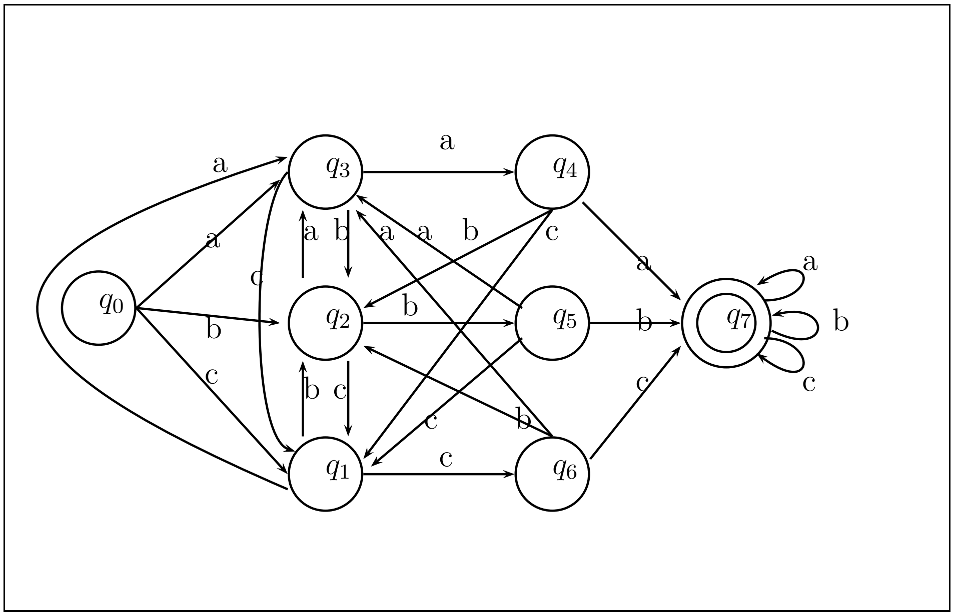
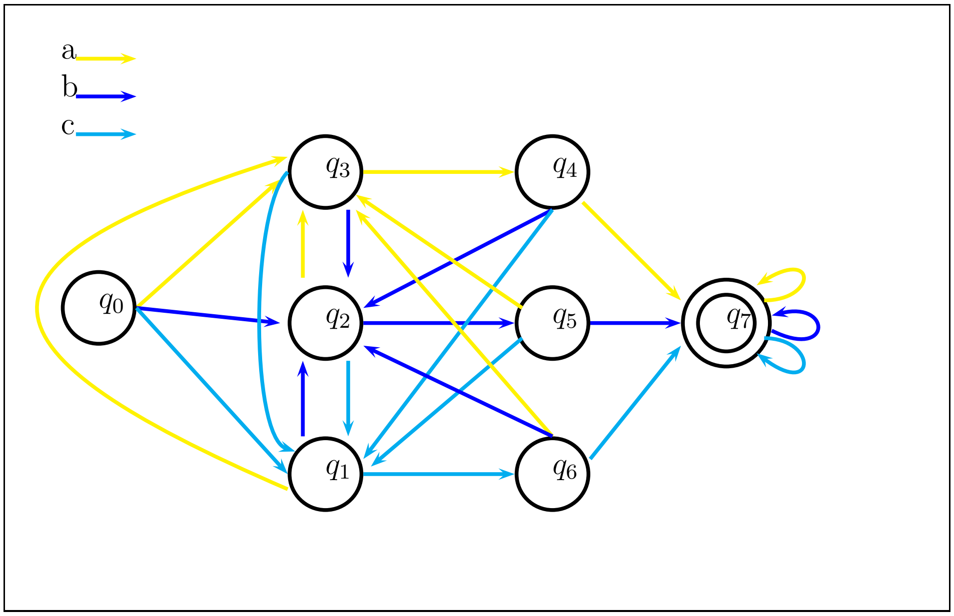
Problem set solutions 5

1. Here is the diagram:



The diagram is a bit cluttered with all the letters labelling arrows, so if you find it easier to read, here is a color coded version (Gold = [Chemical: au] = a, blue = b, cyan = c)



2. Say that some language L is accepted by an FSA F. Let $\{a_1, ..., a_n\}$ be the alphabet. Call the complementary language L*.

