Problem set 7
Phil 303
Posted Mar 8, 2009
Due March 13 2009

1. Give a categorical proof of $[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$

2. Give a categorical proof of $(P \vee \neg P)$ (This is just a bit more challenging than those on the last problem set.)

3. A *derived rule* is a rule whose correctness is proven by showing that it works as an abbreviation of other, basic rules. Prove that the rule of *modus tollens* is a correct derived rule, where *modus tollens* is the rule:

$$
\begin{array}{c|ll}
j & P \to Q & \\
 & \vdots & \\
k & \neg Q & \\
 & \vdots & \\
l & \neg P & \qquad j, k \textit{ modus tollens}
\end{array}
$$

4. Prove that *shortcut* is a correct derived rule, where *shortcut* is the rule:

$$\begin{array}{l|ll} j & P \to Q & \\ & \vdots & \\ k & Q \to R & \\ & \vdots & \\ l & \neg R \to \neg P & j, k \textit{ shortcut} \end{array}$$

5. Prove that *symmetric negation* is a correct derived rule, where *symmetric negation* is the rule:

$$\begin{array}{l|ll} j & \neg P & \\ \hline & \vdots & \\ k & R & \\ & \vdots & \\ l & \neg R & \\ \hline P & & j, k \textit{ symmetric negation} \end{array}$$

6. Prove Q from $(P \to Q)$ and $(\neg P \to Q)$ [Hint: One way of doing this splices in the answer to problem 2.]

7. Derive $\neg A \wedge \neg B$ from $\neg(A \vee B)$.

8. Derive $\neg(A \wedge B)$ from $\neg A \vee \neg B$.

9. Derive $\neg(A \vee B)$ from $\neg A \wedge \neg B$.

10. Derive $\neg A \vee \neg B$ from $\neg(A \wedge B)$. (This one is more challenging.)