

Solutions to Homework Assignment 8

1. a1) Prove that you can derive $\neg P \vee Q$ from $P \rightarrow Q$.

1	$P \rightarrow Q$	hyp
2	$\neg(\neg P \vee Q)$	hyp
3	P	hyp
4	Q	m p 1 3
5	$\neg P \vee Q$	disj intro 4
6	$\neg(\neg P \vee Q)$	reit 2
7	$\neg P$	neg intro 3-5
8	$\neg P \vee Q$	disj intro 7
9	$\neg(\neg P \vee Q)$	reit 2
10	$\neg\neg(\neg P \vee Q)$	neg intro 2-9
11	$\neg P \vee Q$	neg elim. 9

a2) Prove that you can derive $P \rightarrow Q$ from $\neg P \vee Q$.

1	$\neg P \vee Q$	hyp
2	P	hyp
3	$\neg P$	hyp
4	$\neg Q$	hyp
5	P	Reit 2
6	$\neg P$	reit 3
7	$\neg\neg Q$	neg int 4-6
8	Q	neg elim 7
9	Q	hyp
10	Q	reit 9
11	Q	1, 3-8 9-10 disj. elim.
12	$P \rightarrow Q$	2-11 cond intro

2. b1) Prove that you can derive $\neg(P \wedge \neg Q)$ from $P \rightarrow Q$.

1	$P \rightarrow Q$	hyp
2	$(P \wedge \neg Q)$	hyp
3	P	conj elim. 2
4	Q	m p 1 3
5	$\neg Q$	conj elim 2
6	$\neg(P \wedge \neg Q)$	neg intro 2-5

b2) Prove that you can derive $P \rightarrow Q$ from $\neg(P \wedge \neg Q)$.

1	$\neg(P \wedge \neg Q)$	hyp
2	P	hyp
3	$\neg Q$	hyp
4	$(P \wedge \neg Q)$	conj intro 2, 3
5	$\neg(P \wedge \neg Q)$	reit 1
6	$\neg\neg Q$	neg intro 2-5
7	Q	neg elim 6
8	$P \rightarrow Q$	cond intro 2-7

3. A *derived rule* is a rule whose correctness is proven by showing that it works as an abbreviation of other, basic rules. Prove that the rule of *modus tollens* is a correct derived rule.

j	$P \rightarrow Q$	
	\vdots	
k	$\neg Q$	
	\vdots	
l	P	<i>hyp</i>
$l+1$	Q	j, l <i>modus ponens</i>
$l+2$	$\neg Q$	k , <i>reit</i>
$l+3$	$\neg P$	$l - l + 2$, <i>neg intro</i>

4. A *derived rule* is a rule whose correctness is proven by showing that it works as an abbreviation of other, basic rules. Prove that the rule of *disjunctive syllogism* is a correct derived rule.

j	$P \vee Q$	
\vdots		
k	$\neg P$	
\vdots		
l	P	<i>hyp</i>
$l+1$	$\neg Q$	<i>hyp</i>
$l+2$	P	l, \textit{reit}
$l+3$	$\neg P$	k, \textit{reit}
$l+4$	$\neg\neg Q$	$l+1 - l+3, \textit{neg intro}$
$l+5$	Q	$l+4, \textit{neg elim}$
$l+6$	Q	<i>hyp</i>
$l+7$	Q	$l+6, \textit{reit}$
$l+8$	Q	$j, l-l+5, l+6-l+7 \textit{disj elim}$

5. Chapter 5

7 b) $\neg(P \rightarrow P)$

P	(\neg	(P	\rightarrow	P)	
T	F	T	T	T	T
F	F	F	T	F	F

Not valid, Unsatisfiable.

7 d) $((P \rightarrow Q) \rightarrow P) \rightarrow P$

P	Q	((P	\rightarrow	Q)	\rightarrow	P)	
T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	T
F	T	F	T	T	F	F	F
F	F	F	T	F	F	F	F

Not valid, Satisfiable.

7 h) $((P \rightarrow Q) \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

P	Q	((P	\rightarrow	Q)	\rightarrow	(\neg	P	\rightarrow	Q)	
T	T	T	T	T	T	F	T	T	T	
T	F	T	F	T	T	F	T	T	F	Valid. Satisfiable
T	T	F	T	T	F	T	F	T	T	
T	F	F	T	F	F	T	T	F	F	

7 n)

P	Q	(P	→	((P	∨	Q)	∧	(P	∨	¬	Q)))
T	T	T	T	T	T	T	T	T	T	F	T
T	F	T	T	T	T	F	T	T	T	T	F
T	T	F	T	F	T	T	F	F	F	F	T
T	F	F	T	F	F	F	F	F	T	T	F

Valid, Satisfiable

7 t)

P	Q	((P	∧	Q)	∨	(¬	P	∧	Q)	∨	(P	∧	¬	Q))
T	T	T	T	T	T	F	T	F	T	T	T	F	F	T
T	F	T	F	F	T	F	T	F	F	T	T	T	T	F
F	T	F	F	T	T	T	F	T	T	T	F	F	F	T
F	F	F	F	F	F	T	F	F	F	F	F	F	T	F

Not valid, Satisfiable.