   We need to design a machine that will accept L*. The basic idea is simple: There are two ways that a string can fail to be in L: either F reads the whole string but terminates in a non-final state, or it terminates before reading the entire string. Thus we construct our machine F* to accept L* by changing all the final states of F to non-final states of F* and non-final states of F* to final states of F. That will take care of the first issue.

   To take care of the second issue, we'll add a new state $q_{terminate}$, which just reads everything it sees and stays put. We add the tuples $\langle q_{terminate}, a_i, q_{terminate} \rangle$ for every i.

   Make this a final state of F*. When does the machine go into the state $q_{terminate}$? Here's when:

   Terminating Instruction: For any state q of F, say there is a letter a_i for which q has no instructions. That is: there is *no* state q' such that $\langle q, a_i, q' \rangle$. Then we add the tuple $\langle q, a_i, q_{terminate} \rangle$

   Note that every transition of F is a transition of F*; the only new transitions in F* are those added by the "terminating instruction".

Claim: F* accepts a string if and only if the string is not accepted by F.

Proof (Claim): There are two directions we need to take care of.

$\Rightarrow$

Say that $\sigma = \sigma_1 \cdots \sigma_k$ is a string that is accepted by F. Then F reads σ and terminates in some final state $\tilde{q}$. Since every transition of F is a transition of F*, this means that F* reads σ and terminates in $\tilde{q}$. But by the design of F*, since $\tilde{q}$ is a final state of F, it is not a final state of F*. That is, F* does not accept σ.

[Bit of elementary logic: Proving "If not-A then not-B", as we have done here, also proves "If A then B". Hence we have proven the $\Rightarrow$ direction.]

$\Leftarrow$

Say that $\sigma = \sigma_1 \cdots \sigma_k$ is a string that is not accepted by F. Then one of two things happens when F is fed σ: either i) F doesn't read the whole string, or ii) F reads the whole string and terminates in a non-final state $\tilde{q}$. If ii), then since the transitions of F are also transitions of F*, this means that F* reads all of σ and halts in the same state$\tilde{q}$, which by construction of F* is a final state of F*. So F* accepts σ. If i), then on reading some letter σ_j in σ F doesn't read further. Say F is in state $\bar{q}$ when it is reading σ_j. Since F reads no further, there must be no transition $\langle \bar{q}, \sigma_j, q' \rangle$ for any q' in F. Thus by the construction of F* F* has a transition $\langle \bar{q}, \sigma_j, q_{terminate} \rangle$; in $q_{terminate}$, the rest of the string σ is read. Since $q_{terminate}$ is a final state of F*, F* accepts σ in this case too.

3. a) False. The language containing all the concatenations of a and b is regular, since it is the Kleene * of $\{a\} \cup \{b\}$. The set DUPLICATES described on p. 109 - 110 and in lecture is a subset of $(\{a\} \cup \{b\})^*$ but it is not regular. Similarly, the language $\mathcal{L} = \{\underbrace{aa\ldots a}_{n\,a's}\underbrace{bb\ldots b}_{n\,b's}/n \in \mathbb{N}\} = \{\epsilon, ab, aabb, aaabbb, aaaabbbb, \ldots\}$ that we proved not to be regular on p.12 of this week's notes is a subset of $(\{a\} \cup \{b\})^*$.

   b) True. We showed in question 2 that the complement of a regular language is regular. Note that $\{xy/x \in L \text{ and } y \notin L\} = \{x | x \in L\} \circ \{y | y \notin L\}$