Philosophy 303
Symbolic Logic
Winter, 2009

Solutions to Homework Assignment 6

1. Chapter 4, Exercise 3, Part a), f), h)

   a) This application is incorrect, since it discharges *two* assumptions (P and Q) while the rule only allows the discharging of one. The conclusion $P \to (P \wedge Q)$ should still be subordinate to the hypothesis Q.

   f) This application is incorrect, since you would need "$P \vee Q$" in a preceding line to apply disjunction elimination here.

   h) This application is incorrect: If the application of conditional introduction is to discharge the hypothesis Q, then it must put Q as the condition in the resulting if - then statement.

2. (Chapter 4, Exercise 5, Part b), f), g))

   (Chapter 4, Exercise 5, Part b)

1	$P \to (R \to (Q \to R)))$	hyp
2	$\quad R$	hyp
3	$\quad\quad P$	hyp
4	$\quad\quad R \to (Q \to R)$	mp 1, 3
5	$\quad\quad (Q \to R)$	mp 2, 4
6	$\quad P \to (Q \to R)$	cond int 3–5
7	$R \to (P \to (Q \to R))$	cond int 2–6

   (Chapter 4, Exercise 5, part e))

1	$(P \to (Q \to P)) \to R$	hyp
2	$\quad P$	hyp
3	$\quad\quad Q$	hyp
4	$\quad\quad P$	reit 2
5	$\quad Q \to P$	cond intro 3–4
6	$P \to (Q \to P)$	cond intro 2–5
7	R	mp 1,6

(This was not on the problem set, but I accidentally included it in an earlier draft of the solutions. I figured I would leave it in as an extra worked example.) (Chapter 4, Exercise 5, part f))

1	$(P \to Q) \to Q$	hyp
2	$(P \to R)$	hyp
3	$\quad (R \to Q)$	hyp
4	$\quad\quad P$	hyp
5	$\quad\quad R$	mp 2, 4
6	$\quad\quad Q$	mp 3, 5
7	$\quad P \to Q$	cond int 4–6
8	$\quad Q$	mp 1, 7
9	$(R \to Q) \to Q$	cond int 3–8

(Chapter 4, Exercise 5, part g))

1	$(P \to Q) \to Q$	hyp
2	$Q \to R$	hyp
3	$R \to Q$	hyp
4	$\quad P \to R$	hyp
5	$\quad\quad P$	hyp
6	$\quad\quad R$	mp 4,5
7	$\quad\quad Q$	mp 3,6
8	$\quad P \to Q$	cond int 5–7
9	$\quad Q$	mp 1, 8
10	$\quad R$	mp 2, 9
11	$(P \to R) \to R$	

3. Chapter 4, Exercise 6, Part f)

1	$(P \to P) \to Q$	hyp
2	$\quad P$	hyp
3	$\quad P$	reit, 2
4	$P \to P$	cond intro 2–3
5	Q	mp 1,4
6	$((P \to P) \to Q) \to Q$	cond intro 1–6

4. Chapter 4, Exercise 7, Part d), j), o)

d)

1	$(P \to Q) \to R$	hyp
2	$\quad P \wedge Q$	hyp
3	$\quad\quad P$	hyp
4	$\quad\quad Q$	conj. elim 2
5	$\quad P \to Q$	cond. intro 3–4
6	$\quad R$	mp 1,5
7	$(P \wedge Q) \to R$	cond. intro 2–6

j)

1	$(P \to \neg P)$	hyp
2	$\quad P$	hyp
3	$\quad P$	reit 2
4	$\quad \neg P$	mp 1,2
5	$\neg P$	neg intro 2–4

o)

1	$(P \to Q) \to Q$	hyp
2	$Q \to \neg Q$	hyp
3	$\quad Q$	hyp
4	$\quad Q$	reit 3
5	$\quad \neg Q$	mp 2, 3
6	$\neg Q$	neg intro 3–5
7	$\quad \neg P$	hyp
8	$\quad\quad P$	hyp
9	$\quad\quad\quad \neg Q$	hyp
10	$\quad\quad\quad P$	reit 8
11	$\quad\quad\quad \neg P$	reit 7
12	$\quad\quad \neg\neg Q$	neg intro 9–11
13	$\quad\quad Q$	neg elim 12
14	$\quad P \to Q$	cond intro 8 – 13
15	$\quad Q$	mp 1, 14
16	$\quad \neg Q$	reit 5
17	$\neg\neg P$	neg intro 7–16
18	P	neg elim 17

5. Derive R from $P \wedge \neg P$

1	$P \wedge \neg P$	hyp
2	$\quad \neg R$	hyp
3	$\quad P$	reit 2
4	$\quad \neg P$	mp 1,2
5	$\neg\neg R$	neg intro 2–4
6	R	neg elim 5