework Assignment 6

1. Chapter 4, Exercise 3, Part a), f), h)

   a) This application is incorrect, since it discharges *two* assumptions \((P \text{ and } Q)\) while the rule only allows the discharging of one. The conclusion \(P \rightarrow (P \land Q)\) should still be subordinate to the hypothesis \(Q\).

   f) This application is incorrect, since you would need "\(P \lor Q\)" in a preceding line to apply disjunction elimination here.

   h) This application is incorrect: If the application of conditional introduction is to discharge the hypothesis \(Q\), then it must put \(Q\) as the condition in the resulting if - then statement.

2. (Chapter 4, Exercise 5, Part b), f), g))

    (Chapter 4, Exercise 5, Part b)

```
    1  | P \rightarrow (R \rightarrow (Q \rightarrow R)) | hyp
    2  | R                                      | hyp
    3  | P                                      | hyp
    4  | R \rightarrow (Q \rightarrow R)       | mp 1, 3
    5  | (Q \rightarrow R)                     | mp 2, 4
    6  | P \rightarrow (Q \rightarrow R)       | cond int 3–5
    7  | R \rightarrow (P \rightarrow (Q \rightarrow R)) | cond int 2–6

    (Chapter 4, Exercise 5, part e))
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(This was not on the problem set, but I accidentally included it in an earlier draft of the solutions. I figured I would leave it in as an extra worked example.) (Chapter 4, Exercise 5, part f))

1 \[ (P \rightarrow (Q \rightarrow P)) \rightarrow R \] hyp
2 \[ P \] hyp
3 \[ Q \] hyp
4 \[ P \] reit 2
5 \[ Q \rightarrow P \] cond intro 3–4
6 \[ P \rightarrow (Q \rightarrow P) \] cond intro 2–5
7 \[ R \] mp 1,6

(Chapter 4, Exercise 5, part g))

1 \[ (P \rightarrow Q) \rightarrow Q \] hyp
2 \[ (P \rightarrow R) \] hyp
3 \[ (R \rightarrow Q) \] hyp
4 \[ P \] hyp
5 \[ R \] mp 2, 4
6 \[ Q \] mp 3, 5
7 \[ P \rightarrow Q \] cond int 4–6
8 \[ Q \] mp 1, 7
9 \[ (R \rightarrow Q) \rightarrow Q \] cond int 3–8

(Chapter 4, Exercise 5, part g)
3. Chapter 4, Exercise 6, Part f)

1  |  \((P \to Q) \to Q\)  |  hyp  
2  |  \(Q \to R\)  |  hyp  
3  |  \(R \to Q\)  |  hyp  
4  |  \(P \to R\)  |  hyp  
5  |  \(P\)  |  hyp  
6  |  \(R\)  |  mp 4,5  
7  |  \(Q\)  |  mp 3,6  
8  |  \(P \to Q\)  |  cond int 5–7  
9  |  \(Q\)  |  mp 1, 8  
10 |  \(R\)  |  mp 2, 9  
11 |  \((P \to R) \to R\)  

4. Chapter 4, Exercise 7, Part d), j), o)

\[d)\]

1  |  \((P \to Q) \to R\)  |  hyp  
2  |  \(P \land Q\)  |  hyp  
3  |  \(P\)  |  hyp  
4  |  \(Q\)  |  conj. elim 2  
5  |  \(P \to Q\)  |  cond. intro 3–4  
6  |  \(R\)  |  mp 1,5  
7  |  \((P \land Q) \to R\)  |  cond. intro 2–6  

5. Derive $R$ from $P \land \neg P$
<table>
<thead>
<tr>
<th></th>
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<th>(P \land \neg P)</th>
<th>hyp</th>
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<tbody>
<tr>
<td>2</td>
<td></td>
<td>(\neg R)</td>
<td>hyp</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>(P)</td>
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</tr>
<tr>
<td>4</td>
<td></td>
<td>(\neg P)</td>
<td>mp 1,2</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>(\neg \neg R)</td>
<td>neg intro 2–4</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(R)</td>
<td>neg elim 5</td>
</tr>
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