

Solutions to Homework Assignment 6

1. Chapter 4, Exercise 3, Part a), f), h)

- a) This application is incorrect, since it discharges *two* assumptions (P and Q) while the rule only allows the discharging of one. The conclusion $P \rightarrow (P \wedge Q)$ should still be subordinate to the hypothesis Q .
- f) This application is incorrect, since you would need " $P \vee Q$ " in a preceding line to apply disjunction elimination here.
- h) This application is incorrect: If the application of conditional introduction is to discharge the hypothesis Q , then it must put Q as the condition in the resulting if - then statement.

2. (Chapter 4, Exercise 5, Part b), f), g))

(Chapter 4, Exercise 5, Part b)

1	$P \rightarrow (R \rightarrow (Q \rightarrow R))$	hyp		
2	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td style="padding-left: 10px;">hyp</td> </tr> </table>	R	hyp	hyp
R	hyp			
3	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 10px;">hyp</td> </tr> </table>	P	hyp	hyp
P	hyp			
4	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$R \rightarrow (Q \rightarrow R)$</td> <td style="padding-left: 10px;">mp 1, 3</td> </tr> </table>	$R \rightarrow (Q \rightarrow R)$	mp 1, 3	mp 1, 3
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5	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$(Q \rightarrow R)$</td> <td style="padding-left: 10px;">mp 2, 4</td> </tr> </table>	$(Q \rightarrow R)$	mp 2, 4	mp 2, 4
$(Q \rightarrow R)$	mp 2, 4			
6	$P \rightarrow (Q \rightarrow R)$	cond int 3–5		
7	$R \rightarrow (P \rightarrow (Q \rightarrow R))$	cond int 2–6		

(Chapter 4, Exercise 5, part e))

1	$(P \rightarrow (Q \rightarrow P)) \rightarrow R$	hyp
2	<div style="border-left: 1px solid black; padding-left: 10px;"> P </div>	hyp
3	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> Q </div> </div>	hyp
4	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> P </div> </div> </div>	reit 2
5	<div style="border-left: 1px solid black; padding-left: 10px;"> $Q \rightarrow P$ </div>	cond intro 3–4
6	$P \rightarrow (Q \rightarrow P)$	cond intro 2–5
7	R	mp 1,6

(This was not on the problem set, but I accidentally included it in an earlier draft of the solutions. I figured I would leave it in as an extra worked example.) (Chapter 4, Exercise 5, part f))

1	$(P \rightarrow Q) \rightarrow Q$	hyp
2	$(P \rightarrow R)$	hyp
3	<div style="border-left: 1px solid black; padding-left: 10px;"> $(R \rightarrow Q)$ </div>	hyp
4	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> P </div> </div>	hyp
5	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> R </div> </div>	mp 2, 4
6	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> Q </div> </div>	mp 3, 5
7	<div style="border-left: 1px solid black; padding-left: 10px;"> $P \rightarrow Q$ </div>	cond int 4–6
8	<div style="border-left: 1px solid black; padding-left: 10px;"> Q </div>	mp 1, 7
9	$(R \rightarrow Q) \rightarrow Q$	cond int 3–8

(Chapter 4, Exercise 5, part g))

1	$(P \rightarrow Q) \rightarrow Q$	hyp																
2	$Q \rightarrow R$	hyp																
3	$R \rightarrow Q$	hyp																
4	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow R$</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 4,5</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>mp 3,6</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow Q$</td> <td>cond int 5-7</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>mp 1, 8</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 2, 9</td> </tr> </table> </td> <td></td> </tr> </table>	$P \rightarrow R$	hyp	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 4,5</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>mp 3,6</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow Q$</td> <td>cond int 5-7</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>mp 1, 8</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 2, 9</td> </tr> </table>	P	hyp	R	mp 4,5	Q	mp 3,6	$P \rightarrow Q$	cond int 5-7	Q	mp 1, 8	R	mp 2, 9		
$P \rightarrow R$	hyp																	
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R	mp 4,5																	
Q	mp 3,6																	
$P \rightarrow Q$	cond int 5-7																	
Q	mp 1, 8																	
R	mp 2, 9																	
11	$(P \rightarrow R) \rightarrow R$																	

3. Chapter 4, Exercise 6, Part f)

1	$(P \rightarrow P) \rightarrow Q$	hyp				
2	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>reit, 2</td> </tr> </table>	P	hyp	P	reit, 2	
P	hyp					
P	reit, 2					
4	$P \rightarrow P$	cond intro 2-3				
5	Q	mp 1,4				
6	$((P \rightarrow P) \rightarrow Q) \rightarrow Q$	cond intro 1-6				

