Philosophy 303
Symbolic Logic
Fall, 2007

Homework Assignment 8

Pay attention to all the comments interspersed among the problems: this is information you'll be expected to remember.

1. An interesting feature of the boolean connectives is that just a pair of them suffice to define them all. In chapter 5 we'll develop enough of a framework to prove that just two connectives will suffice to define *all possible* boolean connectives. (This will involve developing some conceptual tools, including clarifying what "all possible" means in this context.) Here are a few first steps in that direction:

   a1) Prove that you can derive $\neg P \vee Q$ from $P \rightarrow Q$.
   a2) Prove that you can derive $P \rightarrow Q$ from $\neg P \vee Q$.

   We will say that two sentences are *logically equivalent* when each can be derived from the other. These two results show that any formula containing "$\rightarrow$" is logically equivalent, in our system, to some formula containing no occurrence of "$\rightarrow$". In other words, the symbol for "$\rightarrow$" can be seen as just a convenient abbreviation in a language whose only connectives are "$\vee$", "$\wedge$", and "$\neg$".

2. Similarly, we can prove that "$\rightarrow$" can be eliminated in favor of "$\neg$" and "$\wedge$".

   b1) Prove that you can derive $\neg(P \wedge \neg Q)$ from $P \rightarrow Q$.
   b2) Prove that you can derive $P \rightarrow Q$ from $\neg(P \wedge \neg Q)$.

   Recall that on the last problem set, we proved that $\neg(P \wedge Q)$ is provably equivalent to $\neg P \vee \neg Q$. This tells us that if we had eliminated "$\rightarrow$" from our language, we could also eliminate either "$\vee$" or "$\wedge$". (Make sure you understand why this means we could "in principle" make do with just "$\vee$" and "$\neg$" or just "$\wedge$" and "$\neg$".)

3. A *derived rule* is a rule whose correctness is proven by showing that it works as an abbreviation of other, basic rules. The idea is that when you apply a derived rule, you could have written the longer derivation in terms of basic rules instead, but the derived rule is like a "macro" that saves you time and space.

   Prove that the rule of *modus tollens* is a correct derived rule, where *modus tollens* is the rule:

j	$P \to Q$	
	$\vdots$	
k	$\neg Q$	
	$\vdots$	
l	$\neg P$	j, k *modus tollens*

4. Prove that the rule of *disjunctive syllogism* is a correct derived rule, where *disjunctive syllogism* is the rule:

j	$P \vee Q$	
	$\vdots$	
k	$\neg P$	
	$\vdots$	
l	Q	j, k *disjunctive syllogism*

5. Chapter 5 exercise 7 b), d), h), n) and t)