Philosophy 303
Symbolic Logic
Winter, 2009

Solutions to Homework Assignment 7

1. Give a categorical proof of $[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$

   This problem was answered - without line numbers - in the book. (A reward for paying attention!)

1	$[P \to (Q \to R)]$	hyp
2	$P \to Q$	hyp
3	P	hyp
4	Q	mp 2,3
5	$Q \to R$	mp 1,3
6	R	mp 3–5
7	$P \to R$	cond intro 2–6
8	$(P \to Q) \to (P \to R)$	cond intro 2–7
9	$[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$	cond intro 1–8

2. Give a categorical proof of $(P \vee \neg P)$

1	$\neg(P \vee \neg P)$	hyp
2	P	hyp
3	$(P \vee \neg P)$	disj. intro 2
4	$\neg(P \vee \neg P)$	Reit 1
5	P	neg int 2–4
6	$P \vee \neg P$	disj int 5
7	$\neg(P \vee \neg P)$	reit 1
8	$P \vee \neg P$	neg int 1–7

3. A *derived rule* is a rule whose correctness is proven by showing that it works as an abbreviation of other, basic rules. Prove that the rule of *modus tollens* is a correct derived rule.

j	$P \to Q$	
	$\vdots$	
k	$\neg Q$	
	$\vdots$	
l	$\quad P$	*hyp*
$l+1$	$\quad\quad Q$	j, l *modus ponens*
$l+2$	$\quad\quad \neg Q$	k, *reit*
$l+3$	$\neg P$	k, *neg intro*

4. Prove that *shortcut* is a correct derived rule.

j	$P \to Q$	
	$\vdots$	
k	$Q \to R$	
	$\vdots$	
l	$\quad \neg R$	*hyp*
$l+1$	$\quad\quad P$	*hyp*
$l+2$	$\quad\quad Q$	$j, l+1$, *modus ponens*
$l+3$	$\quad\quad R$	$k, l+2$, *modus ponens*
$l+4$	$\quad\quad \neg R$	l, *reit*
$l+5$	$\quad \neg P$	$l - l+4$, *neg intro*
$l+5$	$\neg R \to \neg P$	$l - l+5$, *cond intro*

5. Prove that *symmetric negation* is a correct derived rule, where *symmetric negation* is the rule:

j	$\neg P$
	$\vdots$
k	R
	$\vdots$
l	$\neg R$

$l{+}1$ $\neg\neg P$	$j - l$ *neg intro*
$l{+}1$ P	$l + 1$ *neg elim*

6. Prove Q from $(P \rightarrow Q)$ and $(\neg P \rightarrow Q)$

   This isn't the cleanest proof, but it illustrates that once you have a proof of one theorem, you can reuse it in other proofs. So we can put the solution to question 2) to work here:

1	$(P \rightarrow Q)$	hyp
2	$(\neg P \rightarrow Q)$	hyp
3	$\neg(P \vee \neg P)$	hyp
4	P	hyp
5	$(P \vee \neg P)$	disj. intro 2
6	$\neg(P \vee \neg P)$	Reit 1
7	P	neg int 2–4
8	$P \vee \neg P$	disj int 5
9	$\neg(P \vee \neg P)$	reit 1
10	$P \vee \neg P$	neg int 1–7
11	P	hyp
12	Q	modus ponens 1,12
13	$\neg P$	hyp
14	Q	modus ponens 2,14
15	Q	disj elim 8, 12-13,14-15

7. Derive $\neg A \wedge \neg B$ from $\neg(A \vee B)$.

1	$\neg(A \vee B)$	hyp
2	$\quad A$	hyp
3	$\quad A \vee B$	disj intro 2
4	$\quad \neg(A \vee B)$	reit 1
5	$\neg A$	neg intro 2-4
6	$\quad B$	hyp
7	$\quad A \vee B$	disj intro 6
8	$\quad \neg(A \vee B)$	reit 1
9	$\neg B$	neg intro 6-8
10	$\neg A \wedge \neg B$	conj intro 5,9

8. Derive $\neg(A \wedge B)$ from $\neg A \vee \neg B$.

1	$\neg A \vee \neg B$	hyp
2	$\quad \neg A$	hyp
3	$\quad\quad A \wedge B$	hyp
4	$\quad\quad A$	conj. elim. 3
5	$\quad\quad \neg A$	reit 2
6	$\quad \neg(A \wedge B)$	neg intro 3-5
7	$\quad \neg B$	hyp
8	$\quad\quad A \wedge B$	hyp
9	$\quad\quad B$	conj. elim. 8
10	$\quad\quad \neg B$	reit 7
11	$\quad \neg(A \wedge B)$	neg intro 8-10
12	$\neg(A \wedge B)$	disj elim 1 2-6 7-11

9. Derive $\neg(A \vee B)$ from $\neg A \wedge \neg B$. The trick here is to note that you can get a contradiction from A and you can get a contradiction from B, but it is a different contradiction each time. So you need to get the same contradiction to apply disjunction elimination usefully.

1	$\neg A \wedge \neg B$	hyp
2	$A \vee B$	hyp
3	A	hyp
4	A	reit 3
5	$\neg A$	conj elim 1
6	$A \wedge \neg A$	conj intro 4,5
7	B	hyp
8	$\neg(A \wedge \neg A)$	hyp
9	B	reit 7
10	$\neg B$	conj elim 1
11	$\neg\neg(A \wedge \neg A)$	neg intro 8-10
12	$A \wedge \neg A$	neg elim 11
13	$A \wedge \neg A$	disj intro 2 3-6 7-12
14	A	conj elim 13
15	$\neg A$	conj elim 13
16	$\neg(A \vee B)$	neg intro 2-13

10. Derive $\neg A \vee \neg B$ from $\neg(A \wedge B)$.

1	$\neg(A \wedge B)$	hyp
2	$\neg(\neg A \vee \neg B)$	hyp
3	$\neg A$	hyp
4	$\neg A \vee \neg B$	disj intro 3
5	$\neg(\neg A \vee \neg B)$	reit 1
6	$\neg\neg A$	neg intro 5
7	A	neg elim 6
8	$\neg B$	hyp
9	$\neg A \vee \neg B$	disj intro 8
10	$\neg(\neg A \vee \neg B)$	reit 1
11	$\neg\neg B$	neg intro 10
12	B	neg elim 11
13	$A \wedge B$	conj intro 7, 12
14	$\neg(A \wedge B)$	reit 1
15	$\neg\neg(\neg A \vee \neg B)$	neg intro 2-14
16	$\neg A \vee \neg B$	neg elim 15