

Homework Assignment 10

There aren't a lot of questions on this problemset, but the material the questions are drawing from is challenging. Make sure you *understand* exactly what is going on in each question.

- Say that we add to our rules of inference the rule of Resolution:

$$\begin{array}{l|l}
 j & P \vee \neg Q \\
 & \vdots \\
 k & Q \vee R \\
 & \vdots \\
 l & P \vee R
 \end{array}
 \qquad
 j, k \text{ Resolution}$$

Prove that the new system is sound. (You only need to prove the additional case in the inductive argument for soundness that would have to be added if you added the rule of resolution to our system of rules.)

- Say that we add to our rules of inference the rules of Axiom1 and Axiom2. Axiom1 says that on any line of a derivation, you may write down the axiom $(P \rightarrow (Q \rightarrow P))$, subject to all and only the hypotheses that governed the previous line. Axiom2 says that on any line of a derivation, you may write down the axiom $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$, subject to all and only the hypotheses that governed the previous line.

Prove that the new system is sound. (You only need to prove the additional case in the inductive argument for soundness that would have to be added if you added the rules Axiom1 and Axiom2 to our system of rules.)

- Say that Γ is \mathbf{V} -saturated, and A and B are formulas of \mathbf{V} . Show that:

$$(\neg A \wedge \neg B) \in \Gamma \text{ if and only if } A \notin \Gamma \text{ and } B \notin \Gamma$$

$$(\neg A \rightarrow B) \in \Gamma \text{ if and only if } A \in \Gamma \text{ or } B \in \Gamma$$