4. Chapter 4, Exercise 7, Part d), j), o)

d)

1	$(P \rightarrow Q) \rightarrow R$	hyp												
2	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \wedge Q$</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>conj. elim 2</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow Q$</td> <td>cond. intro 3-4</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 1,5</td> </tr> </table> </td> <td></td> </tr> </table>	$P \wedge Q$	hyp	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>conj. elim 2</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow Q$</td> <td>cond. intro 3-4</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 1,5</td> </tr> </table>	P	hyp	Q	conj. elim 2	$P \rightarrow Q$	cond. intro 3-4	R	mp 1,5		
$P \wedge Q$	hyp													
<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">P</td> <td>hyp</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Q</td> <td>conj. elim 2</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$P \rightarrow Q$</td> <td>cond. intro 3-4</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">R</td> <td>mp 1,5</td> </tr> </table>	P	hyp	Q	conj. elim 2	$P \rightarrow Q$	cond. intro 3-4	R	mp 1,5						
P	hyp													
Q	conj. elim 2													
$P \rightarrow Q$	cond. intro 3-4													
R	mp 1,5													
7	$(P \wedge Q) \rightarrow R$	cond. intro 2-6												

j)

1	$(P \rightarrow \neg P)$	hyp		
2	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">P</td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	P	hyp	hyp
P	hyp			
3	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 5px;">reit 2</td> </tr> </table>	P	reit 2	reit 2
P	reit 2			
4	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg P$</td> <td style="padding-left: 5px;">mp 1,2</td> </tr> </table>	$\neg P$	mp 1,2	mp 1,2
$\neg P$	mp 1,2			
5	$\neg P$	neg intro 2-4		

o)

1	$(P \rightarrow Q) \rightarrow Q$	hyp				
2	$Q \rightarrow \neg Q$	hyp				
3	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">Q</td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	Q	hyp	hyp		
Q	hyp					
4	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">Q</td> <td style="padding-left: 5px;">reit 3</td> </tr> </table>	Q	reit 3	reit 3		
Q	reit 3					
5	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg Q$</td> <td style="padding-left: 5px;">mp 2, 3</td> </tr> </table>	$\neg Q$	mp 2, 3	mp 2, 3		
$\neg Q$	mp 2, 3					
6	$\neg Q$	neg intro 3-5				
7	$\neg P$	hyp				
8	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	P	hyp	hyp		
P	hyp					
9	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;"> <table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg Q$</td> <td style="padding-left: 5px;">hyp</td> </tr> </table> </td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg Q$</td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	$\neg Q$	hyp	hyp	hyp
<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg Q$</td> <td style="padding-left: 5px;">hyp</td> </tr> </table>	$\neg Q$	hyp	hyp			
$\neg Q$	hyp					
10	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 5px;">reit 8</td> </tr> </table> </td> <td style="padding-left: 5px;">reit 8</td> </tr> </table>	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 5px;">reit 8</td> </tr> </table>	P	reit 8	reit 8	reit 8
<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">P</td> <td style="padding-left: 5px;">reit 8</td> </tr> </table>	P	reit 8	reit 8			
P	reit 8					
11	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg P$</td> <td style="padding-left: 5px;">reit 7</td> </tr> </table>	$\neg P$	reit 7	reit 7		
$\neg P$	reit 7					
12	$\neg\neg Q$	neg intro 9-11				
13	Q	neg elim 12				
14	$P \rightarrow Q$	cond intro 8 - 13				
15	Q	mp 1, 14				
16	$\neg Q$	reit 5				
17	$\neg\neg P$	neg intro 7-16				
18	P	neg elim 17				

5. Derive R from $P \wedge \neg P$

1	$P \wedge \neg P$	hyp									
2	<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg R$</td> <td style="padding-left: 10px;">hyp</td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 5px;">P</td> <td style="padding-left: 10px;">reit 2</td> </tr> <tr> <td style="padding-right: 10px;">5</td> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg P$</td> <td style="padding-left: 10px;">mp 1,2</td> </tr> </table>	3	$\neg R$	hyp	4	P	reit 2	5	$\neg P$	mp 1,2	
3	$\neg R$	hyp									
4	P	reit 2									
5	$\neg P$	mp 1,2									
6	$\neg\neg R$	neg intro 2-4									
7	R	neg elim 5									